3D depth estimation for visual inspection using wavelet transform modulus maxima

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Abstract

A vision based approach for calculating accurate 3D models of the objects is presented. Generally industrial visual inspection systems capable of accurate 3D depth estimation rely on extra hardware tools like laser scanners or light pattern projectors. These tools improve the accuracy of depth estimation but also make the vision system costly and cumbersome. In the proposed algorithm, depth and dimensional accuracy of the produced 3D depth model depends on the existing reference model instead of the information from extra hardware tools. The proposed algorithm is a simple and cost effective software based approach to achieve accurate 3D depth estimation with minimal hardware involvement. The matching process uses the well-known coarse to fine strategy, involving the calculation of matching points at the coarsest level with consequent refinement up to the finest level. Vector coefficients of the wavelet transform-modulus are used as matching features, where wavelet transform-modulus maxima defines the shift invariant high-level features with phase pointing to the normal of the feature surface. The technique addresses the estimation of optimal corresponding points and the corresponding 2D disparity maps leading to the creation of accurate depth perception model.

Keywords: Wavelet transform modulus; Coarse to fine; Disparity; 3D depth

1. Introduction

The 3D reconstruction process can be categorized into three main categories, i.e. calibration: calculating the intrinsic and extrinsic parameters of the cameras [1,2], finding correspondence: finding the corresponding pairs of points projected from the same 3D point on to the two perspective views [3,4], and triangulation: the projection of 2D information back to the 3D space in order to create a 3D depth model [5,6].

A number of algorithms have been proposed to address at least some of these problems in stereo vision [17]. The majority of these algorithms can be categorized into either area based or feature based algorithms. Area
based approaches [4,19] are based on the correlation of two image functions over locally defined regions. Dense depth maps can be achieved by correlating the grey intensities of image regions in the views being considered assuming that image regions possess some similarities in the perspective views. The area based methods perform well in the presence of rich textured areas but their performance could decline in featureless regions resulting in wrong correspondence [22,23]. Another problem with area based algorithms is their poor performance in handling of surface boundaries, which is very important in the context of generating accurate and dense depth maps [22,23]. On the other hand, feature based algorithms [20,21] attempt to establish correspondences between the selected features, which are extracted using some explicit feature extraction algorithms. The main drawback in these feature based techniques is the retrieval of very sparse depth information making it hard to recover the dense depth maps. The feature based algorithms also have the tendency of getting stuck in unnecessary features of the images and are quite dependent on the efficiency of the feature extraction algorithms.

The main concern in the proposed work is the development of a stereovision algorithm capable of estimating accurate 3D depth of metallic die casting engine parts. The objective is to develop a visual inspection system with minimal hardware involvement unlike many existing techniques which involves the use of laser scanners, laser and light pattern projectors [17,18]. The main motivation behind the algorithm is to keep the system simple, in terms of minimum hardware involvement and complexity, in order to make the system cost effective and cumbersome. Due to its minimal hardware dependency it can be implemented in different factory environments without a significant change in the setup.

Considering the involvement of metallic die casting engine parts with poorly-textured shiny areas and some irregular and unwanted marks, the exclusive use of either area based or feature based algorithms would not be a better choice. Area based algorithms find it hard to estimate the accurate depth in the absence of rich textured image regions whereas feature based algorithms could have difficulty due to the presence of unnecessary and unwanted marks on the surfaces. This escorted us to multiresolution analysis based coarse to fine matching strategy where correspondences are established at different scales and resolutions.

In the context of stereo matching using multiresolution analysis, some work has already been done using scalar wavelets [9,10] and quite promising results have been achieved. The theory of multiwavelets evolved in early 1990s from wavelet theory and was enhanced for more than a decade since then. The success of multiwavelets over scalar wavelet bases stems from the fact that they can simultaneously possess the good properties of orthogonality, symmetry, high approximation order and short support [7,8], which are not possible in the scalar case [7,14]. In this work, multiwavelets based multiresolution approach is used to address the problem of stereo image matching. The presented work is, in fact, a continuation of the work in [11,12]. By using the concept of wavelet transform modulus maxima (WTMM) [7] the performance of the algorithm in [12] is improved. It is due to the use of vector coefficients of wavelet transform modulus with magnitude and phase pointing to the normal of the edges.

The rest of the work is organized as the definition of classical wavelet and multiwavelet theory and a brief introduction of WTMM is presented in Section 2. Complete disparity estimation process is presented in Section 3 with brief information, explicitly, on similarity measure and consistency check. Results and discussion of different experiments is given in Section 4 followed by the conclusion.

### 2. Wavelet transform modulus

The classical wavelet theory [7] can be defined by two simple dilation equations as given below

\[
\phi(t) = \sum_k c_k \phi(Mt - k)
\]

\[
\psi(t) = \sum_k w_k \phi(Mt - k)
\]

where \(c_k\), \(w_k\) are coefficients of scaling functions \(\phi(t)\) and wavelets \(\psi(t)\), respectively, with \(M\) and \(k\) representing filter band and integer shifts [24]. Furthermore multiscaling functions \(\Phi(t)\) and multiwavelets \(\Psi(t)\) can be expressed as
\[
\Phi(t) = \begin{bmatrix}
\phi_0(t) \\
\phi_1(t) \\
\vdots \\
\phi_{r-1}(t)
\end{bmatrix}, \quad \Psi(t) = \begin{bmatrix}
\psi_0(t) \\
\psi_1(t) \\
\vdots \\
\psi_{r-1}(t)
\end{bmatrix}
\]

where \( r \) represents the multiplicity of the multiwavelets which is unity in the case of scalar wavelets. Similarly, in the context of multiwavelets \( c_k \) and \( w_k \) in (1) and (2) are real \( r \times r \) matrices of multi-filter coefficients that iteratively and with rescaling define the scaling and wavelet functions, respectively. The wavelet transform modulus (WTM) can then be expressed as

\[
|\text{WT}_s| = |\text{WT}_s| \angle \Theta_W
\]

where \( |\text{WT}_s| \) represents the magnitude of WTM, of \( k \)th coefficient, whereas \( s \) represents the scale of the subspace. Furthermore, the magnitude of WTM can be expressed as

\[
|\text{WT}_s| = \sqrt{|W_{hs,s}^2| + |W_{vs,s}^2|}
\]

where \( W_{hs,s} \) and \( W_{vs,s} \) are horizontal and vertical detail spaces, respectively. Similarly the phase of the WTM can be expressed as

\[
\Theta_W = \begin{cases} 
\alpha(k) & \text{if} \ W_{hs,s} > 0 \\
\pi - \alpha(k) & \text{if} \ W_{hs,s} < 0
\end{cases}
\]

where

\[
\alpha(k) = \tan^{-1}\left(\frac{W_{vs,s}}{W_{hs,s}}\right)
\]

The vector \( \vec{n}(k) \) points to the direction normal to the edge surface as

\[
\vec{n}(k) = [\cos(\Theta_W), \sin(\Theta_W)]
\]

Fig. 1. Top left: original image, top right: wavelet transform modulus, bottom left: wavelet transform modulus phase, bottom right: wavelet transform modulus maxima with phase vectors.
An edge point is the point \( p \) at some scale \( s \) such that \( \text{WT}_s \) is locally maximum at \( k = p \) and \( k = p + \varepsilon n(k) \) for \( |\varepsilon| \) small enough. These maxima are called wavelet transform modulus maxima (WTMM) [7,13] and are shift invariant. An example of the wavelet transform modulus can be found in Fig. 1 with explicit presentation of magnitude and phase of WTMM.

3. Disparity estimation

The matching process starts with scale normalization which minimizes the effect of illuminative variation in each scale that can exist between different scales of perspective views and can be expressed as

\[
\text{NW}_s = \frac{W_{i,s}}{|A_s|}
\]  
(9)

where \( \text{NW}_s \) are the scale normalized wavelet coefficients, \( i \) refers to all detail spaces \( W_i \) and \( A_s \) represents the approximation space within the scale \( s \). After wavelet transformation and the calculation of WTM, we end up with coefficient matrices with features highlighted. The scale normalization process is then followed by correlation-based similarity measure that can be expressed as

\[
C_{x,y,d} = \text{corr}(\text{NW}^1_{s,k}, \text{NW}^2_{s,k}) \quad \text{for} \quad d = 0 \cdots d_{\text{max}}
\]

(10)

where \( \text{NW}^1_{s,k} \) and \( \text{NW}^2_{s,k} \) represents normalized WTMM related to first and second images, respectively. For more comprehensive visualization of the proposed algorithm a block diagram is shown in Fig. 2.

3.1. Probabilistic weighting

To keep the matching candidates consistent, a symbolic tagging procedure is introduced based on probability of occurrence and three thresholds possessing the criteria \( T_1 < T_2 < T_3 \). The values of these thresholds is usually within the ranges of \([0.5 \ 0.7]\), \([0.70 \ 0.85]\) and \([0.85 \ 0.95]\), respectively. Theses values of the \( T_i \) are carefully chosen by trial and error method. Probability of occurrence \( P_{oc} \), which is the probability of selection of any point as a candidate from any of the search space out of \( r^2 \) spaces can be defined as

\[
P_{oc}(C_i) = \frac{n_{C_i}}{r^2} \quad \text{where} \quad 1 \leq n_{C_i} \leq r^2
\]

(11)

Here \( n_{C_i} \) is the number of times a candidate match \( C_i \) is selected out of search spaces \( r^2 \). All matching candidates have equal probability of being selected, i.e. \( 1/r^2 \). The correlation score for each candidate is then weighted with the occurrence probability and can be expressed as

\[
P_{C_i} = P_{oc}(C_i) \sum_n C_{x,y,d}^i \quad \text{where} \quad n \leq r^2
\]

(12)

![Fig. 2. Block diagram of the proposed algorithm.](image-url)
3.2. Symbolic tagging

The candidates are then divided into pools based on thresholds $T_i$. First threshold $T_1$ is applied just after the correlation step to filter out the bad matches. Remaining candidates are then assigned symbolic tags depending on their consistency as given below

$$
C_{x,y,d}^i > T_2, \quad \text{and} \quad PC_i = 1 \Rightarrow Op
$$

$$
C_{x,y,d}^i > T_2, \quad \text{and} \quad 0.5 \leq PC_i < 1 \Rightarrow Cd
$$

$$
C_{x,y,d}^i > T_3, \quad \text{and} \quad PC_i < 0.5 \Rightarrow Cr
$$

As can be seen from the first expression, there is no ambiguity for the matches with tag $Op$ as the probability of consistency is 1, whereas ambiguity does exist for the matches with tags $Cd$ and $Cr$. Ambiguity is the phenomenon where there is more than one candidate available for a single point in the reference image [17,18]. To solve that problem of ambiguity a geometric optimization is performed. The matches fulfilling the geometric topological criteria and having the tag $Cd$ are then promoted to $Op$ whereas the ones with the tag $Cr$ are promoted to $Cd$. After that step, all matched pairs with the tag $Op$ are considered as credible matches whereas the matches with the tag $Cd$ are peer reviewed in the next interpolated level for their credibility. The reason for keeping points with the tag $Cd$ in consideration is to exploit the possibility of their potential in the next resolution level. The rest of the matches, not fulfilling any of the above criteria, are simply discarded, leaving only the most consistent matches.

3.3. Geometric refinement

As there are $r^2$ search spaces, there is a possibility of selection of different points from different search spaces, i.e. ambiguity. To address the issue of ambiguity, a simple geometric topological refinement is introduced in order to extract the optimal candidate matches out of the pool of ambiguous candidate matches. For that purpose, the geometric orientation of the ambiguous points with reference to $Op$ from (13) is checked and the pairs having the closest geometric topology with respect to the $Op$ are selected as optimal candidates. Three geometric features that are relative distance difference, absolute distance difference and slope difference of lines joining the points were selected and calculated to check the geometric orientational similarity. The geometric measurement is then weighted with the correlation score of candidates to keep the previous achievements of the candidates in the consideration as given below

$$
Gc_i = C_{x,y,d}^{ave} \left( e^{-\overline{rd}} + e^{-\overline{ad}} + e^{-\overline{sd}} \right)
$$

where $\overline{rd}$, $\overline{ad}$ are the average of relative and absolute distance differences whereas $\overline{sd}$ is average slope difference. These geometric features are calculated a number of times in order to minimize the effect of any wrong reference candidate. These three geometric features are invariant through many geometric transformations like Affine, Metric, Euclidean etc. that are very common in the applications of stereo vision and 3D depth estimation [3,6]. The candidate having closest geometric topology, i.e. lowest $Gc$ will be selected as an optimal match.

3.4. Scale interpolation

The matching process at the coarsest level ends up with a number of matching pairs, which needs to be interpolated to the finer level. The constellation relation between the coefficients at coarser and finer levels can be visualized by taking the decimation of factor 2 into consideration though in the case of proposed algorithm averaging is performed, instead, over the consecutive pair of coefficients across rows and columns. The reason of averaging, instead of just discarding every other coefficient by factor 2 decimation, is to keep the significance of all coefficients in consideration. Furthermore there might exist WTMM which could be lost by factor 2 decimation instead of averaging. The disparity from coarser disparity $d_c$ to finer disparity $d_f$ can be updated using the relation

$$
d_f = d_c + 2d_c
$$

where $d_c$ is the estimated disparity at the current scale level.
4. Experiments

The algorithm proposed here is the continuation of the work presented in [11,12] and significant improvements can be observed, regarding the quality of estimated disparity maps. To evaluate the performance of the proposed algorithm, two different kind of experiments are performed. In the first type of experiments, the algorithm in Section 3 is applied to a number of different images taken from Middlebury College’s website [15]. These images are well known in the computer vision community due to a variety of texture and intensity based complexities involved in them. The statistical performance check is done by using percentage of bad pixel matches $B$ which can be defined as

![Fig. 3. (a) and (b) Original pair of Map images, (c) ground truth disparity, (d) calculated disparity with occlusion, (e) calculated disparity with occlusion filled, (f) disparity error, (g) occlusion calculated from ground truth, (h) calculated occlusion.](image-url)
\[ B = \frac{1}{N} \sum_{(x,y)} \left| d_c(x,y) - d_{\text{gtb}}(x,y) \right|^2 > \xi \]  \hspace{1cm} (16)

where \( d_c(x,y) \) and \( d_{\text{gtb}}(x,y) \) represent the calculated and ground-truth disparity maps whereas \( \xi \) represents the acceptable disparity difference (usually 0.5). A simple linear interpolation is performed to fill the disparity map gaps where no discrete disparity value could be found, assuming that disparities are found at all discontinuities, i.e. at all wavelet transform modulus maxima.

One of the outcomes of experiments on the images of Middlebury College is shown in Fig. 3 with detailed view and explanation of the disparity estimation and occlusion detection. It seems worth mentioning that no explicit occlusion detection is performed in the proposed algorithm. The detected occluded area is extracted by keeping the matching process highly constrained and promising results were achieved without the involvement of any post optimization procedure. Any match that does not satisfy the consistency checks from Sections 3.1 to 3.3 is simply discarded. It can be seen in Fig. 3(f) where only a few matches were found at the location of occlusion. A simple linear interpolation is then performed by knowing the occlusion region and by the observation that occlusion is on the right side of the higher disparity region. The disparity map after interpolation step is shown in Fig. 3(e) followed by the bad pixel errors in Fig. 3(f). Fig. 3(g) and (h) shows the occluded area related to the ground-truth disparity map shown in Fig. 3(c) and estimated one shown in Fig. 3(d), respectively. The disparity map shown in Fig. 3 is estimated using Chui–Lian multiwavelets bases [14] and is related to right image as right image is taken as a reference image. The calculated error in terms of percentage of bad pixel matches is 2.2%.

The second set of experiments were done on industrial metallic car engine parts. The main motivation behind this project is to develop an algorithm capable of estimating accurate 3D depth of metallic cast engine parts with minimal hardware involvement. The reason is to keep the process simple in terms of hardware complexity so that it can be implemented in different factory environments without a significant change in the setup. As the stereo vision process is not mature enough to solve the problem of depth estimation to high accuracy, external information from a reference 3D model is used. Currently, experiments are performed on metallic pump-covers. The reference model of these pump-covers is produced in Matlab by manually measuring the physical dimensions. In future, 3D reference models from AutoCAD, Solidworks, etc. will also be included as automotive industries usually use these softwares for designing different parts. An example of the reference model, produced in Matlab can be seen in Fig. 4. The information that is needed from the reference image model is the relative location of the discontinuities and the dimensional range of the 3D depth, consequently giving rise to the information regarding the disparity. Relative locations of the different surface edges are the most important as the stereo vision algorithms usually perform poorly in the areas where sharp depth changes occur. It seems wise mentioning here that one reference model is required for each category of objects.

The hardware setup of the presented work is very simple and consists of stereo cameras from Videre Design [16] and simple circular fluorescent light. One example of the estimated depth model is shown in Fig. 5. It can be seen that a very smooth disparity map is obtained after it is optimized and refined using the information extracted from the reference model. One thing that needs to be clarified is that the process is completely inde-
Fig. 5. (a) and (b) Stereo pair of original images, (c) estimated 2D disparity map, (d) 3D depth of model.

Fig. 6. Error plot between real 3D depth and the recovered one.
dependent of the orientation or the location of the object on the image plane. An error graph is shown in Fig. 6. It shows the difference in depth of different pump-cover surfaces, between the reference depth model as shown in Fig. 4 and estimated one as in Fig. 5(d). It can be seen that error lies within the range of [0.65 mm 1.45 mm], which is very promising considering the simplicity of the proposed algorithm. Furthermore, a series of experiments are underway to check the consistency of the algorithm in a number of different environmental and lighting conditions. The objective is to construct an automated fault detection system using depth information. Complete statistical consistency performance evaluation will be presented in the future evaluating the full capabilities of the algorithm.

5. Conclusion

A multiresolution analysis based stereo vision algorithm is presented. The technique addresses the estimation of optimal corresponding points consequently leading towards the estimation of accurate 3D depth. The proposed algorithm is a software based approach with minimal hardware involvement. The efficiency and accuracy of the algorithm lies on existing 3D reference model instead of extra hardware tools like laser scanners, laser and light pattern projectors, which makes the system costly and burdensome. Due to its minimal hardware dependency the system is cost effective and can be deployed in different factory environments with different environmental and illuminative variations and without a significant change in the proposed system parameters.

To keep the whole matching process consistent and resistant to errors two techniques, symbolic tagging and geometric refinement, are introduced and are applied along with the common constraints of uniqueness and continuity. The problem of ambiguity explicitly and occlusion implicitly, is addressed using a geometric topological refinement along with the symbolic labeling. Symbolic tagging procedure keeps track of the different candidates and their potentials to be the optimal match by dividing them into different bins based on the probability of occurrence and threshold levels. The tags of the candidates update with their performances and potential along the procedure instead of just discarding the matches who does not follow the criteria of fixed thresholds as in the winner takes all strategies.

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References

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