Coexisting Attractors, Chaotic Saddles, and Fractal Basins in a Power Electronic Circuit

Soumitro Banerjee

Abstract—We report the observation of coexisting attractors with fractal basin boundaries in the voltage mode controlled buck converter. Long chaotic transients resulting from chaotic saddles are observed for a large range of parameter values. These phenomena may have serious technological implications.

Index Terms—Chaos, nonlinear phenomena, power electronics.

I. INTRODUCTION

In this letter we investigate the nonlinear phenomena in the voltage mode controlled buck converter (Fig. 1). In the past the bifurcation behavior of this circuit has been reported in [1] and [2]. The experimental as well as simulation results show the bifurcation of period-1 behavior to period-2 behavior, followed by a period doubling sequence. But then the attractor seems to expand suddenly into a much larger chaotic orbit.

It is known that a chaotic orbit is composed of an infinite number of unstable periodic orbits. Therefore, in the expanded region of the attractor there are unstable periodic orbits which appear to come into existence all of a sudden with small increase of parameter value. This, however, can not happen in continuous systems which allow only small changes in orbits in response to small changes in parameters. Clearly, something else must be happening.

There is another problem in this system presented by past work. The bifurcation diagram in [1] exhibit narrow bands of chaotic behavior in the period-1 and period-2 zones, which also seems to imply sudden change in system behavior with tiny variation of the bifurcation parameter. By the foregoing argument, this can not happen in continuous systems. Where, then, do the narrow bands of chaos come from?

In this letter we offer definitive answers to these questions. The results open up interesting possibilities in the study of nonlinear phenomena in power electronic circuits.

II. COEXISTING ATTRACTIONERS

We have simulated the system by numerically obtaining a discrete map that gives the state at a clock instant (the beginning of a ramp cycle) in terms of the state at the previous clock instant. This map is then iterated to give the system dynamics. The parameter values are $L = 20$ mH, $C = 47 \mu F$, $R = 220 \Omega$, $T = 400 \mu s$, $V_{C} = 8.2$ V, $V_{L} = 3.8$ V, $V_{ref} = 11.3$ V and the amplifier gain is 8.4. The input voltage is used as the variable parameter.

We have found that for this system two stable behaviors coexist at certain parameter ranges. Fig. 2 shows the coexisting attractors at $V_{in} = 13.6$ V in continuous time. One attractor is period-1 and the other period-6 (there are 6 clock pulses in a cycle). The phase space is divided into two basins of attraction as shown in Fig. 3. If the initial condition is within the dark region the system is attracted to the stable period-1 behavior. If the initial condition is within the white region, the asymptotic behavior is period-6. It is interesting to note that the boundary between the two basins is a fractal.

We study the evolution of these attractors by drawing the bifurcation diagram in a special way. We observe that the attractor sizes are limited within $v = [10, 18]$ V and $i = [0.2, 1.3]$ A. We take a diagonal of this rectangle in the phase space and place 10 equidistant initial conditions on this diagonal. For each parameter value, the iteration starts with each of these initial conditions, 10 000 cycles of transient are eliminated and 20 points are plotted. When there are coexisting attractors, at least one of these initial conditions generally falls within the basin of attraction of each attractor.

The resulting bifurcation diagram is shown in Fig. 4(a). It is found that while the main attractor undergoes period doubling bifurcation, coexisting attractors appear and disappear in two ranges of parameter values—one when the main attractor is period-1 and the other when the main attractor is period-2.

For a closer scrutiny we present a closeup of the region $V_{in} = [12.5, 14.5]$ V in Fig. 4(b). It is seen that a period-3 attractor comes into existence through a saddle-node bifurcation at $V_{in} = 13.39$ V. It immediately bifurcates into a period-6 orbit which bifurcates into...
Fig. 2. Coexisting stable orbits at $V_{in} = 13.6$ V in the phase space (inductor current versus capacitor voltage).

Fig. 3. Basins of attraction at $V_{in} = 13.6$ V in the space of initial conditions at a clock instant. Black and white regions signify the basins of the period-1 and period-6 attractors, respectively.

Fig. 4. (a) Bifurcation diagram showing the evolution of coexisting attractors. (b) Blow up of the region $V_{in} = [12.5, 14.5]$ V.

chaos subsequently. And then the attractor vanishes at around $V_{in} = 13.88$ V. The reason for this phenomenon is the following.

When two attractors are coexisting, they have their own basins of attraction. As the parameter is varied, the basins also undergo change of shape and area. In the present case as $V_{in}$ is increased, the basin of the period-1 attractor approaches the chaotic attractor and at $V_{in} = 13.88$ V they make contact (boundary crisis). This renders the chaotic orbit asymptotically unstable and therefore it vanishes from the bifurcation diagram.

III. Chaotic Saddle

It is important to note that the chaotic behavior continues to exist as an unstable aperiodic orbit or a “chaotic saddle”. Its existence makes the system behavior significantly different from other parameter ranges (say $V_{in} = 13.0$ V) where such things do not exist. Any initial condition falling close to this orbit undergoes very long chaotic transient before the state falls in the basin of the period-1 attractor and is pulled to the periodic orbit. It is observed that transients often continue for more than 5000 cycles in presence of a chaotic saddle while they die down within 70 cycles in other parameter ranges. If a sufficient number of preiterates are not eliminated while plotting the bifurcation diagram, bands of chaos appear for certain parameter values. This explains the “narrow band chaos” observed in [1].

The coexisting attractors are generally not observed in controlled experiments. In such experiments the bifurcation diagram is obtained on a CRO by observing one of the variables in discrete time as a parameter is smoothly varied. Naturally the initial condition for each
parameter value is the final condition for the previous one and the system remains locked to one of the attractors.

However in real circuits operating in industrial environments the random disturbances may sometimes push the state away from the main attractor. If a chaotic saddle is present, the system would experience long chaotic transients. This may cause stresses in the components for which the system was not designed.

IV. SUDDEN EXPANSION OF THE ATTRACTOR

While the main attractor undergoes period doubling cascade as the parameter $V_{in}$ is increased, the chaotic saddle continues to exist and evolve. At around $V_{in} = 30$ V it is joined by another chaotic saddle created from the other coexisting attractor. Then at around $V_{in} = 32.5$ V the main attractor touches the chaotic saddle (interior crisis). This makes the chaotic saddle stable and the resulting attractor now spans the whole range covered by the main attractor as well as the pre-existing unstable orbits. Hence we observe a sudden expansion of the attractor at this parameter value. The unstable periodic orbits in the expanded region actually came from the chaotic saddle.

V. CONCLUSIONS

This letter reports the first observation of simultaneously existing stable orbits in a power electronic circuit. Rather long bursts of erratic behavior may be observed in systems with coexisting attractors and chaotic saddles in presence of occasional random disturbances.

This letter also explains the sudden expansion of the attractor and the narrow bands of chaos reported in previous work.

ACKNOWLEDGMENT

We would like to thank Dr. J. Deane and Dr. D. Hamill for attracting attention to the problem during the author’s visit to the University of Surrey, U.K., in February 1996.

REFERENCES