CHAPTER SIX

Receiver Hardware Considerations

6.1 INTRODUCTION

This chapter discusses the hardware of the receiver. Since the basic design of GPS receiver in this book is software oriented, the hardware presented here is rather simple. The only information needed for a software receiver is the sampled data. These sampled or digitized data will be stored in memory to be processed. For postprocessing the memory size dictates the length of data record. A minimum of 30 seconds of data is needed to find the user position as mentioned in Section 5.5. In real-time processing the memory serves as a buffer between the hardware and the software signal processing. The hardware includes the radio frequency (RF) chain and analog-to-digital converter (ADC). Thus, the signal processing software must be capable of processing the digitized data in the memory at a real-time rate. Under this condition, the size of the memory determines the latency allowable for the signal processing software.

This chapter will include the discussion of the antenna, the RF chain, and the digitizers. Two types of designs will be discussed. One is a single channel to collect real data and the other is an in-phase and quadrature phase (I-Q) channel to collect complex data. In both approaches, the input signals can be either down-converted to a lower intermediate frequency (IF) before digitization or directly digitized at the transmitted frequency. The relation between the sampling frequency and the input frequency will be presented. Some suggestions on the sampling frequency selection will be included. Two hardware setups to collect real data will be discussed in detail as examples. The impact of the number of digitized bits will also be discussed.

A digital band folding technique will be discussed that can alias two or more narrow frequency bands into the baseband. This technique can be used to alias the L1 and L2 bands of the GPS into the baseband, or to alias the GPS L1
frequency and the Russian Global Navigation Satellite System (GLONASS) signals into the baseband. If one desires, all three bands, L1, L2, and the GLONASS, can be aliased into the baseband. With this arrangement the digitized signal will contain the information from all three input bands.

One of the advantages of a software receiver is that the receiver can process data collected with various hardware. For example, the data can be real or complex with various sampling frequencies. A simple program modification in the receiver should be able to use the data. Or the data can be changed from real to complex and complex to real such that the receiver can process them.

6.2 ANTENNA(2–4)

A GPS antenna should cover a wide spatial angle to receive the maximum number of signals. The common requirement is to receive signals from all satellites about 5 degrees above the horizon. Combining satellites at low elevation angles and high elevation angles can produce a low value of geometric dilution of precision (GDOP) as discussed in Section 2.15. A jamming or interfering signal usually comes from a low elevation angle. In order to minimize the interference, sometimes an antenna will have a relatively narrow spatial angle to avoid signals from a low elevation angle. Therefore, in selecting a GPS antenna a trade-off between the maximum number of receiving satellites and interference must be carefully evaluated.

If an antenna has small gain variation from zenith to azimuth, the strength of the received signals will not separate far apart. In a code division multiple access (CDMA) system it is desirable to have comparable signal strength from all the received signals. Otherwise, the strong signals may interfere with the weak ones and make them difficult to detect. Therefore, the antenna should have uniform gain over a very wide spatial angle.

If an antenna is used to receive both the L1 (1575.42 MHz) and the L2 (1227.6 MHz), the antenna can either have a wide bandwidth to cover the entire frequency range or have two narrow bands covering the desired frequency ranges. An antenna with two narrow bands can avoid interference from the signals in between the two bands.

The antenna should also reject or minimize multipath effect. Multipath effect is the GPS signal reflections from some objects that reach the antenna indirectly. Multipath can cause error in the user position calculation. The reflection of a right-handed circular polarized signal is a left-handed polarized signal. A right-handed polarized receiving antenna has higher gain for the signals from the satellites. It has a lower gain for the reflected signals because the polarization is in the opposite direction. In general it is difficult to suppress the multipath because it can come from any direction. If the direction of the reflected signal is known, the antenna can be designed to suppress it. One common multipath is the reflection from the ground below the antenna. This multipath can be reduced because the direction of the incoming signal is known. Therefore, a
6.3 AMPLIFICATION CONSIDERATION

GPS antenna should have a low back lobe. Some techniques such as a specially designed ground plane can be used to minimize the multipath from the ground below. The multipath requirement usually complicates the antenna design and increases its size.

Since the GPS receivers are getting smaller as a result of the advance of integrated circuit technology, it is desirable to have a small antenna. If an antenna is used for airborne applications, its profile is very important because it will be installed on the surface of an aircraft. One common antenna design to receive a circular polarized signal is a spiral antenna, which inherently has a wide bandwidth. Another type of popular design is a microstrip antenna, sometimes also referred to as the patch antenna. If the shape is properly designed and the feed point properly selected, a patch antenna can produce a circular polarized wave. The advantage of the patch antenna is its simplicity and small size.

In some commercial GPS receivers the antenna is an integral part of the receiver unit. Other antennas are integrated with an amplifier. These antennas can be connected to the receiver through a long cable because the amplifier gain can compensate the cable loss. A patch antenna (M/A COM ANP-C-114-5) with an integrated amplifier is used in the data collection system discussed in this chapter. The internal amplifier has a gain of 26 dB with a noise figure of 2.5 dB. The overall size of the antenna including the amplifier is diameter of 3′′ and thickness about 0.75′′. The antenna pattern is measured in an anechoic chamber and the result is shown in Figure 6.1a. Figure 6.1b shows the frequency response of the antenna. The beam of this antenna is rather broad. The gain in the zenith direction is about +3.5 dBiC where ic stands for isotropic circular polarization. The gain at 10 degrees is about −3 dBiC.

6.3 AMPLIFICATION CONSIDERATION \(^{(5,7,10)}\)

In this section the signal level and the required amplification will be discussed. The C/A code signal level at the receiver set should be at least −130 dBm \(^{(5)}\) as discussed in Section 5.2. The available thermal noise power \(N_i\) at the input of a receiver is: \(^{(6)}\)

\[
N_i = kTB \text{ watts} \tag{6.1}
\]

where \(k\) is the Boltzmann’s constant \((= 1.38 \times 10^{-23} \text{ J/K})\) \(T\) is the temperature of resistor \(R\) (\(R\) is not included in the above equation) in Kelvin, \(B\) is the bandwidth of the receiver in hertz, \(N_i\) is the noise power in watts. The thermal noise at room temperature where \(T = 290^\circ\text{K}\) expressed in dBm is

\[
N_i(\text{dBm}) = -174 \text{ dBm/Hz} \text{ or } N_i(\text{dBm}) = -114 \text{ dBm/MHz} \tag{6.2}
\]

If the input to the receiver is an antenna pointing at the sky, the thermal noise is lower than room temperature, such as 50°K.

For the C/A code signal, the null-to-null bandwidth is about 2 (or 2.046)
FIGURE 6.1  Antenna measurements of an M/A COM ANP-C-114-5 antenna.

(a) Spatial pattern.
MHz, thus, the noise floor is at $-111 \text{ dBm} \left( -114 + 10 \log_2 \right)$. Supposing that the GPS signal is at $-130 \text{ dBm}$, the signal is $19 \text{ dB} \left( -130 + 111 \right)$ below the noise floor. One cannot expect to see the signal in the collected data. The amplification needed depends on the analog-to-digital converter (ADC) used to generate the data. A simple rule is to amplify the signal to the maximum range of the ADC. However, this approach should not be applied to the GPS signal, because the signal is below the noise floor. If the signal level is brought to the maximum range of the ADC, the noise will saturate the ADC. Therefore, in this design the noise floor rather than the signal level should be raised close to the maximum range of the ADC.

A personal computer (PC)-based card\(^7\) with two ADCs is used to collect data. This card can operate at a maximum speed of 60 MHz with two 12-bit ADCs. If both ADCs operate simultaneously, the maximum operating speed is 50 MHz. The maximum voltage to exercise all the levels of the ADC is about 100 mv and the corresponding power is:

$$ P = \frac{(0.1)^2}{2 \times 50} = 0.0001 \text{ watt} = 0.1 \text{ mw} = -10 \text{ dBm} \quad (6.3) $$
It is assumed that the system has a characteristic impedance of 50 Ω. A simple way to estimate the gain of the amplifier chain is to amplify the noise floor to this level, thus, a net gain of about 101 dB (−10 + 111) is needed. Since in the RF chain there are filters, mixer, and cable loss, the insertion loss of these components must be compensated with additional gain. The net gain must be very close to the desired value(10) of 101 dB. Too low a gain value will not activate all the possible levels of the ADC. Too high a gain will saturate some components or the ADC and create an adverse effect.

6.4 TWO POSSIBLE ARRANGEMENTS OF DIGITIZATION BY FREQUENCY PLANS(8,9)

Although many possible arrangements can be used to collect digitized GPS signal data, there are two basic approaches according to the frequency plan. One approach is to digitize the input signal at the L1 frequency directly, which can be referred to as direct digitization. The other one is to down-convert the input signal to a lower frequency, called the intermediate frequency (IF), and digitize it. This approach can be referred to as the down-converted approach.

The direct digitization approach has a major advantage; that is, in this design the mixer and local oscillator are not needed. A mixer is a nonlinear device, although in receiver designs it is often treated as a linear device. A mixer usually generates spurious (unwanted) frequencies, which can contaminate the output. A local oscillator can be expensive and any frequency error or impurity produced by the local oscillator will appear in the digitized signal. However, this arrangement does not eliminate the oscillator (or clock) used for the ADC.

The major disadvantage of direct digitization is that the amplifiers used in this approach must operate at high frequency and they can be expensive. The ADC must have an input bandwidth to accommodate the high input frequency. In general, ADC operating at high frequency is difficult to build and has fewer effective bits. The number of effective bits can be considered as the useful bits, which are fewer than the designed number of bits. Usually, the number of effective bits decreases at higher input frequency. In this approach the sampling frequency must be very accurate, which will be discussed in Section 6.15. Another problem is that it is difficult to build a narrow-band filter at a higher frequency, and usually this kind of filter has relatively high insertion loss.

In the down-converted approach the input frequency is converted to an IF, which is usually much lower than the input frequency. It is easy to build a narrow-band filter with low insertion loss and amplifiers at a lower frequency are less expensive. The mixer and the local oscillator must be used and they can be expensive and cause frequency errors.

Both approaches will be discussed in the following sections. Some considerations are common to both designs and these will be discussed first.
6.5 FIRST COMPONENT AFTER THE ANTENNA

The first component following the antenna can be either a filter or an amplifier. If the antenna is integrated with an amplifier, the first component after the antenna is the amplifier. Both arrangements have advantages and disadvantages, which will be discussed in this section.

The noise figure of a receiver can be expressed as:

\[ F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1G_2} + \cdots + \frac{F_N - 1}{G_1G_2\cdots G_N} \]  \hspace{1cm} (6.4)

where \( F_i \) and \( G_i \) (\( i = 1, 2, \ldots, N \)) are the noise figure and gain of each individual component in the RF chain.

If the amplifier is the first component, the noise figure of the receiver is low and is approximately equal to the noise figure of the first amplifier, which can be less than 2 dB. The overall noise figure of the receiver caused by the second component, such as the filter, is reduced by the gain of the amplifier. The potential problem with this approach is that strong signals in the bandwidth of the amplifier may drive it into saturation and generate spurious frequencies.

If the first component is a filter, it can stop out-of-band signals from entering the input of the amplifier. If the filter only passes the C/A band, the bandwidth is around 2 MHz. A filter with 2 MHz bandwidth with a center frequency at 1575.42 MHz is considered high Q. Usually, the insertion loss of such a filter is relatively high, about 2–3 dB, and the filter is bulky. The receiver noise figure with the filter as the first component is about 2–3 dB higher than the previous arrangement. Usually, a GPS receiver without special interfering signals in the neighborhood uses an amplifier as the first component after the antenna to obtain a low noise figure.

6.6 SELECTING SAMPLING FREQUENCY AS A FUNCTION OF THE C/A CODE CHIP RATE

An important factor in selecting the sampling frequency is related to the C/A code chip rate. The C/A code chip rate is 1.023 MHz and the sampling frequency should not be a multiple number of the chip rate. In other words, the sampling should not be synchronized with the C/A code rate. For example, using a sampling frequency of 5.115 MHz (1.023 \( \times \) 5) is not a good choice. With this sampling rate the time between two adjacent samples is 195.5 ns \((1/5.115 \text{ MHz})\). This time resolution is used to measure the beginning of the C/A code. The corresponding distance resolution is 58.65 m \((195.5 \times 3 \times 10^8 \text{ m})\). This distance resolution is too coarse to obtain the desired accuracy of the user position. Finer distance resolution should be obtained from signal processing. With synchronized sampling frequency, it is difficult to obtain fine distance resolution. This phenomenon is illustrated as follows.

Figure 6.2 shows the C/A code chip rate and the sampled data points. Fig-
ures 6.2a and 6.2b show the synchronized and the unsynchronized sampling, respectively. In each figure there are two sets of digitizing points. The lower row is a time-shifted version of the top row.

In Figure 6.2a, the time shift is slightly less than 195.5 ns. These two sets of digitizing data are exactly the same as shown in this figure. This illustrates that shifting time by less than 195.5 ns produces the same output data, if the sampling frequency is synchronized with the C/A code. Since the two digitized data are the same, one cannot detect the time shift. As a result, one cannot derive finer time resolution (or distance) better than 195.5 ns through signal processing.

In Figure 6.2b the sampling frequency is lower than 5.115 MHz; therefore, it is not synchronized with the C/A code. The output data from the time-shifted case are different from the original data as shown in the figure. Under this condition, a finer time resolution can be obtained through signal processing to measure the beginning of the C/A code. This fine time resolution can be converted into finer distance resolution.

As discussed in Chapter 3, the Doppler frequency on the C/A code is about ±6 Hz, which includes the speed of a high-speed aircraft. Therefore, the code frequency should be considered as in the range of $1.023 \times 10^6 \pm 6$ Hz. The sampling frequency should not be a multiple of this range of frequencies. In general, even in the sampling frequency is close to the multiple of this range of frequencies, the time-shifted data can be the same as the original data for a period of time. Under this condition, in order to generate a fine time resolution, a relatively long record of data must be used, which is not desirable.
6.7 SAMPLING FREQUENCY AND BAND ALIASING FOR REAL DATA COLLECTION

If only one ADC is used to collect digitized data from one RF channel, the output data are often referred to as real data (in contrast to complex data). The input signal bandwidth is limited by the sampling frequency. If the sampling frequency is $f_s$, the unambiguous bandwidth is $f_s/2$. As long as the input signal bandwidth is less than $f_s/2$, the information will be maintained and the Nyquist sampling rate will be fulfilled. Although for many low-frequency applications the input signal can be limited to 0 to $f_s/2$, in general, the sampling frequency need not be twice the highest input frequency.

If the input frequency is $f_i$, and the sampling frequency is $f_s$, the input frequency is aliased into the baseband and the output frequency $f_o$ is

$$f_o = f_i - nf_s/2 \quad \text{and} \quad f_o < f_s/2$$

where $n$ is an integer. The relationship between the input and the output frequency is shown in Figure 6.3.

When the input is from $nf_s$ to $(2n + 1)f_s/2$, the frequency is aliased into the baseband in a direct transition mode, which means a lower input frequency translates into a lower output frequency. When the input is from $(2n + 1)f_s/2$ to $(n + 1)f_s$, it is aliased into the baseband in an inverse transition mode, which means a lower input frequency translates into a higher output frequency. Either case can be implemented if the frequency translation is properly monitored.

If the input signal bandwidth is $\Delta f$, it is desirable to have the minimum sampling frequency $f_s$ higher than the Nyquist requirement of $2\Delta f$. Usually, 2.5$\Delta f$ is used because it is impractical to build a filter with very sharp skirt (or a brick wall filter) to limit the out-of-band signals. Thus, for the C/A code the required minimum sampling rate is about 5 MHz. This sampling frequency is adequately separated from the undesirable frequency of 5.115 MHz. The sampling frequency must be properly selected. Figure 6.4a shows the desired frequency aliasing. The input band is placed approximately at the center of the

![Figure 6.3](image-url)  
**FIGURE 6.3** Input versus output frequency of band aliasing.
output band and the input and output bandwidths are equal.

Figure 6.4b shows improper frequency aliasing. In Figure 6.4b, the center frequency of the input signal does not alias to the center of the baseband. The frequency higher than \((2n+1)f_s/2\) and the portion of the frequency lower than \((2n+1)f_s/2\) are aliased on top of each other. Therefore, portion of the output band contains an overlapping spectrum, which is undesirable. When there is a spectrum overlapping in the output, the output bandwidth is narrower than the input bandwidth.

In order to alias the input frequency near the center of the baseband, the following relation must hold,

\[
    f_o = f_i - n(f_s/2) = f_s/4 \quad \text{and} \quad f_s > 2\Delta f
\]

(6.6)

where \(\Delta f\) is the bandwidth of input signal. The first part of this equation is to put the aliasing signal approximately at the center of the output band. The second part states that the Nyquist sampling requirement must hold. If the frequency of the input signal \(f_i\) is known, this equation can be used to find the sampling frequency. Examples will be presented in Sections 6.8 and 6.9.
In this section a down-converted approach to digitize the signal will be discussed. The IF and sampling frequency will be determined, followed by some general discussion. A set of hardware to collect data for user location calculation will be presented.

In this approach the input signal is down converted to an IF, then digitized by an ADC. In Equation (6.6) there are three unknowns: \( n \), \( f_i \), and \( f_s \); therefore, the solutions are not unique. Many possible solutions can be selected to build a receiver. In the hardware design, the sampling frequency of \( f_s = 5 \text{ MHz} \) is selected. From Equation (6.6) \( f_i = \text{IF} = 5n + 1.25 \text{ MHz} \), where \( n \) is an integer. The value of \( n = 4 \) is arbitrarily selected and the corresponding \( \text{IF} = 21.25 \text{ MHz} \), which can be digitized by an ADC.

Of course, one can choose \( n = 0 \) and down convert the input frequency to 1.25 MHz directly. In this approach the mixer generates more spurious frequencies. The input signal is down-converted to from 0.25 to 2.25 MHz, which covers more than an octave bandwidth. An octave bandwidth means that the highest frequency in the band is equal to twice the lowest frequency in the band. A common practice in receiver design is to keep the IF bandwidth under an octave to avoid generation of in-band second harmonics.

There are many different ways to build an RF front end. The two important factors are the total gain and filter installations. Filters can be used to reject out-of-band signals and limit the noise bandwidth, but they add insertion loss. If multiple channels are used, such as in the I-Q channels, filters may increase the difficulty of amplitude and phase balancing. The locations of filters in a receiver affect the performance of the RF front end.

The personal computer–based ADC card discussed in Section 6.3 is used as the ADC. It requires about 100 mv input voltage or \(-10 \text{ dBm}\) to activate all the bits. A net gain of 101 dB is required to achieve this level. If a digital scope is used as the ADC\(^{(8,9)}\) because of the built-in amplifiers in the scope, it can digitize a rather weak signal. In this kind of arrangement, only about 90 dB gain is used.

Two RF front-end arrangements are shown in Figure 6.5. The major difference between Figures 6.5a and b is in the amplifiers. In Figure 6.5a amplifiers 2, 3, and 4 operate at IF, which costs less than amplifiers operating at RF. Filter 1 is used to limit the input bandwidth. Filter 2 is used to limit the spurious frequencies generated by the mixer, and filter 3 is used to limit noise generated by the three amplifiers. Although Figure 6.5a is the preferred approach, in actual laboratory experiments Figure 6.5b is used because of the availability of amplifiers.

In Figure 6.5b, the M/A COM ANP-C-114-5 antenna with amplifier is used. Amplifier 1 is an integrated part of the antenna with a 26 dB gain and a 2.5 dB noise figure. The bias \( T \) is used to supply 5-volt dc to the amplifier at the antenna. Filter 1 is centered at 1575.42 MHz with a 3 dB bandwidth of 3.4 MHz,
which is wider than the desired value of 2 MHz. Amplifiers 2 and 3 provide a total of 60 dB gain. The frequency of the local oscillator is at 1554.17 MHz. The mixer-down converts the input frequency from 1575.42 to 21.25 MHz. In this frequency conversion, high input frequency transforms to high output frequency. The attenuator placed between the mixer and the oscillator is used to improve impedance matching and it reduces the power to the mixer. After the mixer an IF amplifier with 24 dB of gain is used to further amplify the signal. Finally, filter 2 is used to reject spurious frequencies generated by the mixer and limit the noise bandwidth. Filter 2 has a center frequency of 21.25 MHz and bandwidth of 2 MHz. If filter 2 is not used all the noise will alias into the output band and be digitized by the ADC as shown in Figure 6.3. The overall gain from the four amplifiers is 110 dB (26 + 30 + 30 + 24). Subtracting the insertion losses from the filters, bias T, and mixer, the gain is slightly over 100 dB. There is no filter after the mixer because it is not available.
6.9 DIRECT DIGITIZATION FOR REAL DATA COLLECTION

Direct digitization at RF is a straightforward approach. The only components required are amplifiers and two filters. The amplifiers must provide the desired RF gain. One filter is used after the first amplifier to limit out-of-band signal and the second filter is placed in front of the ADC to limit the noise bandwidth. A direct digitization arrangement is shown in Figure 6.6. In this arrangement the second filter is very important. Without this filter the noise in the collected data can be very high and it will affect signal detection.

In the direct sampling case, the frequency of the input signal is fixed; one must find the correct sampling frequency \( f_s \) to avoid band overlapping in the output. In this approach there are two unknowns: \( f_s \) and \( n \), in Equation (6.6). An exact solution is somewhat difficult to obtain. However, the problem can be easily solved if the approximate sampling frequency is known.

Let us use an example to illustrate the operation. In this example, the input GPS L1 signal is at 1575.42 MHz, and the sampling frequency is about 5 MHz. First use Equation (6.5) with \( f_o/ = 1.25 \text{ MHz} \), \( f_i/ = 1575.42 \text{ MHz} \), and \( f_s/ = 5 \text{ MHz} \) to find \( n = 629.66 \). Round off \( n = 630 \) and use \( f_o = f_s/4 \) in Equation (6.6). The result is \( f_s = 5.009 \text{ MHz} \) and the center is aliased to 1.252 MHz.

In one arrangement a scope is used to collect digitized data because the personal computer–based ADC card cannot accommodate the frequency of the input signal. The scope has a specified bandwidth of dc-1000 MHz, but it can digitize a signal at 1600 MHz with less sensitivity. The scope can operate at 5 MHz, but the sampling frequency cannot be fine-tuned. If 5 MHz is used to sample the input frequency, the center frequency will be aliased to 420 KHz. Since the bandwidth of the C/A code is 2 MHz, there is band overlapping with the center frequency at 420 KHz as shown in Figure 6.7.

Actual GPS data were collected through this arrangement. Although there is band overlapping, the data could still be processed, because the overlapping range is close to the edge of the signal where the spectrum density is low. In this arrangement, the overall amplification is reduced because the scope can digitize weak signals.

In another arrangement, an experimental ADC built by TRW is used. The ADC can sample only between approximately 80 to 120 MHz, limited by the circuit around the ADC. In order to obtain digitized data at 5 MHz, the output

![FIGURE 6.6](image-url)
from the ADC is decimated. For example, if the ADC operates at 100 MHz and one data point is kept out of every 20 data points, the equivalent sampling rate is 5 MHz. The actual sampling frequency is selected to be 5.161 MHz; the input signal is aliased to 1.315 MHz, which is close to the center of the output band at about 1.29 (5.161/4) MHz.

With today’s technology, it is easier to build a down-converted approach, but the direct digitization is attractive for its simplicity. There is another advantage for direct digitization, which is to alias more than one desired signal into the baseband. This approach will be discussed in Section 6.11.

6.10 IN-PHASE (I) AND QUADRANT-PHASE (Q) DOWN CONVERSION(10)

In many commercial GPS receivers, the input signal is down converted into I-Q channels. The data collected through this approach are complex and the two sets of data are often referred to as real and imaginary. Since there are two channels, the Nyquist sampling is $f_s = \Delta f$. A common practice is to choose $f_s > 1.25\Delta f$ to accommodate the skirt of the filter. The relation between the input and the output frequencies is

$$ f_o = f_i - nf_s \quad \text{and} \quad f_o < f_s $$

(6.7)

where $n$ is a positive integer. The relation between the input and output band is shown in Figure 6.8. In the I-Q channel digitization, as long as $\Delta f < f_s$ there is no spectrum overlapping in the output baseband.
A common frequency selection is shown in Figure 6.8. The center of the output frequency is placed at zero. This approach usually can be achieved only through a down-converted design, and the input frequency \( f_i \) is usually set to zero or to a multiple of the sampling frequency \( f_s \). In this arrangement, the input frequency is divided into two equal bands. The lower input frequency is aliased to a higher frequency in the baseband and the higher input frequency is aliased to a lower frequency as shown in Figure 6.8. This phenomenon affects the data conversion procedure discussed in Section 6.14.

There are two ways to build an I-Q down converter as shown in Figure 6.9. In Figure 6.9a, a 90-degree phase shift is introduced in the input circuit. In Figure 6.9b, the 90-degree phase shift is introduced in the oscillator circuit. Both approaches are popularly used. If one wants the output frequency to be zero, the local oscillator is often selected as the input signal or at 1575.42 MHz. For a wideband receiver the I-Q approach can double the input bandwidth with the same sampling frequency. Since the GPS receiver bandwidth is relatively narrow, this approach is not needed to improve the bandwidth. This approach uses more hardware because one additional channel is required. The amplitude and phase of the two outputs are difficult to balance accurately. From the software receiver point of view, there is no obvious advantage of using an I-Q channel down converter.

Actual complex data with zero center frequency have been collected. Since the acquisition and tracking programs used in this book can process only real data, the complex data are converted into real data through software. The details will be presented in Section 6.14.

6.11 ALIASING TWO OR MORE INPUT BANDS INTO A BASEBAND

If one desires to receive signals from two separate bands, the straightforward way is to use two mixers and two local oscillators to covert the two input bands
into desired IF ranges such as adjacent bands, combine, and digitize them. If direct digitization is used and the correct sampling frequency is selected, two input bands can be aliased into a desired output band. Figure 6.10 shows the arrangement of aliasing two input bands into the baseband for a real data collection system.

The aliased signals in the baseband can be either overlapped or separated. In Figure 6.10 the two signals in the baseband are separated. Separated bands have better signal-to-noise ratio because the noise in the two bands is separated. Separated spectra occupy a wider frequency range and require a higher sampling rate. The overlapped bands have lower signal-to-noise ratio because the noise of two bands is added together. Overlapped spectra occupy a narrower bandwidth and require a lower sampling rate. The aliasing scheme can be used to fold more than two input bands together and it also applies to complex data collection. Before the input bands can be folded together, analog filters must be used to properly filter the desired input bands.

![FIGURE 6.9 I and Q down converter.](image1)

![FIGURE 6.10 Aliasing two input bands to baseband.](image2)
Let us use three examples to illustrate the band aliasing idea. In the first example the two P code channels from both L1 and L2 frequencies are aliased into two separated bands in the baseband. Since the P code has a bandwidth of approximately 20 MHz, two P code bands will occupy 40 MHz. The minimum sampling frequency to cover these bands is about 100 MHz \((40 \times 2.5)\), if 2.5 rather than the Nyquist sampling rate of 2 is used as the minimum required sampling rate. The two input frequency ranges are 1565.42–1585.42 MHz and 1217.6–1237.6 MHz for L1 and L2 bands, respectively. It is tedious to solve the desired sampling frequency. It is easier to solve the sampling frequency \(f_s\) through trial and error. The output frequency can be obtained from Equation (6.5) by increasing the sampling frequency in 100 KHz steps starting from 100 MHz. When the two output frequency ranges are properly aliased into the baseband, the sampling frequency is the desired one. There are many sampling frequencies that can fulfill this requirement. One of the lower sampling frequencies is arbitrarily selected, such as \(f_s = 107.8\) MHz. With this sampling frequency, the L1 band is aliased to 2.32–22.32 MHz, and the L2 band is aliased to 31.8–51.8 MHz. These two bands are not overlapped and they are within the baseband of 0–53.9 \((f_s/2)\) MHz. Figure 6.11 shows such an arrangement.

In the second example, the same two bands are allowed to partially overlap after they are aliased into the baseband. The sampling frequency can be found through the same approach. The output bandwidth can be as narrow as 20 MHz, when the input bands are totally overlapped. Therefore, the minimum sampling frequency is about 50 MHz, if the sampling frequency \(f_s = 57.8\) MHz, the L1 frequency aliases to 3.8–23.8 MHz and L2 aliases to 4.82–24.82 MHz and they are partially overlapped. The output bandwidth is from 0 to 28.9 \((f_s/2)\) MHz.

The third example is to alias the C/A band of the GPS signal and Russia’s GLONASS signal into separate bands in the baseband. The GLONASS is Russia’s standard position system, which is equivalent to the GPS system of the United States. The GLONASS uses bi-phase coded signals with 24 channels at frequencies \(1602 + 0.5625n\) where \(n\) is an integer representing the channel number. A future plan to revise the frequency channels used will eliminate a number of the upper channels. Therefore, only the 1–12 channels will be considered. The center frequency of these 12 channels is at 1605.656 (1602 +
0.5625 \times 6.5) \text{ MHz}. The total bandwidth is 7.3125 \text{ MHz}. For simplicity let us use a 7.5 \text{ MHz} bandwidth, the signal frequency is approximately 1601.9–1609.4 \text{ MHz}. Including the C/A code of the GPS signal, the overall bandwidth is about 9.5 \text{ MHz}. The minimum sampling frequency can be 23.75 (9.5 \times 2.5) \text{ MHz}. If \( f_s = 35.1 \text{ MHz} \), the GPS channel is aliased to 12.47–14.47 \text{ MHz}, the GLONASS to 4.8–12.35 \text{ MHz}, and both signals are within the 0 to 17.55 (\( f_s/2 \)) \text{ MHz} baseband. Hardware has been built to test this idea. The collected data contain both the GPS and the GLONASS signals.

6.12 QUANTIZATION LEVELS\(^{(11–13)}\)

As discussed in Section 5.3, GPS is a CDMA signal. In order to receive the maximum of signals, it is desirable to have comparable signal strength from all visible satellites at the receiver. Under this condition, the dynamic range of a GPS receiver need not be very high. An ADC with a few bits is relatively easy to fabricate and may operate at high frequency. Another advantage of using fewer bits is that it is easier to process the digitized data, especially when they are processed through hardware. The disadvantage of using fewer bits is the degradation of the signal-to-noise ratio. Spilker\(^{(11)}\) indicated that a 1-bit ADC degrades the signal-to-noise ratio by 1.96 dB and a 2-bit ADC degrades the signal-to-noise by 0.55 dB. Many commercial GPS receivers use only 1- or 2-bit ADCs.

Chang\(^{(12)}\) claims that the degradation due to the number of bits of the ADC is a function of input signal-to-noise ratio and sampling frequency. Low signal-to-noise ratio signal sampled at a higher frequency causes less degradation in a receiver. The GPS signal should belong to the low signal-to-noise ratio because the signal is below the noise. At a Nyquist sampling rate, the minimum degradation is about 3.01 and 0.72 dB for 1- and 2-bit quantizers, respectively. At five times the Nyquist sampling rate, the minimum degradation is 2.18 and .60 dB for 1- and 2-bit quantizers, respectively. These values are slightly higher than the results in reference 11.

The only time that a high number of bits in ADC is required in a GPS receiver is to build a receiver with antijamming capability. Usually, the jamming signal is much stronger than the desired GPS signals. An ADC with a small number of bits will be easily saturated by the jamming signal. Under this condition, the GPS signals might be masked by the jamming signal and the receiver cannot detect the desired signals. If an ADC with a large number of bits is used, the dynamic range of the receiver is high. Under this condition, the jamming signal can still disturb the operation; however, the weak GPS signals are preserved in the digitized data. If proper digital signal processing is applied, the GPS signals should be recovered. This problem can be considered in the frequency domain. Assume that there are two signals, a strong one and a weak one, and they are close in frequency. In order to receive both signals, the receiver must have enough instantaneous dynamic range, which is defined as
the capability to receive strong and weak signals simultaneously. If the ADC
does not have enough dynamic range, the weak signal may not be received.
Reference 12 provides more information on this subject.

6.13 HILBERT TRANSFORM

In this book a single channel is used to collect data and the software is written
to process real data. If a software receiver is designed to process complex data
and only real data are available, the real data can be changed to complex data
through the Hilbert transform. A detailed discussion on the Hilbert transform
will not be included. Only the procedure will be presented here.

First the Hilbert transform from Matlab will be presented. The approach is
through discrete Fourier transform (DFT) or fast Fourier transform (FFT). The
following steps are taken:

1. The DFT result can be written as:

\[
X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}
\]

(6.8)

where \(x(n)\) is the input data, \(X(k)\) is the output frequency components, \(k = 0, 1, 2, \ldots, N-1\), and \(n = 0, 1, 2, \ldots, N-1\). Since the input data are
real, the frequency components have the following properties:

\[
X(k) = X(N-k)^* \quad \text{for} \quad k = 1 \sim \frac{N}{2} - 1
\]

(6.9)

where * represents complex conjugate. If the input data are complex the
relationship in this equation does not exist.

2. Find a new set of frequency components \(X_1(k)\). They have the following
values:

\[
X_1(0) = \frac{1}{2} X(0)
\]

\[
X_1(k) = X(k) \quad \text{for} \quad k = 1 \sim \frac{N}{2} - 1
\]

\[
X_1 \left( \frac{N}{2} \right) = \frac{1}{2} X \left( \frac{N}{2} \right)
\]

\[
X_1(k) = 0 \quad \text{for} \quad k = \frac{N}{2} + 1 \sim N - 1
\]

(6.10)
These new frequency components also have $N$ values from $k = 0$ to $N - 1$.

3. The new data $x_1(n)$ in time domain can be obtained from the inverse DFT of the $X_1(k)$ as:

$$x_1(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_1(k)e^{j\frac{2\pi nk}{N}} \quad (6.11)$$

From this approach, if there are $N$ points of real input data, the result will be $N$ points of complex data. Obviously, additional information is generated through this operation. This is caused by padding the $X_1(k)$ values with zeros as shown in Equation (6.10). Padding with zeros in the frequency domain is equivalent to interpolating in the time domain.\(^{(10)}\)

The above method generates $N$ points of complex data from $N$ points of real data. The new data may increase the processing load without gaining significant receiver performance improvement. Therefore, another approach is presented, which is similar to the above Matlab approach, but generates only $N/2$ points of complex data. In taking real digitized data the sampling frequency $f_s \approx 2.5$ $\Delta f$ is used and the input signal is aliased close to the center of the baseband. Under this condition, the frequency component $X(N/2)$ should be very small. The following steps can be taken to obtain complex data:

1. The first step is the same as step 1 (Equation 6.8) in the Matlab approach to take the FFT of the input signal.

2. The new $X_1(k)$ can be obtained as

$$X_1(k) = X(k) \quad \text{for} \quad k = 0, 1, 2, \ldots, \frac{N}{2} - 1 \quad (6.12)$$

Therefore, only half of the frequency components are kept.

3. The new data in time domain can be obtained as

$$x_1(n) = \frac{2}{N} \sum_{k=0}^{N-1} X_1(k)e^{j\frac{4\pi nk}{N}} \quad (6.13)$$

The final results are $N/2$ points of complex data in the time domain and they contain the same information as the $N$ points of real data. These data cover the same length of time; therefore, the equivalent sampling rate of the complex data is $f_{s1} = f_s/2$. The argument is reasonable because for complex data the Nyquist sampling rate is $f_{s1} = \Delta f$.\(^{(9)}\)
6.14 CHANGE FROM COMPLEX TO REAL DATA

In this section changing complex data to real data will be discussed. The approach basically reverses the operation in Section 6.13. However, the IF of the down conversion is very important in this operation. The detail operation depends on this frequency. One of the common I-Q converter designs is to make the IF at zero frequency as shown in Figure 6.8. Under this condition, the center frequency of the input signal is determined by the Doppler shift. For this arrangement the following steps can be taken:

1. Take the DFT of $x(n)$ to generate $X(k)$ as shown in Equation (6.8),

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi nk/N}$$ (6.14)

where $x(n)$ is complex, $k = 0, 1, 2, \ldots, N-1$, and $n = 0, 1, 2, \ldots, N-1$.

2. Generate a new set of frequency components $X_1(k)$ from $X(k)$ as

$$X_1(k) = X\left(\frac{N}{2} + k\right) \quad \text{for} \quad k = 0 \sim \frac{N}{2} - 1$$

$$X_1(k) = X\left(-\frac{N}{2} + k\right) \quad \text{for} \quad k = \frac{N}{2} \sim N - 1$$ (6.15)

In Figure 6.8, the lower input frequency is converted into a higher output frequency and the higher frequency is converted into a lower frequency. This operation puts the two separate bands in Figure 6.8 into the correct frequency range as the input signals. Or one can consider that it shifts the center of the input signal from zero to $f_s/2$. If the IF is not at zero frequency a different shift is required. If the IF of the I-Q channels is at $f_s/2$, no shift is required and this step can be omitted, because the input signal will not split into two separate bands.

3. Generate additional frequency components for $X_1(k)$ as

$$X_1(N) = 0$$

$$X_1(N + k) = X_1(N - k)^* \quad \text{for} \quad k = 1 \sim N - 1$$ (6.16)

Including the results from Equation (6.15) there are total $2N$ frequency components from $k = 0 \sim 2N - 1$.

4. The final step is to find the new data in the time domain through inverse FFT as
The $N$ points of complex data generate $2N$ points of real data and they contain the same amount of information. These $2N$ points cover the same time period; therefore, the equivalent sampling frequency is doubled or $f_{s1} = 2f_s$.

Actual complex data are collected from satellites with I-Q channels of zero IF from Xetron Corporation. The sampling frequency is 3.2 MHz and the Nyquist bandwidth is also 3.2 MHz. The operations from Equations (6.14) to (6.17) are used to change these data to real data with IF = 1.6 MHz. The number of real data is double the number of complex data and the equivalent sampling frequency is 6.4 MHz. These data are processed and the user position has been found.

### 6.15 EFFECT OF SAMPLING FREQUENCY ACCURACY

Although the sampling frequency discussed in this chapter is given a certain mathematical value, the actual frequency used in the laboratory usually has limited accuracy. The effect of this inaccuracy will be discussed as follows.

The first impact to be discussed is on the center frequency of the digitized signal. For the down-converted approach, the sampling frequency inaccuracy causes a small error in the output frequency. For example, if the IF is at 21.25 MHz and the sampling frequency is at 5 MHz, the digitized output should be at 1.25 MHz. This value can be found from Equation (6.5) and the corresponding $n = 4$. If the true sampling frequency $f_s = 5,000,100$ Hz, there is an error frequency of 100 Hz. The center frequency of the digitized signal is at 1,249,600 Hz. The error frequency is 400 Hz, which is four times the error in the sampling frequency because in this case $n = 4$. This frequency error will affect the search range of the acquisition procedure.

For a direct digitization system, the error in the sampling frequency will create a larger error in the output frequency. As discussed in Section 6.9, if the sampling frequency $f_s = 5,161,000$ Hz, with the input signal at 1575.42 MHz, the output will be 1.315 MHz with $n = 305$ from Equation (6.5). If $f_s = 5,161,100$ Hz, which is off by 100 Hz from the desired value, the input will be aliased to 1,284,500 Hz, which is off by 30,500 Hz because $n = 305$. The frequency error will have a severe impact on the acquisition procedure. Therefore, for the direct digitization approach the accuracy of the sampling frequency is rather important.

The second impact of inaccurate sampling frequency is on the processing of the signal. In a software receiver, both the acquisition and tracking programs
take the sampling frequency as input. If the actual sampling frequency is off by too much the acquisition program might not cover the anticipated frequency range and would not find the signal. For a small error in sampling frequency, it will not have a significant effect on the acquisition and the tracking programs. For example, the sampling frequency of 3.2 MHz used to collect the complex data must be off slightly because all the Doppler frequencies calculated are of one sign. If the correct sampling frequency is used in the program, the Doppler frequency should have both positive and negative values because the receiving antenna is stationary. From these experimental results, no obvious adverse effect on the acquisition and tracking is discovered due to the slight inaccuracy of the sampling frequency.

The most important effect of sampling frequency inaccuracy may be the pseudorange measured. The differential pseudorange is measured by sampling time, which will be discussed in Section 9.9. If the sampling frequency is not accurate, the sampling time will be off. The inaccuracy in the pseudorange will affect the accuracy of the user position measured.

6.16 SUMMARY

This chapter discusses the front end of a GPS receiver. The antenna should have a broad beam to receive signals from the zenith to the horizon. It should be right handed circularly polarized to reduce reflected signals. The overall gain of the amplifier chain depends on the input voltage of the ADC. Usually the overall gain is about 100 dB. The input signal can either be down converted and then digitized or directly digitized without frequency translation. Although direct digitization seems to have some advantages, with today’s technology a down conversion approach is simpler to build. It appears that I-Q channel down conversion does not have much advantage over a single channel conversion for a software GPS receiver. Direct digitization can be used to alias several narrow input signals into the same baseband. Several experimental setups to collect data are presented. The number of quantization bits is discussed. One or two bits may be enough for GPS application with degradation of receiver sensitivity. If antijamming is of concern, a large number of bits needed. The conversion of data from real to complex and from complex to real are discussed. Finally, the impact of sampling frequency accuracy is discussed.

REFERENCES


