Blind separation of vibration components: Principles and demonstrations

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Abstract

Blind source separation is the issue of recovering the various independent sources exciting a system given only the measurements of the outputs of that system. It has recently become the focus of intensive research work due to its high potential in many applications. However when it comes to vibration signals, blind source separation faces a number of difficulties which seriously hinder its feasibility. In this paper we first draw a list of these difficulties. We then propose to somehow lessen the ambitions of BSS to more realistic goals, and in particular to separate vibration signal into contributions of (1) periodic, (2) random stationary, and (3) random nonstationary sources. Although not ideal, such a separation can provide substantial information in many practical cases of interest. Towards this aim we propose two robust separation techniques based on the short-time Fourier transform. They are perfectly suited to handle convolutive mixtures of sources and they lead to fast and efficient implementation.

Keywords: Blind source separation; Vibration signals; Periodic/random separation; Stationary/nonstationary separation; Spectral kurtosis; Short-time Fourier transform

1. Introduction

Research in information theory and signal processing has recently witnessed tremendous efforts for developing the theory of “blind source separation” (BSS). BSS is the issue of extracting individual but physically different sources from output measurements where only a mixture of the sources is observed. This issue is recurrent to many physical experimentations, and therefore encounters a rich potential of applications. Not less obvious is its importance to vibration analysis, where it is commonplace to deal with measurements resulting from mixtures of many excitation forces. However, the newly developed BSS algorithms face a number of difficulties with vibration signals, which hardly lend them to fit conventional hypotheses. The first difficulty arises from the fact that a mixture of vibrations is most often of the convolutive type. The second difficulty is that the number of individual sources in the mixture is hardly ever known, and is often very high—e.g. as in the case of rotating machinery. The third difficulty is that vibration measurements are often very...
noisy. The last but not least difficulty is that the sources are usually distributed instead of being punctual—a sine qua non condition for BSS to work. These common difficulties are enough to seriously hinder—if not defeat—most BSS algorithms proposed thus far.

The aim of this paper is to reformulate the BSS with less ambitious objectives when it comes to deal with vibration signals. A first simplification comes from the fact that recovering the original sources themselves is not so crucial: the contributions of the sources at the output measurements will be satisfying enough in many instances. A second simplification comes from recognising that separating the sources associated with all the components in a mechanical system is, although ideal, neither realistic nor necessarily what the engineer aims at: regrouping together sources sharing similar characteristics may be sufficient, for instance harmonic sources and transient sources.

In this communication, we introduce such techniques for decomposing a vibration signal into its periodic, random, transient, and stationary components. The proposed techniques require few conditions to work and, in that sense, are much more robust and universal than other BSS alternatives. Moreover, they are all based on efficient numerical processing based on the short-time-Fourier-transform. We believe that the decomposition they provide is relevant enough to be of value in many vibration problems. We support this belief by convincing examples of application to the diagnostics of gearboxes and rolling element bearings.

2. Principles of blind source separation

Many experiments in vibration engineering can be described by the “systemic” approach where a linear homogeneous system subjected to a number of excitation forces delivers a number of responses—see Fig. 1—the objective being to find the experimental relationships between the set of inputs and the set of outputs. Whereas the outputs of the system are issuing from carefully selected measurement points, the inputs are often very difficult to be measured directly for they correspond to internal forces.

Complex machines provide typical examples of the issue at hand—see Fig. 2. In this context, the objective of BSS is to gain access to each individual excitation force from processing the output measurements only.

Fig. 1. BSS aims at recovering the multiple unknown input sources to a system, from output-measurements only.

Fig. 2. Typical industrial example where BSS is of interest.
may be seen as an inverse problem, which can be tackled from a deconvolution point of view. However, classical deconvolution requires the physical transfer paths from the inputs to the outputs to be known exactly before inversion. The objective of BSS is more ambitious in the sense that the transfers are unknown. The only assumption is that the excitation forces are physically independent, that is they correspond to different sources. However difficult this issue seems to be, it has recently been given an elegant solution in the field of signal processing [1]. The idea is to find those sources that reproduce exactly the output measurements, subjected to the constraint that they be as statistically independent as possible. Provided that the concept of “statistical independence” is understood in a large enough framework, the so-obtained solution can then be shown to be unique—within two indeterminacies which will be mentioned later.

Example 1. In order to briefly explain the mechanism of BSS, let us consider the massless system depicted in Fig. 3. This is a two-input two-output system described by the set of equations:

\[
\begin{bmatrix}
  x \\
  \theta
\end{bmatrix} = \begin{bmatrix}
  1/k & 1/k \\
  -l/\kappa & l/\kappa
\end{bmatrix} \begin{bmatrix}
  F_1 \\
  F_2
\end{bmatrix},
\]

(1)

where \( k \) and \( \kappa \) denote the stiffnesses of the linear and torsional springs, respectively, and \( l \) is the half-length of the bar.

Supposing that only displacement \( x \) and rotation \( \theta \) are measured, the goal is to recover the unknown forces \( F_1 \) and \( F_2 \) from assuming their physically independence only, and without knowledge of the stiffness matrix \( [k] = [1/k, 1/k; -l/\kappa, -l/\kappa]^{-1} \). Physical independence actually implies several degrees of statistical independence. The first implication is that \( F_1(t) \) and \( F_2(t) \)—in terms of functions of time \( t \)—are uncorrelated, i.e. \( \langle F_1(t), F_2(t) \rangle = 0 \) where \( \langle \cdot \rangle \) denotes the time averaging operator. Generally this is not a strong enough requirement to yield a unique solution. For instance it is easy to check that combinations \( \{F_1(t) + F_2(t)\} \) and \( \{F_1(t) - F_2(t)\} \) are also uncorrelated while obviously not being the expected solution. Other degrees of independence must therefore be used to solve this indeterminacy. A popular criterion is to additionally force the nullity of the fourth-order statistics \( \langle |F_1(t)|^2, |F_2(t)|^2 \rangle - \langle |F_1(t)|^2 \rangle \langle |F_2(t)|^2 \rangle \) \[2\], an operation which may be physically understood as searching for sources with uncorrelated variations of energy—another possible criterion would be to search for sources with uncorrelated envelopes using the criterion \( \langle |F_1(t)|, |F_2(t)| \rangle \langle |F_1(t)|^2 \rangle \langle |F_2(t)|^2 \rangle \) \[3\]. The result of this process is displayed in Fig. 4. Note that the excitation forces are correctly recovered within (i) a scaling factor and (ii) their labelling (the 2 forces are permuted). These two indeterminacies turn out to be fundamental to BSS and cannot be resolved without further information (they relate to quantities that can be equivalently assigned to the sources or to the transfer paths).

3. Blind separation of vibration sources: a list of difficulties

The potential of BSS is appealing in a number of mechanical applications for it offers a new way to solve traditionally intractable problems. However, the recent enthusiasm risen by BSS should be somehow
moderated due to the fact that vibration signals are much more complex than communication signals for which BSS was originally devised. This section aims at listing the main difficulties BSS is faced with when dealing with vibration signals; most of these difficulties have never been explicitly recognised before.

3.1. Instantaneous versus convolutive mixtures

Probably the first difficulty arises from the fact that mechanical systems are rarely described by a simple algebraic relationship (static coupling) as illustrated in the previous example. Mechanical systems actually include inertia and dynamic coupling effects which give rise to a system of differential (rather than algebraic) equations. This is for instance the case as soon as the mass of the bar in the previous example is taken into account; then in brief, and using vector notations, Eq. (1)

\[
\frac{1}{2}m\ddot{x} + \frac{1}{2}k\dot{x} = F
\]

becomes

\[
[m][\ddot{x}] + [k][\dot{x}] = \{F\}.
\]

Such systems are said to be described by a dynamic or convolutive mixture (of sources), as opposed to a static or instantaneous mixture. Convolutive mixtures of sources are a much more difficult to tackle, although the same statistical principles apply as for instantaneous mixtures. There is currently an endeavour of research in this direction; yet no fully satisfying algorithms have been proposed so far. The main difficulty is that very long impulse-responses have to be dealt with when it comes to separate vibration sources, often with hundreds or thousands of coefficients. Identifying separating filters with so many unknowns is obviously a complicated task.

3.2. Fundamental indeterminacies

Solving instantaneous mixtures of sources has been mentioned to find a unique solution within two fundamental indeterminacies relating to (i) the scaling factor and (ii) the labelling of the sources. In the case of convolutive sources, the scaling factor indeterminacy turns into a more dramatic ambiguity: the scale becomes a function of frequency so that the sources are only recoverable up to a filtering operation. Imposing the
recovered sources to be white or to have a pre-defined colour can sometimes counter this difficulty. However such a solution is artificial and mainly aims at masking a serious deficiency of BSS when applied to convolutive mixtures. Nevertheless, a possible approach is to relax the ambition of BSS by seeking for the individual contributions of the sources at the outputs rather than the sources themselves. More will be said on this in Section 4.

3.3. Physical relevance of the “sources

Thus far the terminology “source” has been understood as the time history of the excitation intensity to a system, i.e. as a signal. This is the usual BSS (statistical) meaning. Yet the concept of a source has a much more precise meaning in physics since it is not only an intensity, but it also has a location and a direction. In brief, whereas a source is understood as a signal in the statistical meaning, it must be understood as a vector in the physical meaning. In the case of very simple systems with a finite number of degrees-of-freedom, the two concepts may exactly coincide as illustrated in Example 1. However, in the general case it rapidly turns out that the actual excitation forces cannot be inferred from the recovered source signals alone. The example in Fig. 5 illustrates this idea on a two-input two-output continuous system. With no additional information, any of the configurations in Fig. 5(b–d) are possible candidates to represent the two separated source signals. In brief, BSS does not provide information on:

- the nature of the sources (force, torque),
- the points of action of the sources,
- the lines of action the sources.

![Diagram](image)

Fig. 5. Typical example where the two blindly recovered sources in (a) can be equivalently represented by any of the pair of force/torque in (b–d).
3.4. **BSS assumes point-to-point relationships**

BSS as originally introduced in the signal processing community implicitly assumes the signals to be functions of a single variable, e.g. time. In the current state of the art BSS is unable to handle signals that are both distributed in time and in space. This means that both the sources and the measurements must be perfectly located in space, i.e. they must be point-wise. While this condition naturally holds for the measurements, it is very unlikely to be met by the sources: most real world systems actually involve spatially distributed sources, as illustrated in Fig. 6.

Note however that one derogation to this constraint is when the force distribution of the sources is separable, i.e. when \( f(x, t) = f(x)s(t) \) with \( x \) standing for space and \( t \) for time. Then it is easy to verify that density \( f(x) \) can be replaced by an equivalent point-wise force.

3.5. **BSS requires a (small) finite number of sources**

So far existing BSS algorithms can only handle a finite number of sources. This is reminiscent to the previous remark which required the sources to be point-wise. Into addition, robust separation most often requires the number of sources to be less than or equal to the number of measurement points. This is very restrictive when it comes to considering industrial systems, where tens or hundreds of sources are commonplace—see Fig. 2 and Fig. 8. Last but not least, it should be emphasised again that a “mechanical” source (i.e. an excitation force/torque) possesses more degrees of freedom than a “statistical” source (i.e. a signal). As a matter of fact it decomposes into 6 components—3 forces and 3 torques—which in spite of being highly interrelated are rarely fully dependent (in the sense that the knowledge of one component implies the exact knowledge of the others). In brief, one should bear in mind that a mechanical source may embody up to 6 statistical sources. Hence the number of statistical sources in mechanical systems is likely to be much larger than expected. This is illustrated in Fig. 7.

3.6. **The exact number of sources should be known a priori**

So far most existing BSS algorithms requires the exact number of sources impinging on the system to be known a priori. This is because the approach to separate the sources from each other is intrinsically a global approach. Again, in most instances this requirement is unrealistic—see Fig. 8. One naïve approach to counter

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**Fig. 6.** Typical example where conventional BSS algorithms does not apply due to spatially distributed sources.

**Fig. 7.** Typical example where the number of sources impinging on the system is larger than expected. Up to four possible independent sources (b) should be considered here and not only two (a).
the problem is to consider the less dominant sources as having a cumulative effect similar to that of additive noise. The flaw to this approach is that the frontier between sources and “noise” is necessarily a fuzzy and arbitrary one, which hardly lends itself to formalisation into the algebraic framework of BSS. In addition, existing BSS algorithms can hardly cope with significant additive noise, apart from the idealistic “spatially white and temporally white” assumption.

3.7. System invertibility

A sensible requirement for BSS is that the system be invertible, or in other words, that the sources be all observable from the measurements—this is also the reason why many BSS algorithms require the number of measurements to be greater than or equal to the number of (statistical) sources. The ability of a system to be invertible strongly depends on the source and measurement locations. For instance a classical pitfall is when a sensor is positioned on a vibration node, thus leading to an inefficient measurement point. Another pitfall is when the distance between two sensors is significantly less than the wavelengths of interest, thus leading to information deficiency. The same phenomenon occurs from the source side: too closely located sources cannot be separated—see Fig. 9. Incidentally Fig. 10 shows the example of a system that is not invertible for more than 3 (statistical) sources due to a structural bottleneck.

4. Restating the objectives of BSS

The previous section has described a list of difficulties associated with the “blind” application of BSS to vibration signals. Maybe the more stringent of these difficulties are related with:

(1) the necessity of dealing with convolutive mixtures,
(2) the impossibility to fully recover mechanical sources (as opposed to statistical sources),
(3) and the impossibility to assume the existence of a finite number of point-wise sources.
Despite these fundamental limitations, BSS still is appealing in its essence and surely deserves a serious attention from the mechanical community. However, rather than persevering in the classical approach, we argue that the objectives of BSS should be first restated when mechanical applications are of concern. In particular, we introduce the concepts of “blind component separation” (also coined “blind signal separation”) and of “a deflation approach” to separation. This should alleviate points (2) and (3) above, which are the main points why BSS often fails short. In particular the proposed approach applies to single-output measurements, whatever the number of sources. In the next section, we will also propose Fourier-domain algorithms based on the short-time Fourier transform to achieve these goals. This should solve point (1).

4.1. Blind component separation versus blind source separation

One possible approach to relax the ambition of BSS is to seek the individual contributions of the sources at the system outputs rather than seeking the sources themselves. This means recovering the responses of the system to every single source, simultaneously, as if all other sources were switched off. In other words, this also boils down to splitting a multiple-input multiple-output (MIMO) system into an equivalent set of single-input multiple-output (SIMO) systems, as illustrated in Fig. 11. It can be shown that this approach provides the source contributions with the exact scaling/filtering factors, thus solving the difficulty related in Section 3.2. At the same time, it eliminates the ambiguity linked with the concept of mechanical sources since only their effects are of concern, thus solving the difficulty of Section 3.3. We shall refer to this approach as “blind component separation” (BCS)\(^1\) in order to stress the difference with blind source separation. In spite of being more restrictive than BSS, BCS should however be satisfying enough in many instances.

4.2. A deflation approach to separation

Rather then trying to separate the contributions of all sources at once, we argue that a much better approach is to separate the most informative of them, in a iterative way. This should solve the difficulty of a priori knowing the number of sources in a system and should also allow coping with an arbitrary large number of insignificant sources—see Sections 3.5 and 3.6 above. One argument supporting this idea is that separating the sources from all the components in a mechanical system is, although ideal, not necessarily what the engineer aims at. Indeed, regrouping together sources sharing similar characteristics may be sufficient in many instances. For such an approach to hold, we must define a strategy according to which a first source contribution is separated, then a second one, and so on. A very useful separation criterion for vibration signals is to distinguish between harmonic and random components, and then between random stationary and random nonstationary components, as illustrated in Fig. 12. This is a natural classification that perfectly describes most

\(^1\)Also sometimes referred to as “blind signal separation”.

Fig. 10. Typical example of a system that is not invertible for recovering more than three sources.
vibration signals. As will be explained hereafter, such a separation naturally extracts signal components stemming from different physical origins [4], while at the same time it suppresses the ambiguity associated with the definition of a statistical source: here the sources are clearly defined as what is periodic, stationary, and nonstationary in the signals. Also of importance is that the so-defined sources are not restricted to be point-wise forces/torques and can have any spatial distribution. These redefinitions thus alleviate the difficulties mentioned in Sections 3.3 and 3.4. Of importance also is the fact that the proposed approach allows separation on single-output measurements in contrast to most existing BSS techniques.

Fig. 11. Blind component separation, as opposed to blind source separation, is the process of splitting a MIMO system into a set of equivalent SIMO systems.

Fig. 12. The idea of the deflation approach is to progressively decompose the signal into its most informative and statistically independent components. We propose here a decomposition based on periodic, random, transient, and stationary components.
4.2.1. The periodic/random dichotomy

Periodic vibration signals relate to kinematical forces induced by the rotating mechanisms in a machinery (unbalances, misalignments, eccentricities, meshing forces, etc). Strictly speaking, periodicity should be understood with respect to an angular variable describing the rotation of the machine. In particular if the machine does not rotate at constant speed, then the signals should be resampled on an angular basis in order to track speed variations. Techniques for angular resampling vibration signals are described in Ref.[5]. Once the periodic components have been extracted from the additive background noise, it is then an easy matter to identify individual sources by conventional spectral analysis (provided the frequency resolution is fine enough) because the rotating parts of a machine can be precisely assigned to known characteristic frequencies. A large literature exists on the subject.

Interestingly enough, the periodic/random dichotomy is supported by a theoretical result in the theory of time series known as Wold’s decomposition. Wold’s decomposition states that any signal can be uniquely decomposed into a summation of a predictable (deterministic) part and a regular (purely random) part. For vibration signals the predictable part essentially embodies periodic components. Then the regular part is what remains after extraction of the predictable part.

4.2.2. The stationary/nonstationary dichotomy

After extraction of the periodic components, the random residual part can be assigned to non-periodic mechanisms such as contact and friction forces, fluid motions, wears, clearances, etc. This residual part clearly asks for further separation in order to distinguish between the many sources it gathers. One sensible approach is to first extract those source contributions that have the highest degree of impulsiveness, because they are likely to relate to abnormal chocks, frictions and other transient phenomena of interest in vibration analysis. The remaining part can then be assigned to the cumulative effect of more “gentle” sources, i.e. to those sources commonly classified as “background noise”. This actually boils down to achieving a separation in terms of stationary and nonstationary components. Interestingly enough, the stationary/nonstationary dichotomy closely coincides with the idea of information as understood in signal processing, i.e. nonstationarity conveys information as opposed to stationarity which is noise.

5. BSS Fourier-domain algorithms

The previous section has suggested lessening the ambitions of BSS to more realistic goals such as separating a signal into contributions of periodic, random stationary, and random nonstationary sources. In this section, we propose two algorithms based on the short-time Fourier transform to achieve these tasks. The motivation behind the Fourier-domain approach (rather than the time-domain approach) is to be able to handle convolutive mixtures characterised by very long impulse-responses—as a matter of fact the differential equations governing elastic mechanical structures theoretically lead to infinite impulse-responses. As well-known, switching to the Fourier-domain turns convolutive products into conventional products and enables designing arbitrary long filters [6]. Referring back to Example 1:

\[
[m][x] + [k][x] = [F] ↔ [x(t)] = [h(t)]* [F(t)]
\]

\[
↔ [X(f)] = [H(f)] \cdot [F(f)],
\]

where * refers to convolution, t and f to the time and frequency variable respectively, and capital letters to Fourier transforms. In the following we will make an intensive use of this property. The first proposed algorithm aims at achieving the periodic/random separation. The second one is concerned with the stationary/nonstationary separation.

5.1. Algorithm for the blind extraction of periodic components

Our algorithm for the blind extraction of periodic components is based on the use of a prediction filter [7]. Let us consider two N-long segments of sampled data \(x_t(n) = x(t + n)\) and \(x_{t+\Delta}(n) = x(t + n + \Delta)\), \(n = 0, \ldots, N-1\) anchored at time t and \(t + \Delta\), respectively, where the delay \(\Delta\) is set greater than N so that
the segments do not overlap. Therefore segment $x_{t+\Delta}(n)$ may be seen as the future version of segment $x_t(n)$ and, as such, it is legitimate to wonder if it can be linearly predicted from the latter. On the one hand, if $x(n)$ contains a periodic component $p(n)$, then any future segment of $p(n)$ is perfectly predictable in the future, whatever the value of $\Delta$, because the correlation time of a periodic signal is infinite. On the other hand, if $x(n)$ contains a random component $r(n)$, then the prediction is less and less accurate as $\Delta$ becomes large, and ultimately the best predicted value becomes zero—actually as soon as $\Delta$ has become larger than the correlation time of $r(n)$. The idea is illustrated in Fig. 13.

In order to tailor a BSS algorithm based on this idea, let us introduce the STFT $X_w(t,f)$ and $X_w(t+\Delta,f)$ defined as the (windowed) Fourier transforms of the data segments $x_t(n)$ and $x_{t+\Delta}(n)$, respectively. That is

$$X_w(t,f) = \sum_{n=t}^{t+N-1} x(n)w(n-t)e^{-j2nf}$$

where $w(n)$ is a smooth data-window over the interval $n = 0, \ldots, N-1$. Then, the best transfer function $G(f)$ that predicts $X_w(t+\Delta,f)$ from $X_w(t,f)$—in the sense of minimising the mean-square error between $X_w(t+\Delta,f)$ and $G(f)X_w(t,f)$—can be shown to have a complex gain given by

$$G(f) = \frac{\langle X_w(t+\Delta,f)X^*_w(t,f) \rangle}{\langle |X_w(t,f)|^2 \rangle},$$

where $\langle \cdot \rangle$ stands for the time averaging operator—see Fig. 14. It can be shown that the so-defined transfer function naturally tunes itself into a comb-filter with nearly unitary values at those frequencies where harmonics are present and nearly zero values elsewhere. It therefore provides a simple and yet efficient mean for blindly extracting periodic components. Details of the implementation are given in Ref. [7].

5.2. Algorithm for the blind extraction of transient components

Our algorithm for the blind extraction of transient components is based on the use of the spectral kurtosis [8]. As before, let $X_w(t,f)$ be the $N$-long STFT of signal $x(n)$ anchored at time $t$. Next, let us define the $n$th
order empirical spectral moment $S_{nX}(f)$ of $X_w(t, f)$ as

$$S_{nX}(f) = \langle |X_w(t, f)|^n \rangle.$$  (7)

For instance $S_{2X}(f)$ defines the power spectral density. The spectral kurtosis is defined as the normalised fourth-order spectral moment:

$$K_X(f) = \frac{S_{4X}(f)}{S_{2X}(f)^2} - 2.$$  (8)

It can be shown that the spectral kurtosis measures the deviation from Gaussianity of a signal at frequency $f$: the spectral kurtosis of a stationary Gaussian signal is zero, whereas that of a nonstationary signal is strictly positive [8]. Indeed, the higher the “peakiness” of the complex envelope of $x$ at frequency $f$, the larger its spectral kurtosis. In this sense, the spectral kurtosis is used as a frequency dependent measure of the nonstationarity of signal $x$. This is illustrated in Fig. 15.

To see how the spectral kurtosis can be used in BSS, let us consider the compounded signal $x(t) = s(t) + n(t)$, where $s(t)$ is a nonstationary component and $n(t)$ is some additive stationary Gaussian noise resulting from the cumulated effects of many minor sources. It can be checked that the spectral kurtosis of this signal is

$$K_X(f) = \left[ \frac{K_S(f)}{[1 + \rho(f)]^2} \right],$$  (9)

where $K_S(f)$ is the SK of $s(t)$ and $\rho(f)$ is the noise-to-signal ratio $S_n(f)/S_s(f)$ with $S_n(f)$ and $S_s(f)$ the power spectral densities of $n(t)$ and $s(t)$, respectively. Hence, $K_X(f)$ will tend to $K_S(f)$ at those frequencies where the signal-to-noise is high, and will be close to zero at those frequencies where the masking effect of noise is strong. Very interestingly, this behaviour is exactly that of the optimal Wiener filter

$$W(f) = \frac{1}{1 + \rho(f)}$$  (10)

that best extracts the signal $s(t)$ from the noisy measurement $x(t)$ (optimal in the sense that, among all possible linear filters $w(t)$, it minimises the mean square error between $s(t)$ and $w(t) \ast x(t)$). Assuming that $K_S(f)$ is more or less a constant function of frequency, the square root $\sqrt{K_X(f)}$ of the spectral kurtosis then offers a unique opportunity to blindly identify the optimal filter $W(f)$ (within a scaling factor) that best extracts the nonstationary component from other stationary contributions. Details of the implementation are given in Ref. [9].

6. Applications

This section demonstrates our BCS approach on actual signals. The results of the separations are systematically displayed by means of the four following components:

1. the raw signal denoted by $x(t)$,
2. the extracted components relating to periodic sources denoted by $p(t)$,

$^2$The factor $-2$ instead of $-3$ comes from the fact that $X_w(t, f)$ is complex circular.
3. the residual random part denoted by $r_1(t)$,
4. the extracted components relating to transient sources denoted by $t(t)$,
5. and the final residual part denoted by $r_2(t)$.

These signals are interrelated by the following trivial relationships:

$$
\begin{align*}
    x(t) &= p(t) + r_1(t) \\
    r_1(t) &= t(t) + r_2(t) \Rightarrow x(t) = p(t) + t(t) + r_2(t).
\end{align*}
$$

(11)

The first application is on a single-stage gearbox test-rig wherein a fault was introduced in one of the rolling element bearings supporting the primary shaft. Several accelerometers were placed at different locations on the gearbox. A first analysis revealed that the measurements were strongly dominated by the vibrations coming from the gearmesh, and which completely masked the bearing fault. BCS was then applied in order to extract the different vibration components of interest. Results are displayed in Fig. 16. One can clearly see that the periodic components $p(t)$ accounts for a large part of the signal energy. However the residual random part $r_1(t)$ is still very noisy and does not show further information. The transient random part $t(t)$ extracted form the last separation step ultimately reveals the desired information: the bearing fault appears as a series of transients whose rate of occurrence can now be further investigated in order to find which part of the bearing is damaged. Note that in this first experience the periodic components $p(t)$ are unambiguously assigned to gears operation and the random transient part to bearings operation. This is an observation we have been able to verify in many other circumstances [4].

A second experiment similar to the first one was reiterated, but with a different bearing fault. Results of the separation are displayed in Fig. 17. Here again, the bearing fault better shows off after the periodic/random separation, and then the stationary/nonstationary separation nicely finishes the job.

The last experiment concerns the application of BCS to an industrial system. Several acceleration signals were measured on a train vehicle during a condition-monitoring test. The results are displayed as before. First the periodic/random separation shows that most of the signal energy is due to periodic sources, most likely related to shaft rotations and gearmesh forces. The residual part at first seems insignificant. However, the stationary/nonstationary separation then reveals important information conveyed by transient sources in the form of a series of impacts. These can be checked to originate from a bearing fault. Note that in the present experiment the extracted transient part has a RMS value about 50 as small as that of the original signal, thus proving the benefit of BCS over other classical techniques (Fig. 18).

The above examples have illustrated the use of the proposed BCS approach. Apart form separating bearing signals from gear signals—which was the author’s main interest—we believe that the approach is general

![Fig. 16. First example of separation. $x$ = raw vibration signal, $p$ = extracted periodic components, $r_1$ = random residual part, $t$ = extracted transient components, $r_2$ = stationary random residual part.](image-url)
enough to apply in many other circumstances. For instance the periodic/random separation has been found very relevant for discriminating between operational and modal modes in output-only modal analysis [10]. Other applications are currently under investigation with minor modifications of the proposed approach.

7. Conclusion

The aim of this paper was 2-fold. First we have briefly introduced the principles of BSS, a technique which has recently gained considerable interest in the signal processing community. We have next listed a number of difficulties related with the inherent assumptions of BSS when it comes to be applied on vibration signals. We have then suggested lessening the ambitions of BSS to more realistic goals, and in particular to separate in a signal the contributions of (1) periodic, (2) random stationary, and (3) random nonstationary sources. Although not ideal, such as a separation has been shown to provide considerable information in many practical cases of interest. We have also advocated the use of Fourier-domain separation techniques based on
the short-time Fourier transform, which are perfectly suited to handle convolutive mixtures of sources and also lead to fast and very efficient implementations. Our algorithms have been demonstrated to perform well on actual examples where other BSS algorithms typically fail short.

One drawback of the proposed “blind component separation” approach is that it only returns three subsets of components; therefore other separation techniques may be necessary to further distinguish between independent contributions within the periodic, transient, and nonstationary subsets (although classical spectral analysis is often found satisfying enough to achieve this in the first subset, and time-frequency analysis in the second subset).

Finally the proposed algorithms have dealt only with the single-output case. Obviously better results are to be expected when several outputs can be processed together. Extension of our algorithms to the multi-output case will be presented in a future publication.

References