Blind source separation of internal combustion engine piston slap from other measured vibration signals

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Abstract

Internal combustion engines have several vibration sources, such as combustion, fuel injection, piston slap and valve operation. For machine condition monitoring or design improvement purposes, it is necessary to separate the vibration signals caused by different sources and then analyse each of them individually. However, traditional frequency analysis techniques are not very useful due to overlap of the different sources over a wide frequency range. This paper attempts to separate the vibration sources, especially piston slap, by using blind source separation techniques with the intention of revealing the potential of the new technique for solving mechanical vibration problems. The BSS method and the Blind least mean square algorithm using Gray’s variable norm as a measure of non-Gaussianity of the sources is briefly described and separation results for both simulated and measured data are presented and discussed.

Keywords: Blind source separation; Engine combustion pressure; Piston slap; Convolutive models; IC engine diagnostics; Gray’s variable norm

1. Introduction

Reducing internal combustion engine vibration, such as piston slap effects, can reduce noise and wear of piston and cylinder, and hence enable better design and operation of internal combustion engines. There have been many research papers published aiming to study the mechanism [1–3] and model [4,5] of piston slap or its effects. Piston slap is due to the secondary motion caused by clearance between piston and cylinder, the imbalance of forces acting on the piston such as cylinder pressure, forces from the connecting rod and the cylinder inner surface, and the inertia force of the piston, etc. The piston slap force can neither be calculated nor measured directly because of its randomness, its impact character and the small amount of energy it contributes to the engine vibration. However, piston slap can explain most of the sound level when the engine is cold or when the piston clearance is excessive.

In recent years, blind source separation (BSS) techniques have emerged. Because they separate or recover the system and the input sources by using only the measured system output data, they offer a promising
approach to tackle signal processing problems such as in communication, image processing, speech acoustics etc. The research on BSS has led to the publication of several books [6–9] but there are only a few papers published on applying BSS method to mechanical vibration problems [10–12]. Here the BSS method using the blind least mean square (BLMS) algorithm [7,13] with Gray’s variable norm [14] as cost function is briefly described and separation results are presented.

This paper expands on results presented at the ACAM2005 conference [15]. The results can be compared with those from other papers attempting a similar blind separation task on engine vibration signals using different approaches [16,17].

2. Blind source separation method

BSS has emerged as a new research area in recent years [18–20]. It is an exciting area bringing together concepts from many other areas such as signal processing, neural networks, information theory, probability and statistics, group theory, etc., to solve the challenging problems outlined in Fig. 1. There are \( M \) unknown original sources \( s \) and an unknown system \( A \) or \( H \). Because they are both unknown, the calculation of the sources and system cannot be carried out deterministically but in something of a blind and adaptive way. The only known data are the system outputs in which the information for both the sources and the system are hidden. There are two mathematical models in relation to the properties of the mixing system: the instantaneous mixing model and the convolved mixing model as shown in Eqs. (1) and (2). Note in this paper, unless otherwise indicated, matrices are denoted by capital bold letters; vectors are denoted by small bold letters; scalars are denoted by italic letters.

### Instantaneous mixing model

\[
x(t_n) = A s(t_n), \quad n = 0, 1, \ldots, T,
\]

where \( A = [a_{ij}]_{N \times M} \), matrix of scalars as mixing system; \( s(t_n) = (s_i(t_n)), i = 1, 2, \ldots, M \), original sources as input to the system; \( x(t_n) = (x_j(t_n)), j = 1, 2, \ldots, N \), observed signals as system output.

### Convolved mixing model

\[
x(t_n) = H * s(t_n),
\]

where \( H = [h_{ij}]_{N \times M} \), matrix of impulse responses as mixing system; \( * \) convolution between \( H \) and \( s(t_n) \).

In general, there is no definite solution for \( A \) or \( H \) and \( s \) by only knowing \( x \). What makes BSS so exciting is that there is a solution within a permutation matrix \( P \) and a scale factor matrix (diagonal) \( A \) under some weak assumptions, such that the sources are independent from each other. Hence, the instantaneous mixing model is usually referred to as independent component analysis (ICA) [15]. To express this indeterminacy mathematically for the instantaneous mixing model, Eq. (3) exists:

\[
A s(t_n) = (A P^{-1}) (A P^{-1}) s(t_n),
\]

\[
H * s(t_n) = (H * A) * (A^{-1} * P^{-1}) s(t_n).
\]

For the convolved model there is another indeterminacy, time shift or delay, and in most cases there is also some filtering effect as indicated in (4), where \( A \) is a diagonal filtering matrix, i.e. a diagonal matrix with each

![Fig. 1. Illustration of the blind source separation problem and algorithm.](image-url)
diagonal entry as an undetermined filter. In cases where the sources are white, the filters reduce to a time shift or delay. However, there are many situations where the sources are not white, for example the combustion and piston slap sources in the current application. In that case, what is recovered is a filtered version of each source.

The method for recovering the mixing system and the independent sources is to construct an objective or cost function and an adaptation method to learn the inverse system $W$ adaptively until it converges. The output $\hat{s}(t_n)$ afterwards will be the recovered sources. If a perfect inverse system $W$ could be recovered in the instantaneous model, its product with the mixing system $A$ results in an identity matrix $I$. Because of the undetermined scale factors and permutation of blind separation as indicated in (3), the product of the recovered inverse system and true mixing system $A$ leads to a permuted scale factor matrix as in (5). In the convolved mixing model case, the convolution of the $W$ and $H$ results a permuted diagonal filtering matrix as in (6), where $z$ refers to the fact that they are expressed in the form of the $z$ transform of each filter:

$$WA = P^{-1}A^{-1},$$

$$W(z)H(z) = P^{-1}A^{-1}(z).$$

Most practical mechanical problems such as in acoustics and vibration belong to the convolved model, which is more complicated and is more difficult to solve than the instantaneous model. One class of BSS algorithms is based on the Central Limit Theorem which states that a number of independent signals with any distribution of finite variance will sum up to have a distribution closer to a Gaussian distribution. Hence a measure of distance by Kullback–Leibler divergence between the distribution of a recovered source $\hat{s}_i$ and its corresponding Gaussian distribution of the same mean and variance can be used as an objective function as in (7). It is also called a cost function for the reason of simulating the supervised LMS algorithm cost; however, it is maximised in the blind algorithm:

$$D(f_{\hat{s}_i}, f_G) = \int f_{\hat{s}_i}(\hat{s}) \log \frac{f_{\hat{s}_i}(\hat{s})}{f_G(\hat{s})} \, d\hat{s}_i.$$

An objective function can be constructed as a sum of the divergence of each separated source as in (8), and the update algorithm can be expressed as in (9)–(10) for maximisation of the objective function with respect to the filter coefficients in matrix $W(z)$. The item $g(\hat{s}_i(n))$ is usually called the Bussgang non-linearity function [21], which is the part within (11) as expressed in (12), where the variance and probability density function of $s_i$ can be directly used if they are known. The entry $x_j(n)$ is the section of the last $L$ samples of the $j$th mixture component in time-reversed order as indicated in (13), and $L$ is the filter length. Please note also that the symbol $w_j(n)$ here denotes the FIR filter vector of length $L$ at the $i$th row and $j$th column of the inverse system $W(z)$ for iteration step $n$. The $T$ in Eq. (10) denotes the transpose of the row vector $x_j(n)$:

$$J = \sum_{i=1}^{M} D(f_{\hat{s}_i}, f_G),$$

$$w_j(n + 1) = w_j(n) + \mu \frac{\partial J}{\partial w_j(n)},$$

$$\hat{s}_i(n + 1) = \sum_{j=1}^{M} w_j(n + 1)x_j^T(n + 1),$$

where

$$\frac{\partial J}{\partial w_j(n)} = \frac{\partial J}{\partial \hat{s}_i(n)} \frac{\partial \hat{s}_i(n)}{\partial w_j(n)}$$

$$= (\hat{s}_i(n) - g(\hat{s}_i(n)))x_j(n),$$

$$g(\hat{s}_i(n)) = -E[\hat{s}_i|^2f'(\hat{s}_i(n))/f(\hat{s}_i(n))],$$

$$x_j(n) = [x_j(n)x_j(n - 1), \ldots, x_j(n - L + 1)].$$
The Kullback–Leibler divergence calculation is complicated, because it involves integration of the source distribution. Gray’s variable norm [14] in (14) is a parametric approximation of (7), and can be used by simply choosing the value of parameter \( a \) according to the assumption of the source distribution as being sub or super-Gaussian. Using Gray’s variable norm, the BLMS algorithm can be constructed as in (9)–(10) by Lambert [7,13], where the non-linearity function (12) is simplified as in (15):

\[
O_s^2(\hat{s}_i) = \frac{E[|\hat{s}_i|^2]}{(E[|\hat{s}_i|^2])^{\frac{1}{2}}} = D(f_\hat{s}_i, f_G(\hat{s}_i))
\]

and

\[
g(\hat{s}_i(n)) = \frac{E[|\hat{s}_i|^2]}{E[|\hat{s}_i|^2]} \hat{s}_i(n)|\hat{s}_i(n)|^{a-2}.
\]

For super-Gaussian sources with high kurtosis, such as piston slap, let \( a = 1 \). For sub-Gaussian sources such as cylinder pressure, let \( a = 4 \). The algorithm will drive the recovered sources as far away from Gaussian distribution as possible. However, note that the cylinder pressure contains two sources, the compression/expansion and combustion. While the part caused by compression/expansion is sub-Gaussian, the combustion part is super-Gaussian. More details are given in Section 5.

3. Blind source separation simulation

A simulation has been carried out to verify the ability of the BLMS algorithm. Two generated sources of 100,000 samples in length are used to simulate the piston slap as impulses and the pressure change as a uniform distribution. Fig. 2 shows the first 300 samples of the original sources and Fig. 3 the mixed sources as the observed system output. The mixing system \( H \) in Fig. 5 is a randomly generated filter matrix of length 64, multiplied by a double exponentially decaying curve. The true inverse system in Fig. 7 is calculated so that \( H^*H = I \), where \( I \) is the identity filter matrix as shown in Fig. 9. The impulsive source is a super-Gaussian signal and source two is a sub-Gaussian signal. The mixed system outputs \( x \) are more like Gaussian signals and have a time delay about equal to the filter length 32 because of convolution, as can be seen by comparing Figs. 2 and 3.
Fig. 4 shows the separation result obtained by the BLMS method after 500 sweep iterations by using an inverse system $W$ of the same filter length as $H$ (Fig. 5). Source one is almost completely recovered with barely noticeable noise, a scale factor a little greater than one with a minus sign, and a time delay; source two is almost perfectly recovered with only a time delay. Figs. 6, 7 show the BSS identified inverse system $W$ and the calculated true inverse system $H^{-1}/C_0$. It is seen that $W$ and $H^{-1}/C_0$ are very similar, but there are delays, scale factors, and inverted sign for row one of $W$.

For further verification, the global systems $G = W^*H$ and $I = H^{-1}*H$ are calculated and plotted in Figs. 8, 9. It can be easily seen that the blindly identified global system $G$ in Fig. 8 is really a scaled and delayed version of the true global system $I$ in Fig. 9, with only barely noticeable noise.
The inverse filter $W$ is constrained to be a causal filter, which can be seen from the identified inverse system as in Fig. 6; The significant part of the identified inverse system response is located near the middle of the whole filter length of 64 samples, which is desirable. However, this will cause a delay of about half the filter length. For this reason, a delay of 32 samples has been compensated for when outputting the recovered sources. Note that the delay caused by the mixing process in Fig. 3 is not compensated. This is because in real world blind separation problems, the delay caused by the mixing system or mixing channels is often unknown, and it is better not to adjust for this in order to show that the delay exists, which is apparent by comparing Figs. 2 and 4. In addition the global system resulted from the product $W(z)H(z)$ is plotted in Fig. 8, which is a permuted scale factor matrix with some filtering effects also noticeable. This is consistent with Eq. (6) however the non-zero values on the cross diagonal can only be interpreted as errors.

Fig. 5. Generated mixing system $H$.

Fig. 6. Identified inverse system $W$.
In summary, the BLMS method used here is quite capable of separating the simulated sources and should be a good choice for separating the piston slap events in a practical internal combustion engine.

4. Diesel engine data and blind separation result

Signals from a normal and a faulty four-cylinder diesel engine are measured including vibration acceleration, cylinder pressure, fuel injection and once-per revolution tacho of the crankshaft. Only the sections of the measured signals corresponding to combustion in one cylinder around its piston Top Dead Center is used. During this short time range, the vibration is caused mainly by the operation of this cylinder
such as fuel injection, combustion and piston slap with the rest being background noise. Because of the piston motion, the system is actually a time varying system. However, if only the section of the signals around TDC is being considered, e.g. within $-25-45^\circ$ crank angle, the piston only moves slowly over a short distance and hence the system can be regarded as approximately time invariant.

Figs. 10–12 show the measured vibration acceleration sensor signals $x$, the recovered source $\hat{s}$, the measured fuel injection signal and filtered measured cylinder pressure (fluctuation) $dP$. The BLMS separation results for a cylinder with normal piston at about 1250 rpm are shown in Fig. 10. Using three acceleration vibration signals which is significant within $-20^\circ$ to $-10^\circ$ crank angle, the fuel injection source is recovered in the left column. The impulses at about $-16^\circ$ must be fuel injection since there is no other operation there (apart from the closure of an inlet valve which is approximately located around this angle, but is weak in practice). Using three acceleration signals which are not significant within $-20^\circ$ to $-10^\circ$ crank angle, the recovered source is shown in the right column, which is believed to be a filtered version of the combustion pressure for the following reasons: first it cannot be piston slap as this is a normal engine; Second it cannot be fuel injection as the vibrations caused by fuel injection within $-20^\circ$ to $-10^\circ$ crank angle are very small in the three measured signals; The only event remaining is combustion which causes the cylinder pressure to change extensively within crank angle range 5–20°.

It is interesting to note that the recovered pressure is also a super-Gaussian source and used exactly the same parameter $\alpha = 1$ as used in recovering the fuel injections. This is somewhat contradictory to what we initially thought: that the cylinder pressure is one source no matter whether it is due to compression/expansion or combustion. This separation result shows that they should be considered as two distinct sources and be treated separately. The part of the pressure due to piston movement is smooth and symmetrical about TDC and is a sub-Gaussian source of low frequency with negligible noise; The part of the pressure due to combustion is a super-Gaussian source, an abrupt event which causes high frequency pressure changes. Even further, this pressure change is not uniform but oscillating within the cylinder space. It is called combustion knock and tends to cause more engine noise. Hence, these two sources are of different character and the latter one can be harmful and need to be reduced. In fact research on avoiding combustion knock has always been a topic in the IC engine industry. Fig. 10 in the left column also plots an enlarged band-pass filtered version of cylinder pressure measured by a pressure sensor in the cylinder head. It shows that the blindly recovered pressure $\hat{s}$ resembles the measured band-pass filtered cylinder pressure at around 10° after TDC in terms of relative magnitude, location and frequency.
Piston slap is of interest for the faulty piston case (increased piston clearance) of a faulty engine. Using 3 measured vibration signals for engine operation at about 1210 rpm, the recovered piston slap is shown on the left of Fig. 11. It is seen that the recovered piston slap is of the same general shape as the recovered fuel injection, but the location is at about 9° of crank angle after TDC. The reason that the piston slap is recovered

Fig. 10. Recovered fuel injection and pressure change for the normal engine: $x$ is the measured signals; $s$ is the recovered fuel injection or pressure change; fuel is the fuel injection indicator; $dP$ is band-pass filtered cylinder pressure (enlarged 100 times).

Fig. 11. Recovered piston slap of faulty engine for the combustion case in the left column and no combustion case in the right column: $x$ is the measured signals; $s$ is the recovered sources; fuel is the fuel injection indicator.

Piston slap is of interest for the faulty piston case (increased piston clearance) of a faulty engine. Using 3 measured vibration signals for engine operation at about 1210 rpm, the recovered piston slap is shown on the left of Fig. 11. It is seen that the recovered piston slap is of the same general shape as the recovered fuel injection, but the location is at about 9° of crank angle after TDC. The reason that the piston slap is recovered
instead of fuel injection is due to the fact that the piston slap effect is more significant here. In fact we checked all of the many sets of the measured experimental data for both a normal and a faulty engine. Only the first set of data of the normal engine, shown in Fig. 10, manifests significant fuel injection effects. We have no explanation for this phenomenon; however, more attention will be paid to it in future engine experiments. Also note that there is a vibration component caused by valve operation which is probably the opening of an inlet valve within 40–50° crank angle for x1 on the top left, which is cancelled as noise in the recovered piston slap.

To further verify that the recovered source is really piston slap, nine work cycles of the measured acceleration data with neither fuel injection nor combustion in that cylinder have been processed. The faulty piston slap is recovered as shown on the right of Fig. 11. The recovered piston slap is similar to the case with combustion on the left, but smaller due to the lack combustion (lower pressure). In addition, the recovered piston slap occurs at a delay of about 11° crank angle which can also be explained by the lower and more smoothly changing pressure in the cylinder and the corresponding slower transverse movement of the piston. As a result, it offers further verification that the recovered source is piston slap.

More separation has been done to verify the robustness of the BLMS algorithm under different engine operation conditions. The left column in Fig. 12 shows the separation result for combustion pressure when the normal engine is run at 1000 rpm. It shows similar results to the right column in Fig. 10, but with smaller magnitudes as the engine is running at a lower speed. The right column in Fig. 12 shows a separation result for piston slap when the faulty engine is running at about 1000 rpm. It shows similar results to the left column in Fig. 11, but with smaller magnitudes as the engine is running at a lower speed. Please note that the word similar here refers to the fact that the results can be interpreted as combustion pressure or piston slap in terms of the recovered source location and its shape.

In summary, each time we attempted to recover one of our sources of interest we have used three measured vibration signals that significantly contain the source effect. It is believed that using three measurements is sufficient to extract one source. In the separation process we have also taken advantage of our knowledge about the engine. For example, the normal engine with little or no piston slap is used to extract the fuel

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**Fig. 12.** Recovered pressure change of the normal engine on the left and piston slap of the faulty engine on the right: x is the measured signals; ȳ is the recovered pressure change or piston slap; dP is band-pass filtered cylinder pressure (magnified 200 times); fuel is the fuel injection indicator.
injection force by using the three measured vibration signals that have significant fuel injection values; the faulty piston with significant piston slap is used to extract piston slap by three measured vibration signals. The time (TDC angle) location of each event is used to confirm what the recovered source is, and even a measurement with no fuel injection and combustion is used to verify the separation results. By this strategy, each separation process has been selected to involve essentially one significant source, either fuel injection or piston slap or cylinder pressure, and other insignificant sources such as valve operation etc are regarded as low level noise. In other words despite the blind nature of the BSS techniques we should make use of a priori knowledge to make it as little blind as possible in practical engineering applications.

5. Further discussion

The above BLMS technique only finds one source for each value of $\alpha$. This can be both an advantage and a disadvantage. The advantage is that it is not necessary to know the exact number of sources present as with many BSS methods. The disadvantage is that if the signals contain more than one source with approximately the same value of kurtosis, the stronger source will be found. Even when measurement points near the injector were selected for the case of the faulty piston in the current study, the piston slap signal was separated because it was dominant. For some unknown reason, in the measured vibration signals in this experiment, the vibration component due to the injector operation was in any case weaker for the faulty piston.

Obtaining sensor responses of each source of interest after recovering the sources such as combustion pressure and piston slap is useful for comparing their contribution to noise. The solution for this is to apply another learning algorithm called deflation which extracts source responses from the mixtures using the recovered source. In fact the deflation method can also be combined into the above BLMS algorithm to successively separate further sources. One deflation algorithm has been proposed by Chio and Cichocki [22] by minimising the energy cost function. Please note that because only a limited number of sources are separated, the identified inverse system $W$ is not a square matrix and hence cannot be inverted directly to obtain the mixing system $H$. As a result, it is not possible to apply Eq. (4) to obtain source responses at each sensor locations.

We have currently investigated the use of the deflation algorithm to complement the BLMS algorithm for this purpose and hope to publish the results in a future paper. Here we briefly describe the basic theory on which this deflation algorithm is based: For independent sources, the energy of their sum equals the sum of the energy in each source. This is expressed in Eqs. (16), (17), where $E$ denotes expectation or average and in (17) we invoke the fact that the cross correlation between two independent signals is zero. This theory can be applied to the BSS mixing model for the reason that scaled or filtered sources are still independent sources:

$$x = \sum_{i=1}^{M} s_i,$$

$$Ex^2 = E \left( \sum_{i=1}^{M} s_i \right)^2 = \sum_{i=1}^{M} Es_i^2.$$  

When analysing or using each of the BSS recovered sources in real world situation, one concern is how to deal with the indeterminacies such as scale factors and permutation? In practical application the permutation indeterminacy refers to the task of determining which source the separated signal is. This is often solved by the waveform and timing of the signal, for example in the analysis for deciding between piston slap and fuel injection. The scale factor indeterminacy cannot be precisely solved, but can be solved in a relative sense. For example, we choose unit total energy for the coefficients in each row of the inverse system $W(z)$, which correspond to each source to be separated. The energy (without $\frac{1}{2}$) is calculated in (18) and the coefficient is normalised in (19) for each iteration step or block, where $p$ is the index corresponding to the filter coefficient at the $p$th lag. By this normalisation method, the recovered sources will have a magnitude that is proportional to the true source magnitude and proportional to the measured sensor signal component corresponding
to that source:

\[ E(i) = \sum_{j=1}^{M} \sum_{p=0}^{L-1} (w_{ij}(p))^2, \]  

\[ w_{ij}(p) = w_{ij}(p)/E(i). \]  

Note that in this academic research example the combustion pressure, the fuel injection and once per revolution tacho signals were available to assist evaluation of the blind separation result. However, in the real world situation these measured signals would not all be available. For example, cylinder pressure sensors are not usually available in the practical working situation. However, attaching vibration accelerometer sensors on the engine casing is much easier to do so that BSS techniques have the appealing potential to recover the above sources from only the measured vibration signals on the engine casing.

6. Conclusion

A BSS method has successfully recovered the original sources for simulated data. Gray’s variable norm used as a measure of the non-Gaussianity of the sources is shown to be a suitable cost function. The BSS algorithm is applied to both normal and faulty internal combustion engine vibration signals and reasonable results were obtained for fuel injection, piston slap and combustion pressure variations. The recovered pressure shows that the part due to combustion and the part due to piston movement are two separate sources of different character. The combustion pressure part is super-Gaussian with high frequency and causes much more noise than the second part. Separation verification has also been carried out under different engine operation conditions and confirms that the stated BLMS algorithm is robust and can offer stable separation results. The BSS result shows that it is appropriate to assume that the diesel engine system is approximately time invariant within a small crank angle range around top dead centre. In addition, the knowledge of the approximate time that each event happens and the different sensor locations have enabled reduction of the number of mixed sources and facilitated the recovery of each specific source of interest. A further development using a deflation method to complement the used BSS algorithm and overcome its main disadvantages is briefly introduced and is the subject of ongoing work. How the BSS method can be applied to practical engineering problems is also briefly described. There is still a lot of research needed to be done in developing the BSS theory, constructing better algorithms and applying them to practical engineering problems. However, the BSS results here have shown promising prospects for applying BSS techniques to solve practical mechanical problems.

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