Fundamentals of Global Positioning System Receivers: A Software Approach
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CHAPTER FIVE

GPS C/A Code Signal Structure

5.1 INTRODUCTION(1,2)

In the previous chapters user positions are calculated. In order to perform the user position calculation, the positions of the satellites and pseudoranges to the satellites must be measured. Many parameters are required to calculate the positions of the satellites and they are transmitted in the satellite signals.

This chapter provides the details associated with the GPS signals. Spilker(1,2) not only gives a very good discussion on the signal, it also gives the reasons these signals are selected. The discussion in this chapter is limited to the fundamentals of the signals, such that a receiver design can be based on the signals.

There are basically two types of signals: the coarse (or clear)/acquisition (C/A) and the precision (P) codes. The actual P code is not directly transmitted by the satellite, but it is modified by a Y code, which is often referred to as the P(Y) code. The P(Y) code is not available to civilian users and is primarily used by the military. In other words, the P(Y) code is classified. The P(Y) code has similar properties of the P code. In order to receive the P(Y) code, one must have the classified code. Therefore, only the fundamentals of the P code will be mentioned in this book. The discussion will be focused on the C/A code. In general, in order to acquire the P(Y) code, the C/A code is usually acquired first. However, in some applications it is desirable to acquire the P(Y) code directly, which is known as direct Y acquisition.

The radio frequency (RF) of the C/A code will be presented first, then the C/A code. The generation of the C/A code and its properties will be presented because they are related closely to acquiring and tracking the GPS signals. Finally, the data carried by the signals will be presented. The applications of the data will be briefly discussed.

5.2 TRANSMITTING FREQUENCY(1–4)

The GPS signal contains two frequency components: link 1 (L1) and link 2 (L2). The center frequency of L1 is at 1575.42 MHz and L2 is at 1227.6 MHz. These frequencies are coherent with a 10.23 MHz clock. These two frequencies can be related to the clock frequency as

$$\begin{aligned} \text{L1} &= 1575.42 \text{ MHz} = 154 \times 10.23 \text{ MHz} \\ \text{L2} &= 1227.6 \text{ MHz} = 120 \times 10.23 \text{ MHz} \end{aligned}$$

These frequencies are very accurate as their reference is an atomic frequency standard. When the clock frequency is generated, it is slightly lower than 10.23 MHz to take the relativistic effect into consideration. The reference frequency is off by(3) -4.567×10^{-3} Hz, which corresponds to a fraction of -4.4647×10^{-10} $(-4.567 \times 10^{-3}/10.23 \times 10^6)$. Therefore, the reference frequency used by the satellite is 10.229999995433 MHz $(10.23 \times 10^6 - 4.567 \times 10^{-3})$ rather than 10.23 MHz. When a GPS receiver receives the signals, they are at the desired frequencies. However, the satellite and receiver motions can produce a Doppler effect as discussed in Section 3.5. The Doppler frequency shift produced by the satellite motion at L1 frequency is approximately ±5 KHz.

The signal structure of the satellite may be modified in the future. However, at the present time, the L1 frequency contains the C/A and P(Y) signals, while the L2 frequency contains only the P(Y) signal. The C/A and P(Y) signals in the L1 frequency are in quadrant phase of each other and they can be written as:

$$S_{L1} = A_p P(t) D(t) \cos(2\pi f_1 t + \phi) + A_c C(t) D(t) \sin(2\pi f_1 t + \phi) \tag{5.1}$$

where S_{L1} is the signal at L1 frequency, A_p is the amplitude of the P code, $P(t) = \pm 1$ represents the phase of the P code, $D(t) = \pm 1$ represents the data code, f_1 is the L1 frequency, ϕ is the initial phase, A_c is the amplitude of the C/A code, $C(t) = \pm 1$ represents the phase of the C/A code. These terms will be further discussed in the following sections. In this equation the P code is used instead of the P(Y) code. The P(Y), C/A, and the carrier frequencies are all phase locked together.

The minimum power levels of the signals must fulfill the values listed in Table 5.1 at the receiver. These power levels are very weak and the spectrum is spread, therefore they cannot be directly observed from a spectrum analyzer. Even when the signal is amplified to a reasonable power level, the spectrum of the C/A code cannot be observed because the noise is stronger than the signal.

As discussed in Section 3.3, the received power levels at various points on the earth are different. The maximum difference is about 2.1 dB between a point just under the satellite and a point tangential to the surface of the earth. In order to generate a uniform power over the surface of the earth, the main beam pattern

TABLE 5.1 Power Level of GPS Signals

	P	C/A
L1	−133 dBm	−130 dBm
L2	−136 dBm	−136 dBm*

*Presently not in L2 frequency.

of the transmitting antenna is slightly weaker at the center to compensate for the user at the edge of the beam. The resulting power level versus elevation angle is shown in Figure 3.10. The maximum power is −128 dBm, which occurs at about 40 degrees. Of course, the receiving antenna pattern also contributes to the power level of the receiver. Usually the receiving antenna has a higher gain in the zenith direction. This incorporates the ability of attenuating multipath but loses gain to signals from lower elevation angles. As discussed in Sections 3.3 and 3.10, the minimum required beam width of the transmitting antenna to cover the earth is 13.87 degrees. The beam width of the antenna(2) is 21.3 degrees, which is wider than needed to cover the earth as shown in Figure 5.1.

If the user is in an aircraft, as long as it is in the main beam of the GPS signal and not in the shadow of the earth it can receive the signal. The signals generated by the satellite transmitting antenna are right-hand polarized. There-

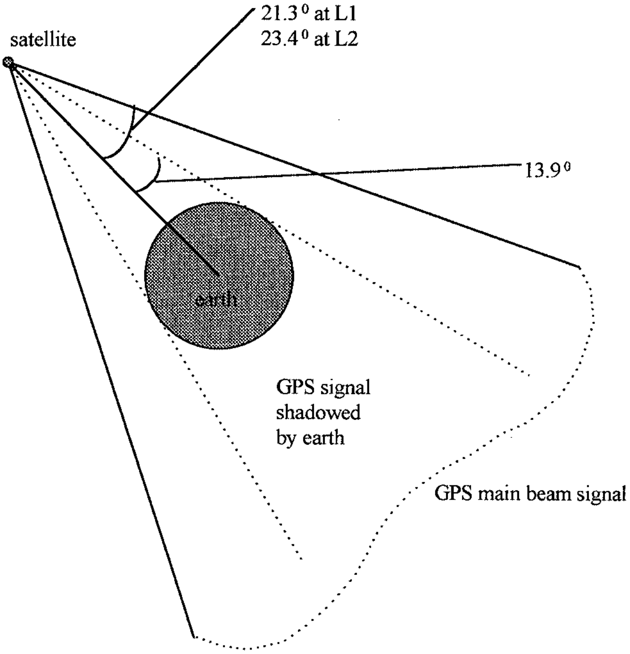


FIGURE 5.1 GPS signal main beam.

fore, the receiver antenna should be right-hand polarized to achieve maximum efficiency.

5.3 CODE DIVISION-MULTIPLE ACCESS (CDMA) SIGNALS

A signal S can be written in the following form:

$$S = A \sin(2\pi f t + \phi) \tag{5.2}$$

where A is the amplitude, f is the frequency, ϕ is the initial phase. These three parameters can be modulated to carry information. If A is modulated, it is referred to as amplitude modulation. If f is modulated, it is frequency modulation. If ϕ is modulated, it is phase modulation.

The GPS signal is a phase-modulated signal with $\phi = \sigma, \pi$; this type of phase modulation is referred to as bi-phase shift keying (BPSK). The phase change rate is often referred to as the chip rate. The spectrum shape can be described by the sinc function ($\sin x/x$) with the spectrum width proportional to the chip rate. For example, if the chip rate is 1 MHz, the main lobe of the spectrum has a null-to-null width of 2 MHz. Therefore, this type of signal is also referred to as a spread-spectrum signal. If the modulation code is a digital sequence with a frequency higher than the data rate, the system can be called a direct-sequence modulated system.

A code division multiple access (CDMA) signal in general is a spread-spectrum system. All the signals in the system use the same center frequency. The signals are modulated by a set of orthogonal (or near-orthogonal) codes. In order to acquire an individual signal, the code of that signal must be used to correlate with the received signal. The GPS signal is CDMA using direct sequence to bi-phase modulate the carrier frequency. Since the CDMA signals all use the same carrier frequency, there is a possibility that the signals will interfere with one another. This effect will be more prominent when strong and weak signals are mixed together. In order to avoid the interference, all the signals should have approximately the same power levels at the receiver. Sometimes in the acquisition one finds that a cross-correlation peak of a strong signal is stronger than the desired peak of a weak signal. Under this condition, the receiver may obtain wrong information.

5.4 P CODE(1,2)

The P code is bi-phase modulated at 10.23 MHz; therefore, the main lobe of the spectrum is 20.46 MHz wide from null to null. The chip length is about 97.8 ns (1/10.23 MHz). The code is generated from two pseudorandom noise (PRN) codes with the same chip rate. One PRN sequence has 15,345,000 chips, which has a period of 1.5 seconds, the other one has 15,345,037 chips, and the difference is 37 chips. The two numbers, 15,345,000 and 15,345,037, are relative prime, which means there are no common factors between them. Therefore, the code length generated by these two codes is 23,017,555.5 (1.5 × 15,345,037) seconds, which is slightly longer than 38 weeks. However, the actual length of the P code is 1 week as the code is reset every week. This 38-week-long code can be divided into 37 different P codes and each satellite can use a different portion of the code. There are a total of 32 satellite identification numbers although only 24 of them are in the orbit. Five of the P code signals (33–37) are reserved for other uses such as ground transmission. In order to perform acquisition on the signal, the time of the week must be known very accurately. Usually this time is found from the C/A code signal that will be discussed in the next section. The navigation data rate carried by the P code through phase modulation is at a 50 Hz rate.

5.5 C/A CODE AND DATA FORMAT(1,2,5)

The C/A code is a bi-phase modulated signal with a chip rate of 1.023 MHz. Therefore, the null-to-null bandwidth of the main lobe of the spectrum is 2.046 MHz. Each chip is about 977.5 ns (1/1.023 MHz) long. The transmitting bandwidth of the GPS satellite in the L1 frequency is approximately 20 MHz to accommodate the P code signal; therefore, the C/A code transmitted contains the main lobe and several sidelobes. The total code period contains 1,023 chips. With a chip rate of 1.023 MHz, 1,023 chips last 1 ms; therefore, the C/A code is 1 ms long. This code repeats itself every millisecond. The spectrum of a C/A code is shown in Figure 5.2.

In order to find the beginning of a C/A code in the received signal only a very limited data record is needed such as 1 ms. If there is no Doppler effect on the received signal, then one millimeter of data contains all the 1,023 chips. Different C/A codes are used for different satellites. The C/A code belongs to the family of Gold codes,(5) which will be discussed in the next section.

Figure 5.3 shows the GPS data format. The first row shows a C/A code with 1,023 chips; the total length is 1 ms. The second row shows a navigation data bit that has a data rate of 50 Hz; thus, a data bit is 20 ms long and contains 20 C/A codes. Thirty data bits make a word that is 600 ms long as shown in the third row. Ten words make a subframe that is 6 seconds long as shown in row four. The fifth row shows a page that is 30 seconds long and contains 5 subframes. Twenty-five pages make a complete data set that is 12.5 minutes long as shown in the sixth row. The 25 pages of data can be referred to as a superframe.

The parameters mentioned in Section 4.10 are contained in the first three subframes of a page. If one can receive the information of these three subframes from four or more satellites, the user location can be found. Theoretically, one can take a minimum of about 18 seconds of data from four satellites and be able to calculate the user position. However, the subframes from each satellite

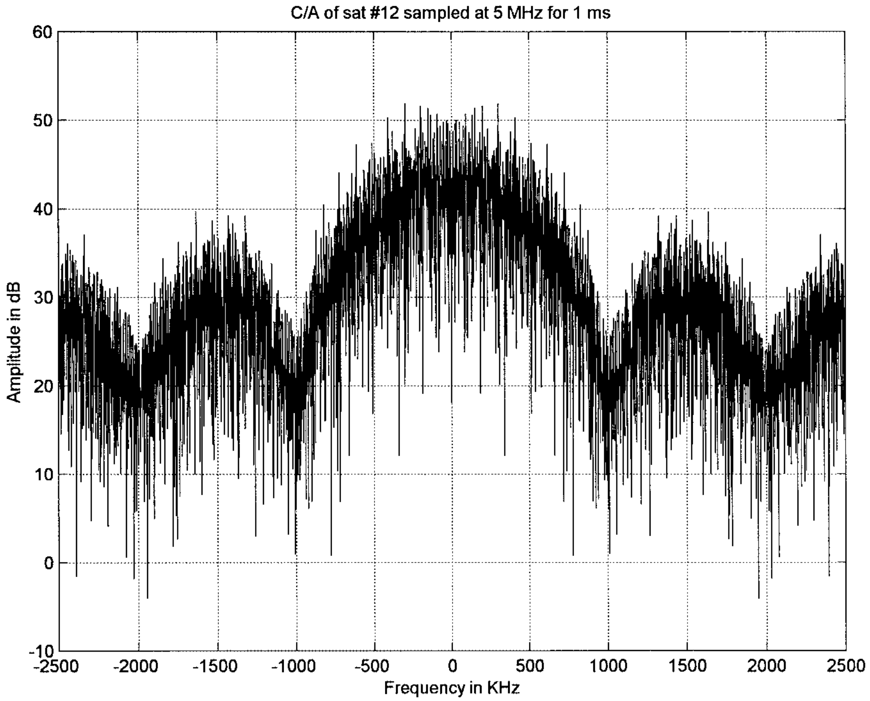


FIGURE 5.2 Spectrum of a C/A code.

will not reach the receiver at the same time. Besides, one does not know when the beginning of subframe 1 will be received. A guaranteed way to receive the first three subframes is to take 30 seconds (or one page) of data. Thus, one can take a minimum of 30 seconds of data and calculate the user position.

5.6 GENERATION OF C/A CODE(1,2,6)

The GPS C/A signals belong to the family of Pseudorandom noise (PRN) codes known as the Gold codes. The signals are generated from the product of two 1,023-bit PRN sequence G1 and G2. Both G1 and G2 are generated by a maximum-length linear shift register of 10 stages and are driven by a 1.023 MHz clock. Figure 5.4 shows the G1 and G2 generators. Figure 5.4a shows the G1 generator and Figures 5.4b and 5.4c show the G2 generator. Figure 5.4c is a simplified notation of Figure 5.4b.

The basic operating principles of these two generators are similar; therefore, only G2 will be discussed in detail. A maximum-length sequence (MLS) generator can be made from a shift register with proper feedback. If the shift register has n bits, the length of the sequence generated is $2^n - 1$. Both shift generators

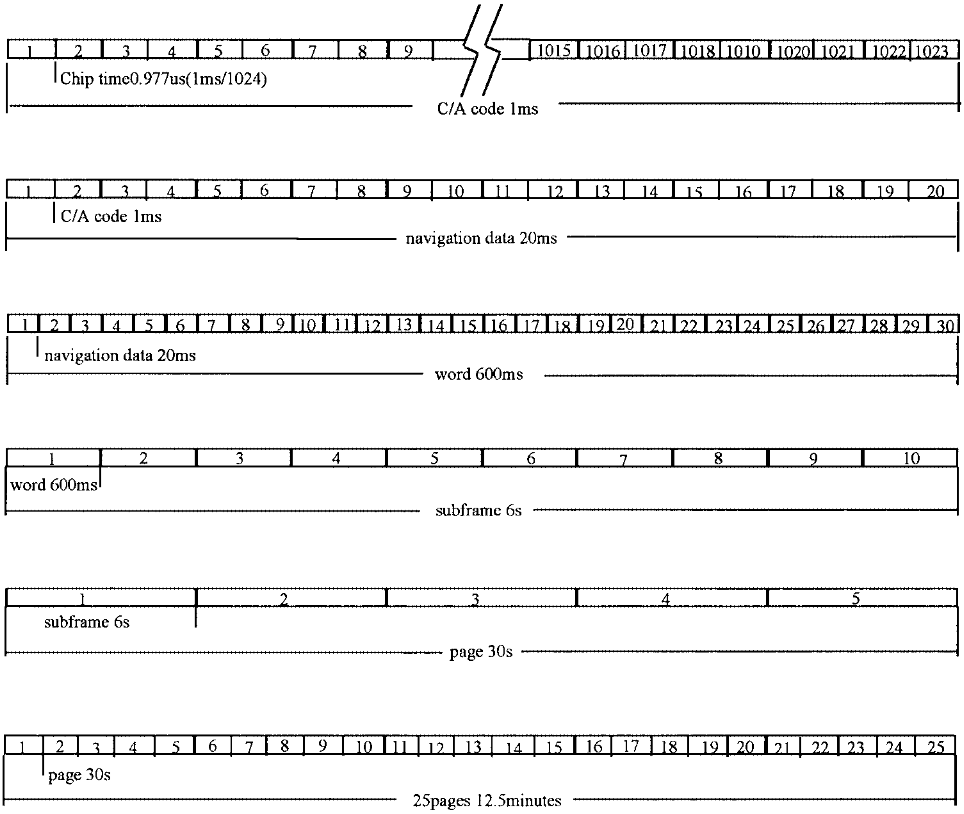
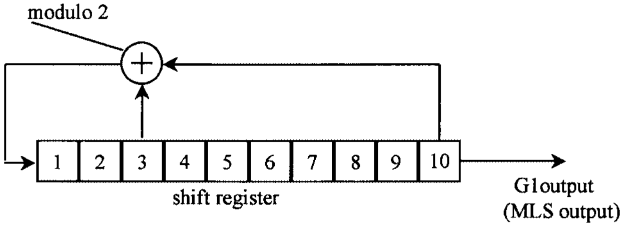


FIGURE 5.3 GPS data format.

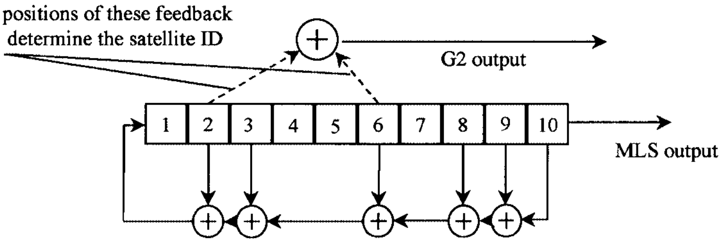
in G1 and G2 have 10 bits, thus, the sequence length is 1,023 ($2^{10} - 1$). The feedback circuit is accomplished through modulo-2 adders.

The operating rule of the modulo-2 adder is listed in Table 5.2. When the two inputs are the same the output is 0, otherwise it is 1. The positions of the feedback circuit determine the output pattern of the sequence. The feedback of G1 is from bits 3 and 10 as shown in Figure 5.4a and the corresponding polynomial can be written as G1: $1 + x^3 + x^{10}$. The feedback of G2 is from bits 2, 3, 6, 8, 9, 10 as shown in Figure 5.4b and the corresponding polynomial is G2: $1 + x^2 + x^3 + x^6 + x^8 + x^9 + x^{10}$.

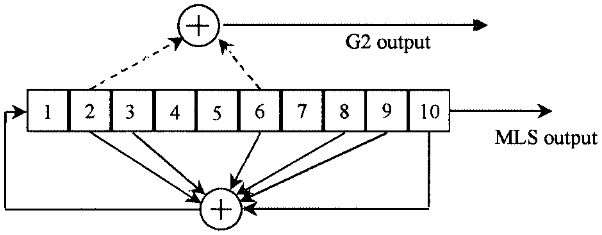
In general, the output from the last bit of the shift register is the output of the sequence as shown in Figure 5.4a. Let us refer to this output as the MLS output. However, the G2 generator does not use the MLS output as the output. The output is generated from two bits which are referred to as the code phase selections through another modulo-2 adder as shown in Figures 5.4b and c. This G2 output is a delayed version of the MLS output. The delay time is determined by the positions of the two output points selected.



(a) G1 generator.



(b) G2 generator.



(c) Simplified notation of G2.

FIGURE 5.4 G1, G2 maximum-length sequence generators.

Figure 5.5 shows the C/A code generator. Another modulo-2 adder is used to generate the C/A code, which uses the outputs from G1 and G2 as inputs. The initial values of the two shift registers G1 and G2 are all 1's and they must be loaded in the registers first. The satellite identification is determined by the

TABLE 5.2 Modulo-2 Addition

Input 1	Input 2	Output
0	0	0
0	1	1
1	0	1
1	1	0

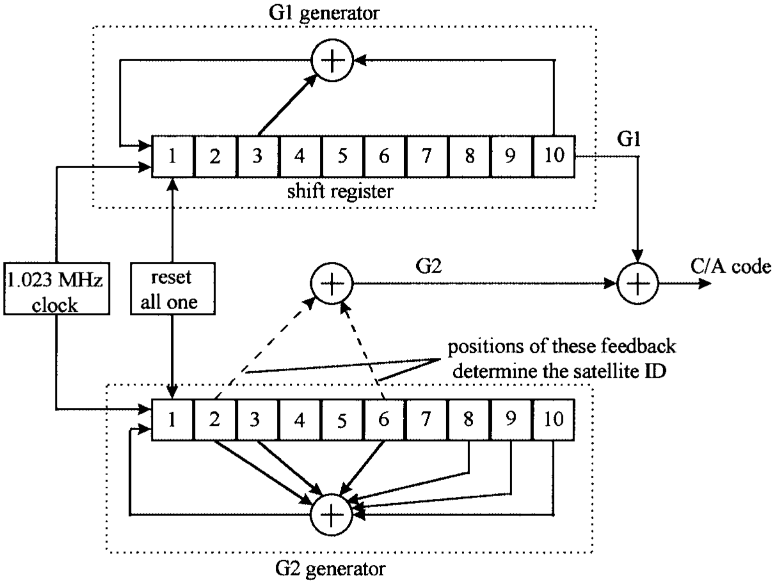


FIGURE 5.5 C/A code generator.

two output positions of the G2 generator. There are 37 unique output positions. Among these 37 outputs, 32 are utilized for the C/A codes of 32 satellites, but only 24 satellites are in orbit. The other five outputs are reserved for other applications such as ground transmission.

Table 5.3 lists the code phase assignments. In this table there are five columns and the first column gives the satellite ID number, which is from 1 to 32.

The second column gives the PRN signal number; and it is from 1 to 37. It should be noted that the C/A codes of PRN signal numbers 34 and 37 are the same. The third column provides the code phase selections that are used to form the output of the G2 generator. The fourth column provides the code delay measured in chips. This delay is the difference between the MLS output and the G2 output. This is redundant information of column 3, because once the code phase selections are chosen this delay is determined. The last column provides the first 10 bits of the C/A code generated for each satellite. These values can be used to check whether the generated code is wrong. This number is in an octal format.

The following example will illustrate the use of the information listed in Table 5.3. For example, in order to generate the C/A code of satellite 19, the 3 and 6 tabs must be selected for the G2 generator. With this selection, the G2 output sequence is delayed 471 chips from the MLS output. The last column is 1633, which means 1 110 011 011 in binary form. If the first 10 bits generated for satellite 19 do not match this number, the code is incorrect.

TABLE 5.3 Code Phase Assignments

Satellite ID Number	GPS PRN Signal Number	Code Phase Selection	Code Delay Chips	First 10 Chips C/A Octal
1	1	$2 \oplus 6$	5	1440
2	2	$3 \oplus 7$	6	1620
3	3	$4 \oplus 8$	7	1710
4	4	$5 \oplus 9$	8	1744
5	5	$1 \oplus 9$	17	1133
6	6	$2 \oplus 10$	18	1455
7	7	$1 \oplus 8$	139	1131
8	8	$2 \oplus 9$	140	1454
9	9	$3 \oplus 10$	141	1626
10	10	$2 \oplus 3$	251	1504
11	11	$3 \oplus 4$	252	1642
12	12	$5 \oplus 6$	254	1750
13	13	$6 \oplus 7$	255	1764
14	14	$7 \oplus 8$	256	1772
15	15	$8 \oplus 9$	257	1775
16	16	$9 \oplus 10$	258	1776
17	17	$1 \oplus 4$	469	1156
18	18	$2 \oplus 5$	470	1467
19	19	$3 \oplus 6$	471	1633
20	20	$4 \oplus 7$	472	1715
21	21	$5 \oplus 8$	473	1746
22	22	$6 \oplus 9$	474	1763
23	23	$1 \oplus 3$	509	1063
24	24	$4 \oplus 6$	512	1706
25	25	$5 \oplus 7$	513	1743
26	26	$6 \oplus 8$	514	1761
27	27	$7 \oplus 9$	515	1770
28	28	$8 \oplus 10$	516	1774
29	29	$1 \oplus 6$	859	1127
30	30	$2 \oplus 7$	860	1453
31	31	$3 \oplus 8$	861	1625
32	32	$4 \oplus 9$	862	1712
**	33	$5 \oplus 10$	863	1745
**	34*	$4 \oplus 10$	950	1713
**	35	$1 \oplus 7$	947	1134
**	36	$2 \oplus 8$	948	1456
**	37*	$4 \oplus 10$	950	1713

*34 and 37 have the same C/A code.

**GPS satellites do not transmit these codes; they are reserved for other uses.

A computer program (p5_1) is listed at the end of this chapter to generate both the MLS and G2 output sequences. The program takes columns 3 and 4 of Table 5.3 as inputs and checks the time delay. If the correct data are used as inputs, the output will show “OK,” otherwise, it will show “not match.”

A program (p5_2) can be used to generate the C/A code. The program is an extension of the program (p5_1) to include the two maximum-length sequence generators. In the program, the delay time listed in Table 5.3 is used as input to generate the G2 signal rather than using the code phase selections in column 3. The first 10 bits of the generated C/A code should be compared with the result listed in the last column of Table 5.3.

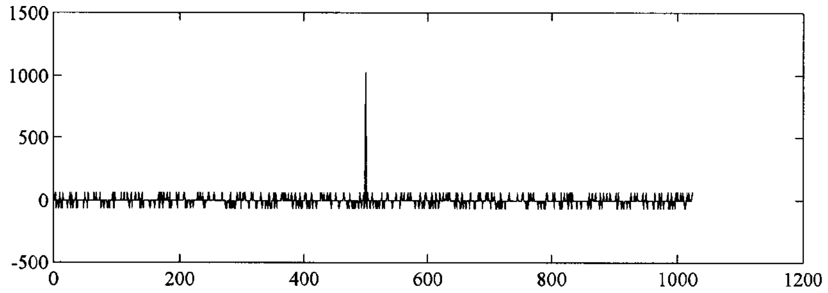
5.7 CORRELATION PROPERTIES OF C/A CODE(1,2)

One of the most important properties of the C/A codes is their correlation result. High autocorrelation peak and low cross-correlation peaks can provide a wide dynamic range for signal acquisition. In order to detect a weak signal in the presence of strong signals, the autocorrelation peak of the weak signal must be stronger than the cross-correlation peaks from the strong signals. If the codes are orthogonal, the cross correlations will be zero. However, the Gold codes are not orthogonal but near orthogonal, implying that the cross correlations are not zero but have small values.

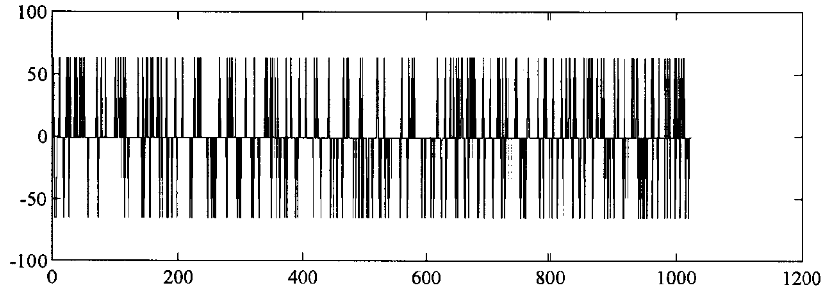
The cross correlation of the Gold code is listed in Table 5.4.(1)

TABLE 5.4 Cross Correlation of Gold Code

Code Period	Number of Shift Register Stages	Normalized Cross Correlation Level	Probability of Level
$P = 2^n - 1$	$n = \text{odd}$	$-\frac{2^{(n+1)/2} + 1}{P}$	0.25
		$-\frac{1}{P}$	0.5
		$\frac{2^{(n+2)/2} - 1}{P}$	0.24
$P = 2^n - 1$	$n = \text{even}$	$-\frac{2^{(n+2)/2} + 1}{P}$	0.125
		$-\frac{1}{P}$	0.75
		$\frac{2^{(n+2)/2} - 1}{P}$	0.125



(a) Autocorrelation of satellite 19.



(b) Cross correlation of satellites 19 and 31.

FIGURE 5.6 Auto and cross correlation of C/A code.

For the C/A code n = even = 10, thus, $P = 1023$. Using the relations in the above table, the cross-correlation values are: $-65/1023$ (occurrence 12.5%), $-1/1023$ (75%), and $63/1023$ (12.5%). The autocorrelation of the C/A codes of satellite 19 and the cross correlation of satellites 19 and 31 are shown in Figures 5.6a and 5.6b respectively. These satellites are arbitrarily chosen.

In Figure 5.6a, the maximum of the autocorrelation peak is 1023, which equals the C/A code length. The position of the maximum peak is deliberately shifted to the center of the figure for a clear view. The rest of the correlation has three values 63, -1, and -65. The cross-correlation shown in Figure 5.6b also has three values 63, -1, -65.

These are the values calculated by using equations in Table 5.4. The difference between the maximum of the autocorrelation to the cross correlation determines the processing gain of the signal. In order to generate these figures, the outputs from the C/A code generator must be 1 and -1, rather than 1 and 0. The mathematical operation to generate these figures will be discussed in the Section 7.7.

5.8 NAVIGATION DATA BITS(2,3,7)

The C/A code is a bi-phase coded signal which changes the carrier phase between 0 and π at a rate of 1.023 MHz. The navigation data bit is also bi-phase code, but its rate is only 50 Hz, or each data bit is 20 ms long. Since the C/A code is 1 ms, there are 20 C/A codes in one data bit. Thus, in one data bit all 20 C/A codes have the same phase. If there is a phase transition due to the data bit, the phases of the two adjacent C/A codes are different by $\pm\pi$. This information is important in signal acquisition. One can perform signal acquisition on two consecutive 10 ms of data. Between two consecutive sets of 10 ms of data there is at most one navigation data bit phase transition. Therefore, one set of these data will have no data bit phase transition and coherent acquisition should produce the desired result. Thirty data bits make a navigation word and 10 words make a subframe. Figure 5.3 shows these relations.

The GPS time is given by the number of seconds in one week and this value is reset every week at the end/start of a week. At end/start of a week the cyclic paging to subframes 1 through 5 will restart with subframe 1 regardless of which subframe was last transmitted prior to end/start of week. The cycling of the 25 pages will restart with page 1 of each of the subframes, regardless of which page was the last to be transmitted prior to the end/start of week. All upload and page cutovers will occur on frame boundaries (i.e., modulo 30 seconds relative to end/start of week). Accordingly, new data in subframes 4 and 5 may start to be transmitted with any of the 25 pages of these subframes.

In the following sections the navigation data will be discussed. Only the limited information used to determine the user position will be included. Detailed information can be found in references 3 and 7.

5.9 TELEMETRY (TLM) AND HAND OVER WORD (HOW)(2,3,7)

As previously mentioned, five subframes make a page. The first two words of all the subframes are the telemetry (TLM) and hand over word (HOW). Each word contains 30 bits and the message is transmitted from bit 1 to bit 30. These two words are shown in Figure 5.7. The TLM word begins with an 8-bit preamble, followed by 16 reserved bits and 6 parity bits. The bit pattern of the preamble is shown in this figure. The bit pattern of the preamble will be used to match the navigation data to detect the beginning of a subframe.

The HOW word can be divided into four parts.

1. The first 17 bits (1–17) are the truncated time of week (TOW) count that provides the time of the week in units of 6 seconds. The TOW is the truncated LSB of the Z count, which will be discussed in the next section.
2. The next two bits (18, 19) are flag bits. For satellite configuration 001 (block II satellite) bit 18 is an alert bit and bit 19 is antispoof. Satellites are procured in blocks. Most block I satellites are experimental ones and all the satellites in orbit are from block II. When bit $18 = 1$, it indicates that the satellite user range accuracy may be worse than indicated in subframe

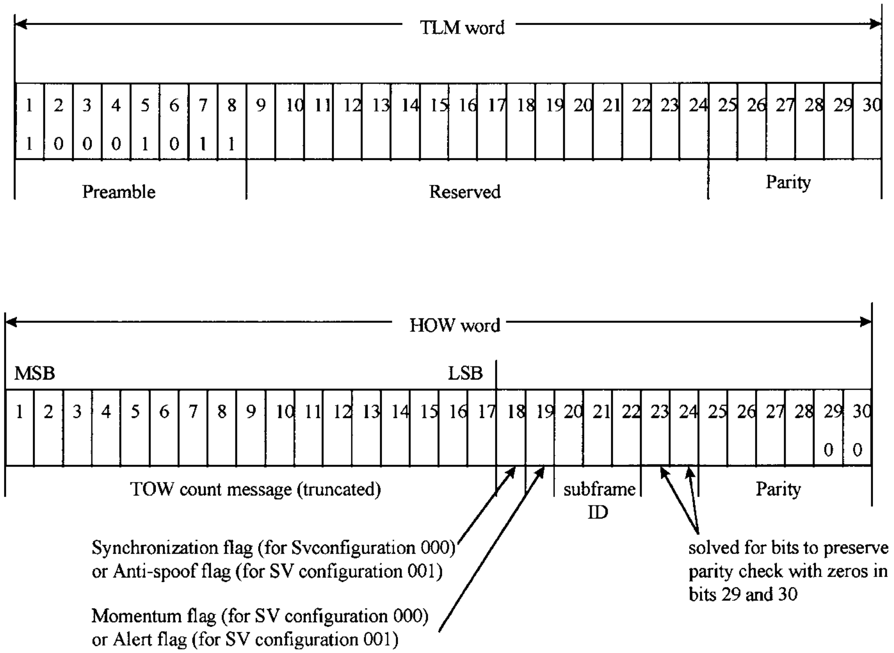


FIGURE 5.7 TLM and HOW words.

1 and the user uses the satellite at the user's own risk. Bit 19 = 1 indicates the antispoof mode is on.

3. The following three bits (20–22) are the subframe ID and their values are 1, 2, 3, 4, and 5 or (001, 010, 011, 100, and 101) to identify one of the five subframes. These data will be used for subframe matching.
4. The last 8 bits (23–30) are used for parity bits.

5.10 GPS TIME AND THE SATELLITE Z COUNT(3)

GPS time is used as the primary time reference for all GPS operation. GPS time is referenced to a universal coordinated time (UTC). The GPS zero time is defined as midnight on the night of January 5/morning of January 6, 1980. The largest unit used in stating GPS time is one week, defined as 604,800 seconds (7 × 24 × 3600). The GPS time may differ from UTC because GPS time is a continuous time scale, while UTC is corrected periodically with an integer number of leap seconds. The GPS time scale is maintained to be within one μs of UTC (modulo of one second). This means the two times can be different by an integer number of seconds. A history of the difference of UTC and GPS time will be shown in Section 5.14.

In each satellite, an internally derived 1.5-second epoch, the Z count, provides

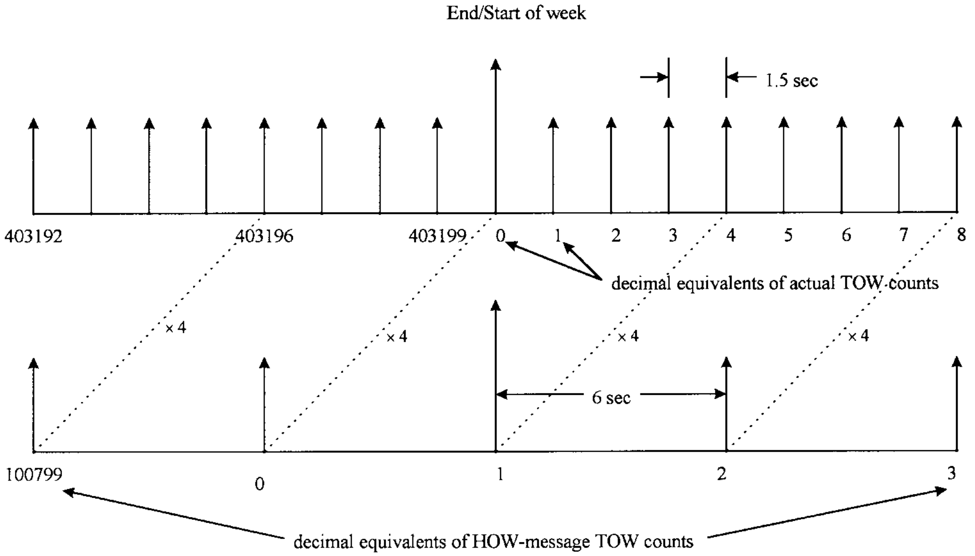


FIGURE 5.8 Z count and TOW count.

a convenient unit for precise counting and communication time. The Z count has 29 bits consisting of two parts: the 19 least-significant bits (LSBs) referred to as the time of the week (TOW) and the 10 most-significant bits (MSBs) as the week number. In the actual data transmitted by the satellite, there are only 27 Z count bits. The 10-bit week number is in the third word of subframe 1. The 17-bit TOW is in the HOW in every subframe as discussed in the previous section. The two LSBs are implied through multiplication of the truncated Z count.

The TOW count has a time unit of 1.5 sec and covers one week of time. Since one week has 604,800 seconds, the TOW count is from 0 to 403,199 because 604,800/1.5 = 403,200. The epoch occurs at approximately midnight Saturday night/Sunday morning, where midnight is defined as 0000 hours on the UTC scale, which is nominally referenced to the Greenwich Meridian. Over the years, the occurrence of the zero-state epoch differs by a few seconds from 0000 hours on the UTC scale. The 17-bit truncated version of the TOW count covers a whole week and the time unit is 6 sec (1.5 sec × 4), which equals one subframe time. This truncated TOW is from 0 to 100, 799, because 604,800/6 = 100,800.

The timeline is shown in Figure 5.8. In Figure 5.8 the Z count is at the end and start of a week as shown in the upper part of the figure. The TOW count consists of the 17 MSBs of the actual 19-bit TOW count at the start of the next subframe as shown in the lower part of the figure. It is important to note that since the TOW count shows the start of the next subframe its value is 1 rather than 0 at the end and start of the week. Multiplying the truncated 17-bit TOW count by 4 converts to the actual 19-bit

TOW count as shown in Figure 5.8. This operation changes the truncated TOW from 0 to 100,799 to from 0 to 403,199, the full range of the Z count.

The 10 MSBs of the Z count is the week number (WN). It represents the number of weeks from midnight on the night of January 5, 1980/morning of January 6, 1980. The total range of WN is from 0 to 1023. At the expiration of GPS week, the GPS week number will roll over to zero. Users must add the previous 1,024 weeks into account when converting from GPS time to a calendar date.

5.11 PARITY CHECK ALGORITHM$^{(3,7)}$

In this section the operation of parity bits will be discussed. From Figure 5.9 (in the following section) one can see that each word has 30 bits and 6 of these are parity bits. These parity bits are used for parity check and to correct the polarity of the navigation bits. If the parity check fails, the data should not be used. In order to check parity, 8 parity bits are used. The additional two bits are the last two bits (also the last two parity bits) from the previous word.

Let D_i represent the data bits in a word received by a receiver where $i = 1, 2, 3, \ldots, 24$ represent the source data and $i = 25, 26, \ldots, 30$ represent the parity bits. The parity encoding equations are listed in Table 5.5, where D_{29}^* and D_{30}^* are the twentyninth and thirtieth data of the previous word, $\oplus$ is the modulo-2 addition and its operation rule is listed in Table 5.2, D_{25} through D_{30} are the parity data.

TABLE 5.5 Parity Encoding Equations

$$
\begin{aligned}
d_1 &= D_1 \oplus D_{30}^* \\
d_2 &= D_2 \oplus D_{30}^* \\
d_3 &= D_3 \oplus D_{30}^* \\
&\vdots \\
d_{24} &= D_{24} \oplus D_{30}^* \\
D_{25} &= D_{29}^* \oplus d_1 \oplus d_2 \oplus d_3 \oplus d_5 \oplus d_6 \oplus d_{10} \oplus d_{11} \oplus d_{12} \oplus d_{13} \oplus d_{14} \oplus d_{17} \oplus d_{18} \\
&\quad \oplus d_{20} \oplus d_{23} \\
D_{26} &= D_{30}^* \oplus d_2 \oplus d_3 \oplus d_4 \oplus d_6 \oplus d_7 \oplus d_{11} \oplus d_{12} \oplus d_{13} \oplus d_{14} \oplus d_{15} \oplus d_{18} \oplus d_{19} \\
&\quad \oplus d_{21} \oplus d_{24} \\
D_{27} &= D_{29}^* \oplus d_1 \oplus d_3 \oplus d_4 \oplus d_5 \oplus d_7 \oplus d_8 \oplus d_{12} \oplus d_{13} \oplus d_{14} \oplus d_{15} \oplus d_{16} \oplus d_{19} \\
&\quad \oplus d_{20} \oplus d_{22} \\
D_{28} &= D_{30}^* \oplus d_2 \oplus d_4 \oplus d_5 \oplus d_6 \oplus d_8 \oplus d_9 \oplus d_{13} \oplus d_{14} \oplus d_{15} \oplus d_{16} \oplus d_{17} \oplus d_{20} \\
&\quad \oplus d_{21} \oplus d_{23} \\
D_{29} &= D_{30}^* \oplus d_1 \oplus d_3 \oplus d_5 \oplus d_6 \oplus d_7 \oplus d_9 \oplus d_{10} \oplus d_{14} \oplus d_{15} \oplus d_{16} \oplus d_{17} \oplus d_{18} \\
&\quad \oplus d_{21} \oplus d_{22} \oplus d_{24} \\
D_{30} &= D_{29}^* \oplus d_3 \oplus d_5 \oplus d_6 \oplus d_8 \oplus d_9 \oplus d_{10} \oplus d_{11} \oplus d_{13} \oplus d_{15} \oplus d_{19} \oplus d_{22} \\
&\quad \oplus d_{23} \oplus d_{24}
\end{aligned}
$$

In using Table 5.5, the first 24 calculations must be carried out first. The purpose is to generate a new set of data d_i for $i = 1$ to 24. If $D_{30}^* = 0$, then from the relation in Table 5.2 $d_i = D_i$ (for $i = 1$ to 24), which means there is no sign change. If $D_{30}^* = 1$, then $D_i = 0$ changes to $d_i = 1$ and $D_i = 1$ changes to $d_i = 0$ (for $i = 1$ to 24). This operation changes the signs of the source bits. These values of d_i are used to check the parity relation given through D_{25} to D_{30}.

In a receiver the polarity of the navigation data bits is usually arbitrarily assigned. The operations listed in Table 5.5 can automatically correct the polarity. If $D_{30}^* = 0$, the polarity of the next 24 data bits does not change. If the D_{30}^* = 1, the polarity of the next 24 data will change. This operation takes care of the polarity of the bit pattern.

The equations listed in Table 5.5 can be calculated from a matrix operation. This matrix is often referred as the parity matrix and defined as(7)

$$H = \begin{array}{c} \begin{array}{cccccccccccccccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 & 21 & 22 & 23 & 24 \end{array} \\ \hline \left[\begin{array}{cccccccccccccccccccccccc} 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \end{array}\right] \end{array} \tag{5.3}$$

This matrix matches the last six equations in Table 5.5. If a certain d_i is present a 1 will be placed in the matrix. If a certain d_i does not exist, a zero will be placed in the matrix. Note that each row in H is simply a cyclic shift of the previous row except for the last row. In order to use the parity matrix, the following property must be noted. The similarity between modulo-2 and multiplication of +1 and −1 must be found first. The results in Table 5.2 are listed in Table 5.6 again for comparison.

It appears that in order to use +1, −1 multiplication to replace the modulo-2 addition the input should be converted as $0 \Rightarrow +1$ and $1 \Rightarrow -1$. This operation can extend to more than two inputs. This designation contradicts the conventional approach from $0 \Rightarrow -1$ and $1 \Rightarrow +1$. The following steps can be taken to check parity:

1. Arbitrarily represent the data D_i by 1 and 0 and change $1 \Rightarrow -1$ and $0 \Rightarrow +1$.
2. Change the signs of D_i ($i = 1$ to 24) by multiplying them with D_{30}^*. These new data are as d_i for $i = 1$ to 24.
3. These values of d_i for $i = 1$ to 24 are used to multiply each row of the H matrix element by element. The results are 6 rows and each row has 24 elements. Each element can be one of the three values +1, 0, and −1. The nonzero terms, which are +1 and −1, are multiplied together and the

TABLE 5.6 Comparison of Modulo-2 Addition and +1, −1 Multiplication

Modulo-2 Addition			Multiplication		
Input 1	Input 2	Output	Input 1	Input 2	Output
0	0	0	+1	+1	+1
0	1	1	+1	−1	−1
1	0	1	−1	+1	−1
1	1	0	−1	−1	+1

new results should be either +1 or −1. These new results are multiplied either by D^*_{29} or D^*_{30} according to last six equations in Table 5.5.

4. The final results should equal to $[D_{25}\ D_{26}\ D_{27}\ D_{28}\ D_{29}\ D_{30}]$.
5. The last step is to convert +1, −1 back to 0 and 1 for further processing.

Subframe matching will be discussed in Section 9.4. A program with subframe matching and parity check will be listed at the end of Chapter 9.

5.12 NAVIGATION DATA FROM SUBFRAME 1(3,7)

The data contained in the first three subframes are shown in Figure 5.9. The minimal parameters required to calculate the user position are contained in these three subframes.

The data used for calculations of locations of the satellites and the user are discussed below.

1. *Week number (61–70):* These ten bits are discussed in Section 5.9. Thus represents the MSB of the Z counts and indicates the number of weeks from midnight on the night of January 5, 1980/morning of January 6, 1980. Users must count the rollover if it is over 1,023 weeks.
2. *User range accuracy (73–76):* These four bits give the predicted user range accuracy and its value N ranges from 0–15. The accuracy value X is:
   - If N is 6 or less, $X = 2^{(1+N/2)}$ (rounded-of-values $N = 1$, $X = 2.8$; $N = 3$, $X = 5.7$; $N = 5$, $X = 11.3$).
   - If N is 6 or more, but less than 15, $X = 2^{(N-2)}$.
   - $N = 15$ will indicate the absence of an accuracy prediction and will advise the user to use that satellite at the user's risk.
3. *Satellite health (77–82):* These six bits represent the health indication of the transmitting satellite. The MSB (bit 77) indicates a summary of the health of the navigation data, where bit 77 equals:

   0 = All navigation data are OK.

   1 = Some or all navigation data are bad.

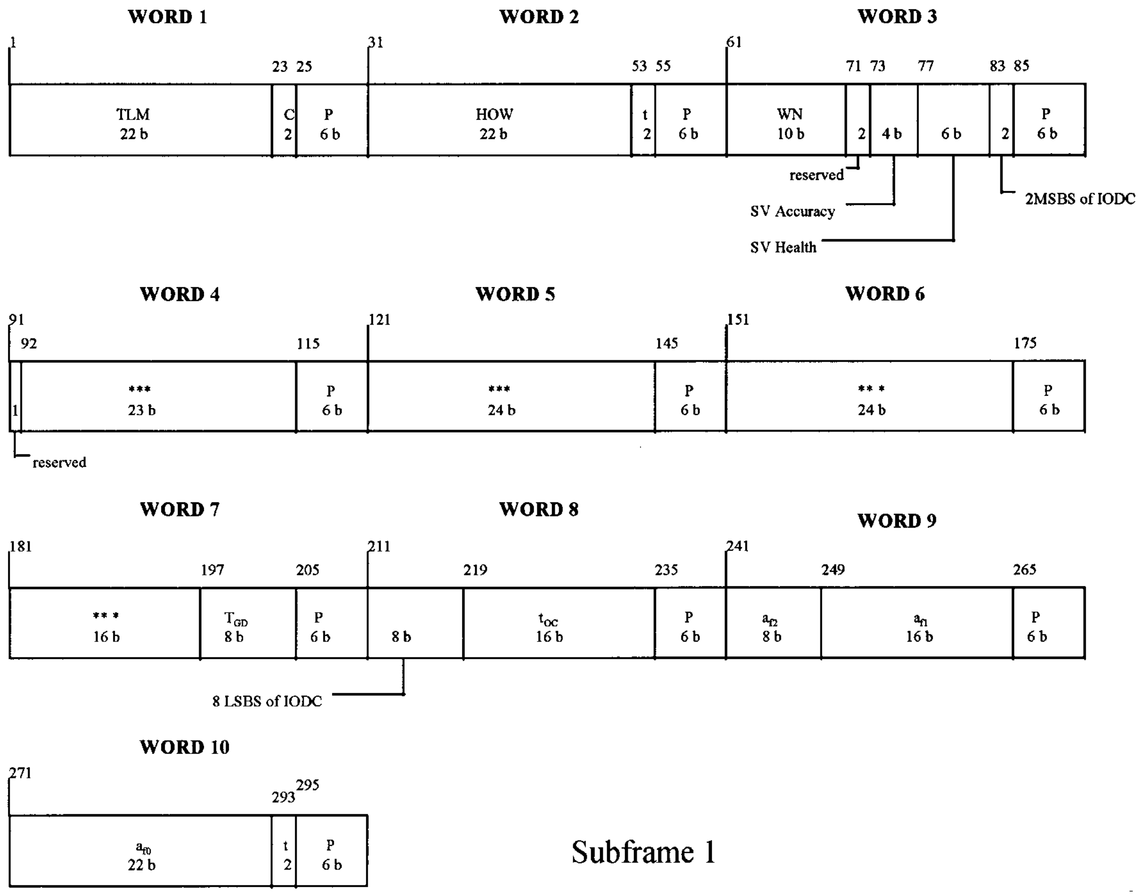


FIGURE 5.9 Data in subframes 1, 2, and 3. ***Reserved. p: Parity bits. t: Two noninformation-bearing bits used for parity computation.

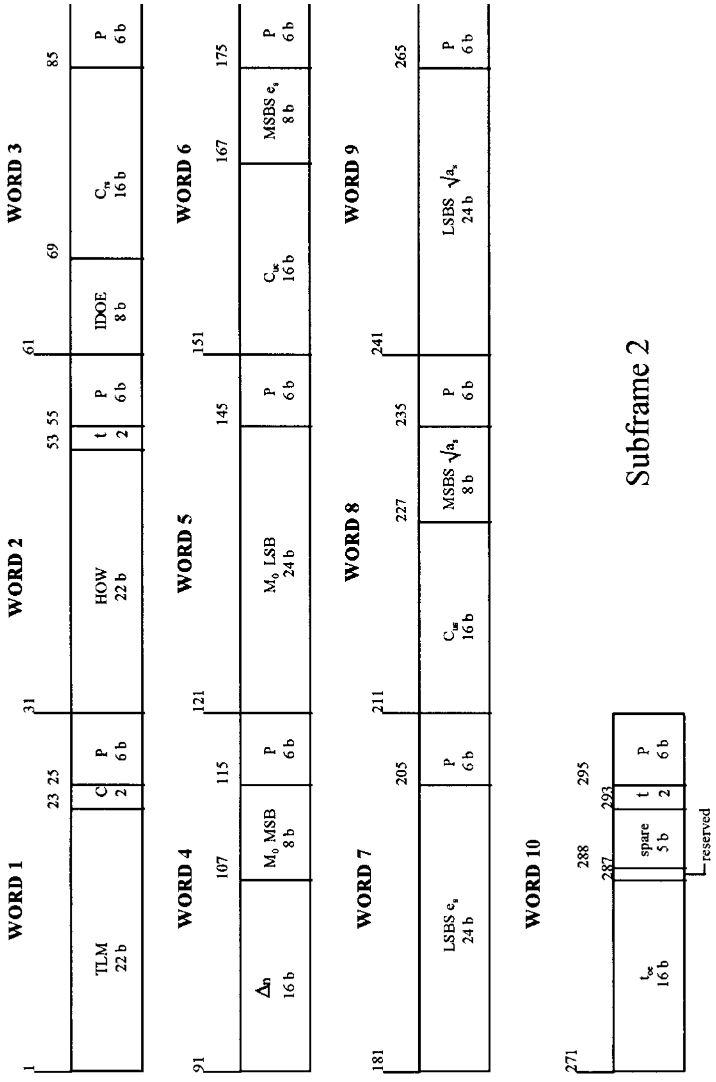


FIGURE 5.9 Continued.

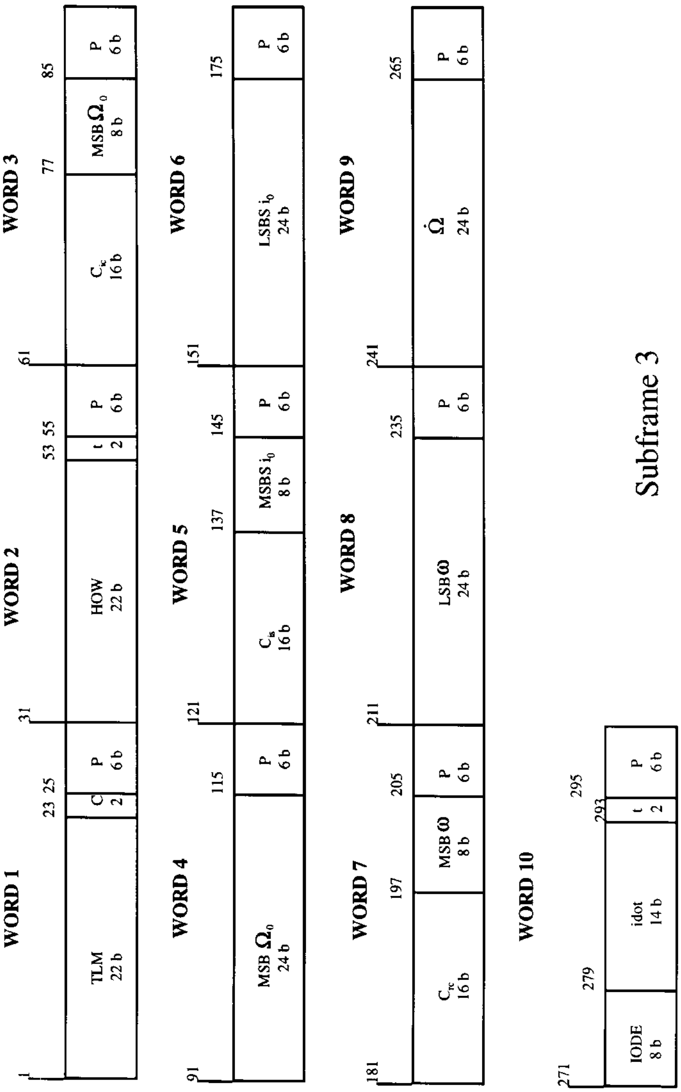


FIGURE 5.9 Continued.

TABLE 5.7 Codes for Health of Satellite Signal Components

MSB				LSB		
0	0	0	0	0	⇒	All signals OK.
1	1	1	0	0	⇒	Satellite *is* temporarily out—do not use this satellite during current pass.
1	1	1	0	1	⇒	Satellite *will be* temporarily out–use with caution.
1	1	1	1	0	⇒	Spare
1	1	1	1	1	⇒	More than one combination would be required to describe anomalies.
All other combinations					⇒	Satellite experiencing code modulation and/or signal level transmission problem—modulation navigation data valid; however, user may experience intermittent tracking problems if satellite is acquired.

The five LSBs indicate the health of the signal components in Table 5.7. Additional satellite health data are given in subframes 4 and 5. The data given in subframe 1 may differ from that shown in subframes 4 and/or 5 of other satellites, since the latter may be updated at a different time.

4. *Issue of data, clock (IODC) (83–84 MSB, 211–218 LSB):* These 10-bit IODC data indicate the issue number of the data set and thereby provide the user with a convenient means of detecting any change in the correction parameters. The transmitted IODC will be different from any value transmitted by the satellite during the preceding seven days. The relationship between IODC and IODE (in both subframes 2 and 3) will be discussed in the next section.
5. *Estimated group delay differential T_{GD} (197–204):* This eight-bit information is a clock correction term to account for the effect of satellite group delay differential. It is used in Equation (4.38).
6. *Satellite clock correction parameters:* This subframe also contains the four additional satellite clock correction parameters: t_{oc} (219–234), a_{f0} (271–292), a_{f1} (249–264), and a_{f2} (241–248). They are used in Equation (4.37).
7. In subframe 1 there are some reserved data fields and their locations are 71–72; 91–114; 121–144; 151–174; 181–196. All reserved data fields support valid parity within their respective words.

The ephemeris parameters in subframe 1 are listed in Table 5.8.

5.13 NAVIGATION DATA FROM SUBFRAMES 2 AND 3(3,7)

Figures 5.9b and c show the following ephemeris data contained in subframes 2 and 3:

TABLE 5.8 Ephemeris Parameters in Subframe 1

Parameter	Location	Number of Bits	Scale Factor (LSB)	Effective Range**	Units
WN: Week number	61-70	10	1		week
Satellite accuracy	73–76	4			
Satellite health	77–82	6	1		
IODC: Issue of data, clock	83–84 211–218	10			
T_{GD}: Satellite group delay differential	197–204	8*	2^{-31}		seconds
t_{oe}: Satellite clock correction	219–234	16	2^4	604,784	seconds
a_{f2}: Satellite clock correction	241–248	8*	2^{-55}		sec/sec^2
a_{f1}: Satellite clock correction	249–264	16*	2^{-43}		sec/sec
a_{f0}: Satellite clock correction	271–292	22*	2^{-31}		seconds

*Parameters so indicated are two's complement, with the sign bit (+ or −) occupying the MSB.
**Unless otherwise indicated in this column, effective range is the maximum range attainable with indicated bit allocation and scale factor.

1. *The issue of data, ephemeris (IODE):* This parameter has 8 bits and is in both subframes 2 (61–68) and 3 (271–278). The IODE equals the 8 LSB of the IODC, which has 10 bits. The IODE provides the user with a convenient means for detecting any change in the ephemeris representation parameters. The transmitted IODE will be different from any value transmitted by the satellite during the preceding six hours. Whenever these three terms, two IODEs from subframes 2, 3 and the 8 LSBs of the IODC, do not match, a data set cutover has occurred and new data must be collected.

   Any change in the subframe 2 and 3 data will be accomplished in concert with a change in both IODE words. Cutovers to new data will occur only on hour boundaries except for the first data set of a new upload. The first data set may be cut in at any time during the hour and therefore may be transmitted by the satellite for less than one hour. Additionally, the t_{oe} value for at least the first data set transmitted by a satellite after an upload will be different from that transmitted prior to the cutover.

2. *The rest of the ephemeris data:* These are listed in Tables 5.9 and 5.10.

3. *Spare and reserved data fields:* In subframe 2 bit 287 is reserved and bits 288–292 are spared. All spare and reserved data fields support valid parity within their respective words. Contents of spare data fields are alternating ones and zeros until they are allocated for a new function. Users are cautioned that the contents of spare data fields can change without warning.

TABLE 5.9 Ephemeris Parameters in Subframe 2

Parameter	Location	Number of Bits	Scale Factor (LSB)	Effective Range	Units
IODE	61–68	8			(see text)
C_{rs}: Amplitude of the sine harmonic correction terms to the orbit radius	69–84	16	2^{-5}		meters
Δn: Mean motion difference from computed value	91–106	16*	2^{-43}		semicircles/ sec
M_0: Mean anomaly at reference time	107–114; 121–144	32*	2^{-31}		semicircle
C_{uc}: Amplitude of the cosine harmonic correction term to the argument of latitude of argument oflatitude	151–166	16*	2^{-29}		radians
e_s: Eccentricity	167–174; 181–204	32	2^{-33}	0.03	dimensionless
C_{us}: Amplitude of the sine harmonic correction term to the argument of latitude	211–226	16*	2^{-29}		radians
$\sqrt{a_s}$: Square root of the semimajor axis	227–234; 241–264	32	2^{-19}		meters$^{1/2}$
t_{oe}: Reference time ephemeris	277–286	16	2^4	604,784	seconds

5.14 NAVIGATION DATA FROM SUBFRAMES 4 AND 5—SUPPORT DATA(3,7,8)

Both subframes 4 and 5 are subcommutated 25 times each. The 25 versions of these subframes are referred to as pages 1 to 25 of each superframe. With the possible exception of "spare" pages and explicit repeats, each page contains different data in words 3 through 10, which are from bits 91–300. Subframe 4 has six different formats but only five of them are shown in Figure 5.10a. Five pages, 1, 6, 11, 16, 21, are in one format. Six pages, 12, 19, 20, 22, 23, 24, are in one format. Page 18 is in one format. Page 25 is in one format, and pages 13, 14, 15, and 17 are in one format. There are a total of 17 pages. Pages 2, 3, 4, 5, 7, 8, 9, and 10 are not shown because they have the same format as page 1 through 24 of subframe 5. Subframe 5 has two different formats as shown in Figure 5.10b.

The information in subframes 4 and 5 and its applications are listed below:

TABLE 5.10 Ephemeris Parameters in Subframe 3

Parameter	Location	Number of Bits	Scale Factor (LSB)	Effective Range	Units
C_{ic}: Amplitude of the cosine harmonic correction term to angle of inclination	61–76	16*	2^{-29}		radians
Ω_e: Longitude of ascending node of orbit plane at weekly epoch	77–84; 91–114	32*	2^{-31}		semicircles
C_{is}: Amplitude of the sine harmonic correction term to angle of inclination	121–136	16*	2^{-29}		radians
i_0: Inclination angle at reference time	137–144; 151–174	32*	2^{-31}		semicircles
C_{rc}: Amplitude of the sine harmonic correction term to the orbit radius	181–196	16*	2^{-5}		meters
ω: Argument of perigee	197–204; 211–234	32*	2^{31}		semicircles
$\dot{\Omega}$: Rate of right ascension	241–264	24*	2^{-43}		semicircles/ sec
IDOE	271–278				(see text)
idot: Rate of inclination angle	279–292	14*	2^{-43}		semicircles/ sec

1. *Subframe 4:*
   - Pages 2, 3, 4, 5, 7, 8, 9, and 10 contain the almanac data for satellite 25 through 32. These pages may be designated for other functions. The satellite ID of that page defines the format and content.
   - Page 17 contains special messages.
   - Page 18 contains ionospheric and universal coordinated time (UTC).
   - Page 25 contains antispoof flag, satellite configuration for 32 satellites, and satellite health for satellites 25–32.
   - Pages 1, 6, 11, 12, 16, 19, 20, 21, 22, 23, and 24 are reserved.
   - Pages 13, 14, and 15 are spares.

2. *Subframe 5:*
   - Pages 1–24 contain almanac data for satellites 1 through 24.
   - Page 25 contains satellite health for satellites 1 through 24, the almanac reference time, and the almanac reference week number.

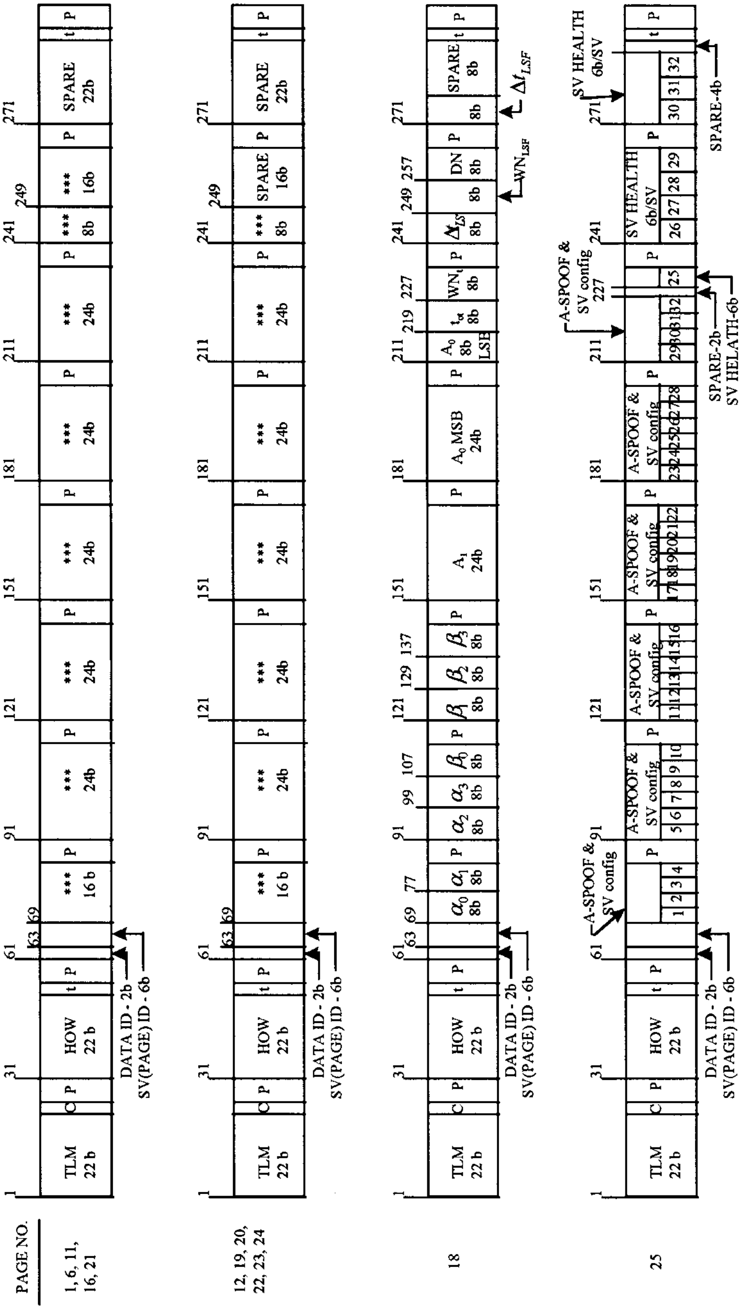
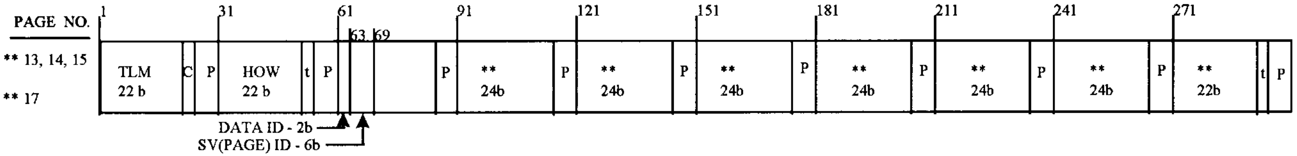
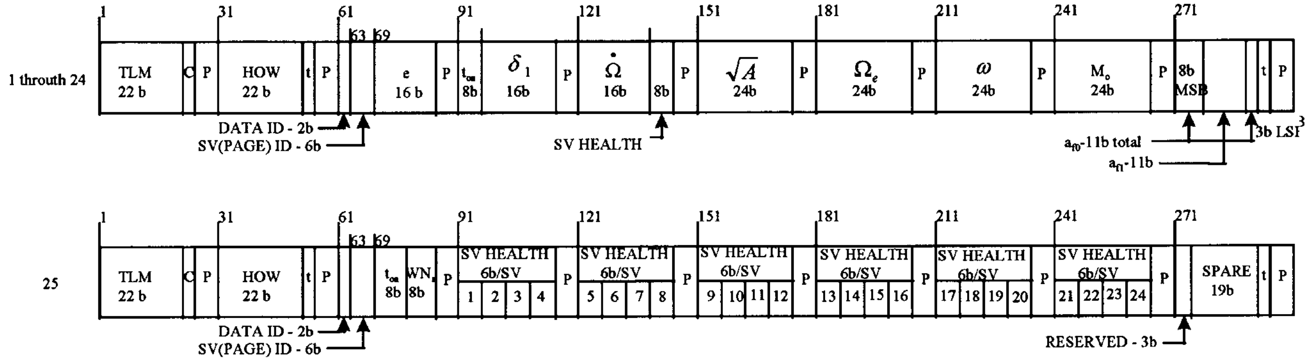


FIGURE 5.10 Data format for subframes 4 and 5.



(a) Five of six formats for subframe 4.



(b) Two formats for subframe 5.

FIGURE 5.10 Continued. **The indicated portions of words 3 through 10 of pages 13, 14, and 15 are spares, while those of page 17 are reserved for special messages. ***Reserved. P: Six parity bits. *t*: Two non-information-bearing bits used for parity computation. C: TLM bits 23 and 24 are reserved. Note: Pages 2, 3, 4, 5, 7, 8, 9, and 10 of subframe 4 have the same format as pages 1 through 24 of subframe 5.

TABLE 5.11 UTC Parameters

Parameter[**]	Number of Bits	Scale Factor	Effective Range[***]	Units
A_0	32[*]	2^{-30}		seconds
A_1	24[*]	2^{-50}		sec/sec
Δt_{LS}	8	1		seconds
t_{ot}	8	2^{12}	602,112	seconds
WN_r	8	1		weeks
WN_{LSF}	8	1		weeks
DN	8[****]	1	7	days
Δt_{LSF}	8[*]	1		seconds

[*] Parameters so indicated are two's complement, with the sign bit (+ or −) occupying the MSB.

[**] See Figure 5.9a for bit allocation in subframe 4.

[***] Unless otherwise indicated in this column, effective range is the maximum range attainable with indicated bit allocation and scale factor.

[****] Right justified.

3. *Almanac data:* The almanac parameters provided in subframes 4 and 5 are: e_s, t_{oa}, $\dot{\Omega}$, $\sqrt{a_s}$, Ω_e, ω, M_0, a_{f0}, and a_{f1}. The almanac data are much less accurate than the detailed ephemeris data of subframes 2 and 3. However, the almanac data are valid for longer periods of time and do not require frequency updates.
4. *Translation of GPS time to UTC time:* In page 18 of subframe 4 the parameters in Table 5.11 are included.

The GPS/UTC time relationship is given by:(3,7)

$$t_{UTC} = (t_E - \Delta t_{UTC})\{\text{modulo } 86400 \text{ seconds}\} \tag{5.4}$$

where t_{UTC} is in seconds and

$$\Delta t_{UTC} = \Delta t_{LS} + A_0 + A_1[t_E - t_{ot} + 604800(WN - WN_t)] \text{ seconds} \tag{5.5}$$

t_E: GPS time as estimated by the user on the basis of correcting t_{SV} for factors given in the subframe 1 clock correction discussion as well as for ionospheric and satellite (dither) effects.

t_{SV}: effective satellite pseudorange code phase time at message of transmission time.

Δt_{LS}: delta time due to leap seconds.

A_0, A_1: constant and first-order terms of polynomial.

t_{ot}: reference time for UTC data.

WN: current week number (derived from subframe 1).

WN_t: UTC reference week number.

The estimated GPS time (t_E) is in seconds relative to end/start of week. The reference time for UTC data (t_{ot}) is referenced to the start of that week whose number (WN_t) is given in bits (227–234) of page 18 subframe 4 representing the 8 LSB of the week. The user must account for the truncated nature of the week number.

Whenever the user's current time falls within the time span of $DN + 3/4$ to $DN + 5/4$, proper accommodation of the leap second event with a possible week number transition is provided by the following expression for UTC:

$$t_{UTC} = W[\text{modulo}(86400 + \Delta t_{LSF} - \Delta t_{LS})] \text{ seconds} \tag{5.6}$$

where

$$W = (t_E - \Delta t_{UTC} - 43200)[\text{modulo } 86400] + 43200 \text{ seconds} \tag{5.7}$$

The definition of Δt_{UTC} given in Equation (5.4) applies throughout the transition period. Note that when a leap second is added, unconventional time values of the form 23 : 59 : 60.xxx are encountered. Some user equipment may be designed to approximate UTC by decrementing the running count of time within several seconds after the event, thereby promptly returning to a proper time indication. Whenever a leap second event is encountered, the user equipment must consistently implement carries or borrows into any year/week/day counts. Table 5.12 gives the past history of the difference between the GPS and the UTC times.(8) In 19 years the difference is 13 seconds.

TABLE 5.12 Difference Between GPS and UTC Times

Date	GPS-UTC Time (sec)
6 Jan 1980	0 (Start of GPS system time)
1 Jul 1981	1
1 Jul 1982	2
1 Jul 1983	3
1 Jul 1985	4
1 Jan 1988	5
1 Jan 1990	6
1 Jan 1991	7
1 Jul 1992	8
1 Jul 1993	9
1 Jul 1994	10
1 Jan 1996	11
1 Jul 1997	12
1 Jan 1999	13

The tendency is that most of the modern navigation equipment uses GPS time as the time base. Therefore, the translation from GPS time to UTC time may no longer be needed in modern equipment.

5. *Ionospheric data:* In page 18 subframe 4, there are eight ionospheric data: α_0 (69–76), α_1 (77–84), α_2 (91–98), α_3 (99–106), β_0 (107–114), β_1 (121–128), β_2 (129–136), β_3 (137–144). These data can be used to correct the time received from the satellite for ionospheric effect. The applications of these data are discussed in the next section.

5.15 IONOSPHERIC MODEL(3,7–10)

The atmosphere around the earth will affect the traveling speed of the GPS signal and cause measurement errors. These errors should be corrected. For GPS application, the atmosphere is usually divided into two portions: the ionosphere and the troposphere. Troposphere is the closer of the two to the surface of the earth while ionosphere is above the troposphere. The troposphere contains neutral particles and ionosphere contains free ions. The ionosphere will cause a code delay but a carrier phase advance.(10) This section presents a correction model for the ionospheric error.

Besides the selectivity availability (SA), which will be discussed in the next section, the ionospheric effect can cause one of the most significant position errors in a GPS receiver. If a receiver operates on both the L1 and L2 frequencies, such as in a military receiver, the time delay Δt_1 at frequency L1 caused by the ionospheric effect can be calculated as(9)

$$\Delta t_1 = \frac{f_2^2}{f_1^2 - f_2^2}\,\delta(\Delta t) \tag{5.8}$$

where f_1 and f_2 are the frequencies at L1 and L2 respectively, $\delta(\Delta t)$ is the measured time difference between frequencies f_1 and f_2 from the same satellite. This Δt_1 can be considered as the measured value.

In most commercial GPS receivers only the L1 frequency is available. The ionospheric data collected from subframe 4 can be used to reduce the ionospheric effect; this is often referred to as the single-frequency ionospheric model. Using this model one can reduce the user root mean square (rms) position error caused by ionospheric effect at least by 50 percent.(7)

The ionospheric model is(3,7)

$$T_{iono} = \left\{ \begin{array}{ll} T * \left[5.0 * 10^{-9} + (AMP)\left(1 - \dfrac{x^2}{2} + \dfrac{x^4}{24}\right)\right] & \text{if } |x| < 1.57 \\ T * (5.0 * 10^{-9}) & \text{if } |x| \geq 1.57 \end{array} \right\} \text{(sec)} \tag{5.9}$$

where T_{iono} is the addition delay time and

$$AMP = \begin{Bmatrix} \sum_{n=0}^{3} \alpha_n \phi_m^n & \text{if } AMP \geq 0 \\ \text{if } AMP < 0 & AMP = 0 \end{Bmatrix} \text{(sec)} \tag{5.10}$$

$$x = \frac{2\pi(t - 50400)}{PER} \text{ (radians)} \tag{5.11}$$

$$PER = \begin{Bmatrix} \sum_{n=0}^{3} \beta_n \phi_m^n & \text{if } PER \geq 72,000 \\ \text{if } PER < 72,000 & PER = 72,000 \end{Bmatrix} \text{(sec)} \tag{5.12}$$

$$T = 1.0 + 16.0[0.53 - \xi]^3 \tag{5.13}$$

where α_n and β_n with (n = 0, 1, 2, 3) are the ionospheric data obtained from the satellite and ξ is the elevation angle between the user and satellite.

Other equations that must be solved are

$$\phi_m = \phi_i + 0.064 \cos(\lambda_i - 1.617)\text{(semicircles)} \tag{5.14}$$

$$\lambda_i = l_u + \frac{\psi \cos A}{\cos \phi_i} \text{ (semicircle)} \tag{5.15}$$

$$\phi_i = \begin{Bmatrix} L_u + \psi \cos A & \text{if } |\phi_i| \leq 0.416 \\ +0.416 & \text{if } \phi_i > +0.416 \\ -0.416 & \text{if } \phi_i < -0.401 \end{Bmatrix} \text{(semicircle)} \tag{5.16}$$

$$\psi = \frac{0.00137}{\xi + 0.11} + 0.022\text{(semicircle)} \tag{5.17}$$

$$t = 4.32 \ * \ 10^4 \lambda_i + GPS \text{ time(sec)} \tag{5.18}$$

where $0 \leq t < 86400$; therefore, if $t \geq 86400$ seconds, subtract 86400 seconds; if $t < 0$ seconds, add 86400 seconds.

The terms used in computation of ionospheric delay are as follows:

Satellite-Transmitted Terms

α_n: The coefficients of a cubic equation representing the amplitude of the vertical delay (4 coefficients—8 bits each).

β_n: the coefficients of a cubic equation representing the period of the model (4 coefficients—8 bits each).

Receiver-Generated Terms

ξ: elevation angle between the user and satellite (semicircles).

A: azimuth angle between the user and satellite, measured clockwise positive from true North (semicircles).

L_u: user geodetic latitude (semicircle).

l_u: user geodetic longitude (semicircle).

GPS time: receiver-computed system time.

Computed Terms

x: phase (radians).

T: obliquity factor (dimensionless).

t: local time (sec).

ϕ_m: geomagnetic latitude of the earth projection of the ionospheric intersection point (mean ionospheric height assumed 350 km) (semicircle).

λ_i: geomagnetic latitude of the earth projection of the ionospheric intersection point (semicircle).

ϕ_i: geomagnetic latitude of the earth projection of the ionospheric intersection point (semicircle).

ψ: earth's central angle between user position and earth projection of ionospheric intersection point (semicircles).

5.16 TROPOSPHERIC MODEL(11)

Compared with the ionospheric effect, the tropospheric effect is about an order of magnitude less. The satellites do not transmit any data to correct for the tropospheric effect. There are many models to correct the error. Here only a simple model will be presented. The delay in meters is given by:(11)

$$\Delta = \frac{2.47}{\sin\ \xi + 0.0121} \text{ meters} \tag{5.19}$$

where ξ is the elevation angle between the user and satellite.

5.17 SELECTIVITY AVAILABILITY (SA) AND TYPICAL POSITION ERRORS(8,12–15)

The selectivity availability is aimed to degrade the performance of the GPS. It was put in effect on March 25, 1990. In accordance with the current policy of the U.S. Department of Defense, the signal available from the GPS is

TABLE 5.13 Observed GPS Positioning Errors with Typical Standard Positioning Service (SPS) Receiver(15)

Error Source	Typical Range Error Magnitude (meters 1σ)
Selective availability	24.0
Ionospheric*	7.0
Tropospheric**	0.7
Satellite clock & ephemeris	3.6
Receiver noise	1.5

*After applying Ionospheric model. Actual values can range between approximately 1–30 m.
**After applying tropospheric model.

actually a purposefully degraded version of the C/A code. The signal degradation is achieved by dithering the satellite clock frequency and providing only a coarse description of the satellite ephemeris. This policy, known as selective availability, effectively raises the value of the user range error by a factor of four or more. The selectivity availability affects only the performance of a GPS receiver, it does not impact the design of the receiver. A Presidential Decision Directive (PDD) released in March 1996 states that the selectivity availability will be turned off within 10 years.

Some typical position errors caused by different effects are listed below. The distance error given in meters for one standard deviation (1σ) is listed in the Table 5.13.

5.18 SUMMARY

In this chapter the C/A code signal of the L1 frequency is discussed. The radio frequency and the C/A code length are important information for performing acquisition and tracking by a receiver. The navigation data are in five subframes. In order to obtain the data, the beginning of the subframe must be found. There are parity data that must be checked before the data can be used. The information in the first three subframes is enough to find the user position. The information in the fourth and fifth subframes is support data. Ionospheric and tropospheric models are introduced to improve receiver accuracy. Selectivity availability is introduced in the L1 C/A code signal to deliberately degrade the user position accuracy.

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```
% p5_1.m generates MLS and G2 outputs and checks their delay time

% input
% k1, k2: positions of two taps
% k3: delay time
```

```
%
k=input('enter [k1 k2 k3] = ');
inp1=-ones(1,10); % initial condition of register
for j=1:1023;
  mlsout(j)=inp1(10); % MLS output
  modulo=inp1(2)*inp1(3)*inp1(6)*inp1(8)*inp1(9)*inp1(10);
  inp1(2:10)=inp1(1:9);
  inp1(1)=modulo;
  g2(j)=inp1(k(1))*inp1(k(2)); % G2 output
end
if mlsout==g2([k(3):1023 1:k(3)-1])
  disp('OK')
  else
  disp('not match')
end
% p5_2.m generates one of the 32 C/A codes written by D. Akos, modified
by J. Tsui
svnum=input ('enter the satellite number = '); % the Satellite's ID
number
% ca : a vector containing the desired output sequence
% the g2s vector holds the appropriate shift of the g2 code to generate
% the C/A code (ex. for SV#19 - use a G2 shift of g2s(19)=471)
g2s = [5;6;7;8;17;18;139;140;141;251;252;254;255;256;257;258;
469;470;471; ...
472;473;474;509;512;513;514;515;516;859;860;861;862];
g2shiftg2s(svnum,1);
% Generate G1 code
  % load shift register
    reg = -1*ones(1,10);
  %
  for i = 1:1023,
    g1(i) = reg(10);
    slave1 = reg(3)*reg(10);
    reg(1,2:10) = reg(1:1:9);
    reg(1) = save1;
  end,
%
% Generate G2 code
%
  load shift register
    reg = -1*ones(1,10);
  %
  for i = 1:1023,
    g2(i) = reg(10);
    save2 = reg(2)*reg(3)*reg(6)*reg(8)*reg(9)*reg(10);
```

```
    reg(1,2:10) = reg(1:1:9);
    reg(1) = save2;
  end,
  %
  % Shift G2 code
  %
  g2tmp(1,1:g2shift)=g2(1,1023-g2shift+1:1023);
  g2tmp(1,g2shift+1:1023)=g2(1,1:1023-g2shift);
  %
  g2 = g2tmp;
%
% Form single sample C/A code by multiplying G1 and G2
%
ss_ca = g1.*g2;
ca = ss_ca;
% Change to 1 0 outputs
ind1=find(ca==-1);
ind2=find(ca==1);
ca(ind1)=ones (1,length(ind1));
ca(ind2)=zeros (1,length(ind2));
ca(1:10) %print first 10 bits
```