SPACECRAFT ATTITUDE DETERMINATION BY ADAPTIVE KALMAN FILTERING

*Zi Ma and Alfred Ng

Canadian Space Agency, 6767 route de l’Aeroport
St. Hubert, Quebec J3Y 8Y9 Canada

*Current address : NRC Integrated Manufacturing Technologies Institute,
800 Collip Circle, London Ontario, Canada, N6G 4X8
e-mail:zi.ma@nrc.ca

ABSTRACT
Combining adaptive technology and the Extended Kalman filtering (EKF), the spacecraft attitudes are determined based on the measurements from sun sensors, earth sensors and magnetometer. The adaptive choice of the measurements noise covariance matrix and process noise covariance matrix is the main contribution of this proposed paper. The approach is applied to the attitude estimation system of Quicksat simulator successfully.

Keywords: Adaptive Kalman filter; spacecraft attitude estimation; satellite; simulation.

1. INTRODUCTION
As well known, the application of Kalman filtering to real problems has to depend on the assumptions about having the priori knowledge for the mean square variance in both of the state and observation processes (see Kalman 1961). In the most application of Kalman filter, by the empirical data or computer simulation, a priori statistics are selected as constants (see Jazwinski 1970). In this case, the non-adaptive estimation may be satisfactory over some global operating regimes, the bad results may appear if the priori statistics are locally as a time function, specially, in the presence of unknown system disturbances. In general optimal state estimators such as Kalman filter, the optimal estimation for the mean-square variance matrices of the state and observation processes does not exist, for improving the problem, many suboptimal schemes have been presented, which estimate one or several parameters of the mean-square variance matrices of the state and observation processes with

states (see Weiss 1970). However, the most of these developments are either too restrictive for nonlinear system such as satellite attitude estimation, or are computationally demanding (see Abramson 1968). Similar to the Abramson’ works, an adaptive sequential approach was presented (see Myers 1976). In the Myers’s work, the sequential estimators are derived for suboptimal adaptive state estimation, however, the scheme was only used to obtain good state model off line.

The EKF have been applied in some spacecraft attitude determination systems (see Clement 1999). Some tuning methods for covariance matrices of the system model error and measurements noises are used, however, these methods such as automatic tuning approach and genetic algorithm are so complicated that its running needs more than one week. These methods only can be used off line so that they are not suitable to the systems with randomly disturbances. In the proposed work, the Myers’ work is developed to implement spacecraft attitude adaptive estimation on line. For demonstrating the effectiveness of the approach, it is applied to the attitude estimation system of Quicksat simulator successfully, which is a satellite simulation system with nonlinearity and unknown priori knowledge about measurement noise and process noise.
2. ATTITUDE ESTIMATION USING EKF

Assume the random process to be estimated can be modeled in form

\[
X_{k+1} = \Phi_k X_k + W_k
\]  

(2.1)

The observation of the processor is assumed to occur at time discrete point in accordance with the linear relationship

\[
z_k = H_k x_k + v_k
\]  

(2.2)

where state vector \( x_k \in \mathbb{R}^n \) and measurement vector \( z_k \in \mathbb{R}^m \), the covariance matrices for the zeros mean vector \( W_k \) and \( V_k \) are given

\[
\text{if } i = k; \quad E[W_i W_i^T] = Q_i; \quad \text{else } E[W_i W_i^T] = 0
\]  

(2.3)

\[
\text{if } i = k; \quad E[V_i V_i^T] = R_i; \quad \text{else } E[V_i V_i^T] = 0
\]  

(2.4)

\[
E[W_i V_i^T] = 0; \quad \text{for all } k \text{ and } i
\]  

(2.5)

It is assumed that the system is completely observable and controllable. The action of optimal Kalman filtering is to estimate the system states based on the measurements information.

Define the estimation error and its covariance matrix to be

\[
e_{k+1} = x_k - \hat{x}_{k|k-1}
\]  

(2.6)

\[
P_{k|k-1} = E[e_{k|k-1} e_{k|k-1}^T]\]

(2.7)

The Kalman filter is follows

\[
\hat{x}_k = \hat{x}_{k|k-1} + P_{k|k-1} (z_k - H_k \hat{x}_{k|k-1})
\]  

(2.8)

\[
K_k = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1}
\]  

(2.9)

\[
P_{k+1} = E[e_{k+1} e_{k+1}^T] = \Phi_k P_{k|k-1} \Phi_k^T + Q_k
\]  

(2.10)

\[
P_k = P_{k|k-1} - K_k (H_k P_{k|k-1} H_k^T + R_k) K_k^T
\]  

(2.11)

In the spacecraft attitude estimation systems, the estimated attitudes include quaternion and angular velocity. The full quaternion cannot be estimated using the ordinary Kalman filter due to nonlinearity. A filter similar to the EKF may be used to estimate the difference between the actual quaternion and its estimate. Each newly updated estimate of this difference will be added to the quaternion estimate to form the newly updated (or current) full quaternion estimate.

A sequence of vectors is measured in two coordinate systems, the two coordinate system are related by

\[
b_{\text{meas}} = [C]_{o\rightarrow s} b_{\text{orb}}
\]  

(2.12)

where \([C]_{o\rightarrow s}\) is the rotation matrix from orbital to spacecraft frame

\[
[C]_{o\rightarrow s} = \begin{bmatrix}
q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2 q_4 + q_3 q_0) & 2(q_1 q_4 - q_3 q_2) \\
2(q_2 q_4 - q_3 q_0) & q_1^2 - q_2^2 + q_3^2 - q_4^2 & 2(q_2 q_0 + q_3 q_1) \\
2(q_1 q_4 + q_3 q_2) & 2(q_2 q_0 - q_3 q_1) & q_1^2 - q_2^2 - q_3^2 + q_4^2
\end{bmatrix}
\]  

(2.13)

\(b_{\text{meas}}\) is attached to a rotating vehicle and system vector, \(b_{\text{orb}}\) is a reference coordinate system vector. The objective is to compute \(\hat{q}\), the minimum variance estimate of \(q\). The dynamic motion equation of an arth orbit satellite and the kinematic equation of the satellite can be found in text book such as (Hughes 1986).

\[
\tilde{T} = [I]_{o\rightarrow s} [\Omega (\hat{\omega}) \times [I]_{o\rightarrow s} (\hat{\omega})] + \Omega (\hat{\omega}) q
\]  

(2.14)

where \([I]_{o\rightarrow s} = \text{diag}[I_{ox}, I_{oy}, I_{oz}]\) is the diagonal inertia matrix whose moments of inertia are the principal moments of inertia, \(\tilde{T}\) is the control torque, \(\tilde{T}_s\) is the gravity gradient torque, also the \(\Omega (\hat{\omega})\) and \(q\) are defined as
\[ \Omega(\hat{\omega}) = \begin{bmatrix} 0 & -(\omega_z) & -(\omega_y) \\ (\omega_z) & 0 & -(\omega_x) \\ (\omega_y) & (\omega_x) & 0 \end{bmatrix} \] (2.15)

\[ q = [q_0 \ q_1 \ q_2 \ q_3]^T \] (2.16)

Note that the subscripts \( i \) and \( o \) in \( \omega \) refers to inertial and spacecraft frames, respectively. They are related by:

\[ \left( \begin{array}{c} \omega_x \\ \omega_y \\ \omega_z \end{array} \right)_i = \left( \begin{array}{c} \omega_x \\ \omega_y \\ \omega_z \end{array} \right)_o + \left[ C \right]_{o \rightarrow i} \left( \begin{array}{c} \omega_x \\ \omega_y \\ \omega_z \end{array} \right)_o \] (2.17)

The state vector estimate obtained from EKF includes the spacecraft angular velocities as well as quaternion with respect to the spacecraft frame. To avoid the matrix singularity, the scalar part of the quaternion \( q_0 \) is not be estimated so that the rank of some relative matrices. To reconstruct the full quaternion, the first element of the quaternion is obtained from follows

\[ q_0 = \sqrt{1 - q_1^2 - q_2^2 - q_3^2} \]

Thus, The estimated state vector is

\[ x = \left[ \begin{array}{c} \omega_x \\ \omega_y \\ \omega_z \\ q_1 \\ q_2 \\ q_3 \end{array} \right]^T \] (2.18)

**CALCULATION OF KALMAN GAIN**

\[ K_{k+1} = P_{k+1/k} H_{k+1}^T \left( R_{k+1} + H_{k+1} P_{k+1/k} H_{k+1}^T \right)^{-1} \] (2.19)

where \( P_{k+1/k} \) is \( 6 \times 6 \) perturbation covariance matrix at time \( k+1 \) given the measurement at time \( k \). \( K_{k+1} \) is Kalman gain matrix, \( R \) is \( 3 \times 3 \) measurement error covariance matrix, and \( H_{k+1} \) is a \( 3 \times 6 \) matrix (for only one observation vector), if sun sensor and/or earth sensor are available, the matrix \( R \) and \( H_{k+1} \) are expanded to \( 6 \times 6 \), they are defined as follows respectively

\[
\begin{pmatrix}
R_{ss} & 0 & 0 \\
0 & R_{mag} & 0 \\
0 & 0 & R_{es}
\end{pmatrix}
\]

or

\[
\begin{pmatrix}
R_{ss} & 0 & 0 \\
0 & R_{mag} & 0 \\
0 & 0 & R_{es}
\end{pmatrix}
\]

where \( R_{mag} \) means a \( 3 \times 3 \) magnetometer measurement noise covariance matrix; \( R_{ss} \) means a \( 3 \times 3 \) sun sensor measurement noise covariance matrix and \( R_{es} \) means a \( 3 \times 3 \) earth sensor measurement noise covariance matrix. Also the \( H_{k+1} \) is defined as (for two sensors are available)

\[ H_{k+1} = \begin{bmatrix} 0_{3 \times 3} & h_{12} & h_{13} & h_{14} \\
0_{3 \times 3} & h_{22} & h_{23} & h_{24} \end{bmatrix} \]

where

\[ h_{ij} = \frac{\partial C(q)}{\partial q_j} \bigg|_{q=q_i, b_{orb,j}}; i = 1,2,3; j = 1,2 \] (2.22)

**UPDATE THE STATE**

The increased quantity of the updated state vector

\[ \Delta \hat{x}_{k+1/k+1} = K_{k+1} (b_{meas,k+1} - C(q_{k+1/k}) b_{orb,k+1}) \] (2.23)

When propagation the state and the attitude quaternion, the full quaternion must be handled carefully to obtain a proper rotation. Therefore, in these steps of the algorithm, the quaternion have to be handled separately from the angular velocity. The propagation of angular velocity vector is

\[ \frac{d}{dt} \omega_{k+1/k+1} = \int f(\hat{\omega}(t), t) dt + \hat{\omega}_{k+1/k} \] (2.24)

The quaternion must be propagated through the transition matrix \( \phi = e^{\frac{1}{2} \hat{\omega}(\Delta t) \hat{\omega}} \) without approximation, resulting in

\[ q_{k+1/k+1} = (\cos(\frac{\Delta T \omega}{2}) + \frac{1}{\omega} \sin(\frac{\Delta T \omega}{2}) \hat{\omega}(\Delta t)) q_{k+1/k} \] (2.25)

where the \( \Delta T \) is the time between two measurements, also \( \omega = \sqrt{\omega_x^2 + \omega_y^2 + \omega_z^2} \).
\[ \omega_{k+1/k+1} = \tilde{\omega}_{k+1/k} + \Delta \tilde{\omega}_{k+1/k} \quad (2.26) \]
\[ \hat{q}_{k+1/k+1} = q' \otimes \hat{q}_{k+1/k} \quad (2.27) \]

where \[
q' = \left[ \frac{\Delta \hat{q}_{k+1/k}}{\sqrt{1 - |\Delta \hat{q}_{k+1/k}|^2}} \right] \quad (2.28)
\]

**CALCULATION COVARIANCE ERROR MATRIX**

\[ P_{k+1/k} = \phi_k P_k \phi_k^T + Q_k \quad (2.33) \]

where the transition matrix is

\[ \phi_k = I + F(t)\Delta T \quad (2.34) \]

In Eqn. (2.34), \( F(t) \) are the linearized equations of motion

\[ F(t) = \frac{\partial f(x(t), t)}{\partial x} \mid_{x_i} \quad (2.35) \]

it is equal to

\[ F(t) = \begin{bmatrix} I_{x\nu}^{-1}(I_{x\nu} \dot{\omega} - \dot{\omega} \Delta T) & 60I_{x\nu}^{-1}F_{ex} \\ \frac{1}{2} I_{x\nu} & -[\Delta T] \end{bmatrix} \quad (2.36) \]

where

\[ \omega^x = \begin{bmatrix} 0 & -\omega_y & \omega_z \\ \omega_y & 0 & -\omega_z \\ -\omega_z & \omega_x & 0 \end{bmatrix} \quad (2.37) \]

\[
F_{ex} = \begin{bmatrix} \sigma_x(\hat{C}_{x_{1x}} \hat{C}_{x_{2x}} - \hat{C}_{x_{1x}} \hat{C}_{x_{2x}}) & \sigma_x(\hat{C}_{x_{1x}} \hat{C}_{x_{3x}} - \hat{C}_{x_{1x}} \hat{C}_{x_{3x}}) & \sigma_x(\hat{C}_{x_{2x}} \hat{C}_{x_{3x}} - \hat{C}_{x_{2x}} \hat{C}_{x_{3x}}) \\ -\sigma_y(\hat{C}_{x_{1x}} \hat{C}_{x_{2x}} - \hat{C}_{x_{1x}} \hat{C}_{x_{2x}}) & \sigma_y(\hat{C}_{x_{1x}} \hat{C}_{x_{3x}} - \hat{C}_{x_{1x}} \hat{C}_{x_{3x}}) & \sigma_y(\hat{C}_{x_{2x}} \hat{C}_{x_{3x}} - \hat{C}_{x_{2x}} \hat{C}_{x_{3x}}) \\ \sigma_z(\hat{C}_{x_{1x}} \hat{C}_{x_{2x}} - \hat{C}_{x_{1x}} \hat{C}_{x_{2x}}) & \sigma_z(\hat{C}_{x_{1x}} \hat{C}_{x_{3x}} - \hat{C}_{x_{1x}} \hat{C}_{x_{3x}}) & \sigma_z(\hat{C}_{x_{2x}} \hat{C}_{x_{3x}} - \hat{C}_{x_{2x}} \hat{C}_{x_{3x}}) \end{bmatrix} \quad (2.38) \]

and

\[ \sigma_x = (I_x^1 - I_x^2); \sigma_y = (I_x^2 - I_x^3); \sigma_z = (I_x^3 - I_x^1) \quad (2.39) \]

**3. ATTITUDE ESTIMATION WITH SELF-TUNING PARAMETERS**

Since the good choice of \( Q \) and \( R \) are important for obtaining good attitude estimates through the EKF approach. However, rigorous approaches for tuning the filter parameters are consuming and ad hoc approaches can lead to inadequate or improper results (see Gemson 1998). Based on Myers’ work, in the above EKF, the following self-tuning method of parameters matrices \( Q \) and \( R \) are used.

For the measurement noise statistics, the linearized observation and updated estimation attitude vector relationship at a given observation time \( k \) is given by

\[ r_k = b_{meas,k} - H_k \hat{x}_k \quad (3.1) \]

The following sample mean is taken as an unbiased estimate for \( r \)

\[ \hat{r} = \frac{1}{N} \sum_{i=1}^{N} r_k \quad (3.2) \]

with covariance

\[ \hat{C}_r = \frac{1}{N} \sum_{i=1}^{N} (r_k - \hat{r})(r_k - \hat{r})^T \quad (3.3) \]

The expected value of the above

\[ E[\hat{r}] = \frac{1}{N} \sum_{i=1}^{N} (H_k P_{k,i-1} H_k + R) \quad (3.4) \]

Here an unbiased estimate of \( R \) can be related as

\[ \hat{R} = \frac{1}{N} \sum_{i=1}^{N} (r_k - \hat{r})(r_k - \hat{r})^T - H_k P_{k,i-1} H_k \quad (3.5) \]

Similarly for the state noise sample at time \( k \), it has

\[ q_{mk} = x_{mk} - \tilde{x}_{mk/k-1} \quad (3.6) \]

where

\[ \hat{q}_m = [\hat{\omega}_m, \hat{\omega}_m, \hat{\omega}_m, \hat{\omega}_m, \hat{\omega}_m, \hat{\omega}_m] \quad (3.7) \]

an unbiased estimate of \( q \) is taken as the sample mean

\[ \hat{q}_m = \frac{1}{N} \sum_{i=1}^{N} q_{mk} \quad (3.8) \]

with covariance

\[ \hat{C}_q = \frac{1}{N} \sum_{i=1}^{N} (q_{mk} - \hat{q}_m)(q_{mk} - \hat{q}_m)^T \quad (3.9) \]

thus, the unbiased estimate for \( Q \) is given by

American Institute of Aeronautics and Astronautics
The above EKF attitude estimation algorithm can be synthesized as the following steps

1. Initialization
2. Input measurements and reference signals.
3. Calculate Kalman gain by Eqn. (2.9)
4. State update by Eqn. (2.8). 
5. Update covariance matrix by Eqn (2.11).
6. Propagation of the state vector by Eqns. (2.26) and (2.27).
7. Propagation covariance matrix by Eqns. (2.33).
8. If the cost are not satisfied, update measurement noise covariance and process noise covariance matrix $R$ and $Q$ by Eqn. (3.5) and (3.10), then, return step 2, and repeat from step 2 to step 8.
9. If the cost are satisfied, return step 2, and repeat step 2 to step 7.

5. SIMULATION RESULTS

The initial process noise covariance matrix $Q$, the initial measurement noise covariance matrix $R$ and the initial state covariance matrix $P$ are chosen as respectively

$$Q_0 = 4.5e^{-5}I_{66}, \quad R_0 = 10^{-5}I_{66}, \quad P_0 = I_{66}$$

The system initial attitude are given as follows

$$\phi_0 = 1.2^\circ; \theta_0 = 1.2^\circ; \psi_0 = 1.2^\circ;$$
$$\omega_{a0} = 1.1^\circ / \text{sec}; \omega_{a0} = 1.1^\circ / \text{sec}; \omega_{a0} = 1.1^\circ / \text{sec}.$$  

The simulation results are shown in Fig. 1, Fig. 3 and Fig. 5. For comparing, the results, which uses the EKF without self-tuning action in the same initial conditions, are also given here in Fig. 2, Fig. 4 and Fig. 6. When the EKF without self-tuning action is used, matrices $R$ and $Q$ are chosen as $R_0$ and $Q_0$, and they are kept to be constant matrices in whole of running process.
Fig. 4 Simulation result of quaternion using EKF without self-tuning action

Fig. 5 Simulation result of angular velocity Using EKF with self-tuning action.

Fig. 6 Simulation result of angular velocity using EKF without self-tuning action

From Fig 2, Fig 4, and Fig 6, if there is not the self-tuning action, the attitude estimation results are not acceptable due to the poor choice of the EKF parameters matrices $Q$ and $R$. From Fig. 1, Fig. 3, and Fig. 5, the bad choice of $R$ and $Q$ are taken as initial matrices, and they can be updated on line, the simulation process is divided into 3 stages, in the first stage, the initial matrix $Q_0$ is kept as a constant matrix, matrix $R$ is updated on line, in the second stage, the updated $R$ matrix is kept as a constant matrix, the matrix $Q$ is updated on line. From the attitude estimation trajectories, the attitude estimation results are better and better as the time, therefore, in the final stage, the self-tuning action is cut down, from 4000th time steps, the matrices $Q$ and $R$ have been keeping as the updated constant matrices. The average squares attitude estimation errors are given in the above table.

6. CONCLUSION

The presented work shows how to take state and observation noise samples generated in Kalman filter algorithm to implement the adaptive spacecraft attitude estimation. The main contribution of the work is that the hard problem, which choose parameters matrices of Kalman filter for the system with unknown measurement noise and process noise model, is solved effectively in the spacecraft attitude estimation system. The effectiveness of the approach has been demonstrated in Quickset simulator.

REFERENCES


Table

<table>
<thead>
<tr>
<th>algorithm</th>
<th>$\frac{1}{m} \sum_{i=n}^{m} \Delta \phi_i^2$</th>
<th>$\frac{1}{m} \sum_{i=n}^{m} \Delta \theta_i^2$</th>
<th>$\frac{1}{m} \sum_{i=n}^{m} \Delta \psi_i^2$</th>
<th>$\frac{1}{m} \sum_{i=n}^{m} \Delta \omega_{x_i}^2$</th>
<th>$\frac{1}{m} \sum_{i=n}^{m} \Delta \omega_{y_i}^2$</th>
<th>$\frac{1}{m} \sum_{i=n}^{m} \Delta \omega_{z_i}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EKF_ST</td>
<td>0.5377</td>
<td>1.3456e-4</td>
<td>0.7968</td>
<td>1.0337e-6</td>
<td>4.6887e-7</td>
<td>5.8232e-4</td>
</tr>
<tr>
<td>EKF</td>
<td>4.6654</td>
<td>3.9499e-4</td>
<td>5.6090</td>
<td>0.0359</td>
<td>0.0543</td>
<td>0.0538</td>
</tr>
</tbody>
</table>

Note that: EKF_ST: Extended Kalman Filtering with self-tuning action
EKF: Kalman Filtering without self-tuning action
$m = 12001; n = 4001$

The unit of angular is degree, and the unit of angular velocity is degree/sec