Abstract

This paper is an attempt to generalize the results obtained earlier and presents the method of sensor fusion based on the Adaptive Fuzzy Kalman Filtering. This method has been applied to fuse position signals from the Global Positioning System (GPS) and Inertial Navigation System (INS) for the autonomous mobile vehicles. The presented method has been validated in 3-D environment and is of particular importance for guidance, navigation, and control of flying vehicles. The Extended Kalman Filter (EKF) and the noise characteristic have been modified using the Fuzzy Logic Adaptive System and compared with the performance of regular EKF.

This paper discusses extensively the GPS and INS measurement covariance and their influence on Kalman Filter performance. It has been demonstrated that the Fuzzy Adaptive Kalman Filter gives better results (more accurate) than the EKF.

Introduction

When navigating and guiding an autonomous vehicle, the position and velocity of the vehicle must be determined. The Global Positioning System (GPS) is a satellite-based navigation system that provides a user with the proper equipment access to useful and accurate positioning information anywhere on the globe [1]. However, several errors are associated with the GPS measurement. It has superior long-term error performance, but poor short-term accuracy. For many vehicle navigation systems, GPS is insufficient as a stand-alone position system. The integration of GPS and Inertial Navigation System (INS) is ideal for vehicle navigation systems. In generally, the short-term accuracy of INS is good; the long-term accuracy is poor. The disadvantages of GPS/INS are ideally cancelled. If the signal of GPS is interrupted, the INS enables the navigation system to coast along until GPS signal is reestablished [1]. The requirements for accuracy, availability and robustness are therefore achieved.

Kalman filtering is a form of optimal estimation characterized by recursive evaluation, and an internal model of the dynamics of the system being estimated. The dynamic weighting of incoming evidence with ongoing expectation produces estimates of the state of the observed system [2]. An extended Kalman filter (EKF) can be used to fuse measurements from GPS and INS. In this EKF, the INS data are used as a reference trajectory, and GPS data are applied to update and estimate the error states of this trajectory. The Kalman filter requires that all the plant dynamics and noise processes are exactly known and the noise processes are zero mean white noise. If the theoretical behavior of a filter and its actual behavior do not agree, divergence problems will occur. There are two kinds of divergence: Apparent divergence and True divergence [3][4]. In the apparent divergence, the actual estimate error covariance remains bounded, but it approaches a larger bound than does predicted error covariance. In true divergence, the actual estimation covariance eventually becomes infinite. The divergence due to modeling errors is critical in Kalman filter application. If, the Kalman filter is fed information that the process behaved one way, whereas, in fact, it behaves another way, the filter will try to continually fit a wrong process. When the measurement situation does not
provide enough information to estimate all the state variables of the system, in other words, the computed estimation error matrix becomes unrealistically small, and the filter disregards the measurement, then the problem is particularly severe. Thus, in order to solve the divergence due to modeling errors, we can estimate unmodeled states, but it add complexity to the filter and one can never be sure that all of the suspected unstable states are indeed model states[3]. Another possibility is to add process noise. It makes sure that the Kalman filter is driven by white noise, and prevents the filter from disregarding new measurement. In this paper, a fuzzy logic adaptive system (FLAS) is used to adjust the exponential weighting of an weighted EKF and prevent the Kalman filter from divergence. The fuzzy logic adaptive controller (FLAC) will continually adjust the noise strengths in the filter's internal model, and tune the filter as well as possible. The FLAC performance is evaluated by simulation of the fuzzy adaptive extended Kalman filtering scheme of Fig.1. This concept has been presented in [5] and applied to robot's navigation and control. For example such a procedure can be used for autonomous vehicle navigation and control [6]. This paper is an attempt to generalize the results obtained in [5] and to established procedures for the adaptive Kalman filter design [see Table 3,4,5]. Many details shown below have been already shown in [5] but are repeated in this paper for better clarity and understanding. Important new results are shown in Tables 3,4,5 as well as in Figures 5 and 6.

**Weighted EKF**

Because the processes of both GPS and INS are nonlinear, a linearization is necessary. An extended Kalman filter is used to fuse the measurements from the GPS and INS. To prevent divergence by keeping the filter from discounting measurements for large k, the exponential data weighting [4] is used.

The models and implementation equations for the weighted extended Kalman filter are:

Nonlinear dynamic model

\[ x_{k+1} = f(x_k, k) + w_k \]  

\[ w_k \sim N(0,Q) \]  

Nonlinear measurement model

\[ z_k = h(x_k, k) + v_k \]  

\[ v_k \sim N(0, R) \]  

Let us set the model covariance matrices equal to

\[ R_k = R \alpha^{-2(k+1)} \]  

\[ Q_k = Q \alpha^{-2(k+1)} \]  

where, \( \alpha > 1 \), and constant matrices Q and R. For \( \alpha > 1 \), as time k increases, the R and Q decrease, so that the most recent measurement is given higher weighting. If \( \alpha = 1 \), it is a regular EKF.

By defining the weighted covariance

\[ P_k^{\alpha^2} = P_k^{\alpha^2} \]  

The Kalman gain can be computed:

\[ K_k = P_k^{\alpha^2} H_k^T (H_k P_k^{\alpha^2} H_k^T + R \alpha^{-2(k+1)})^{-1} \]  

\[ = P_k^{\alpha^2} H_k^T (H_k P_k^{\alpha^2} H_k^T + R \alpha^{-2(k+1)})^{-1} \]  

The predicted state estimate is:

\[ \hat{x}_{k+1} = f(\hat{x}_k, k) \]  

The predicted measurement is:

\[ \hat{z}_k = h(\hat{x}_k, k) \]
The linear approximation equations can be presented in form:

\[ \Phi_k \approx \frac{\partial f(x, k)}{\partial x} \bigg|_{x=x_k} \]  

(9)

The predicted estimate on the measurement can be computed:

\[ \hat{x}_k = \hat{x}_k^- + K_k(z_k - \hat{z}_k) \]  

(10)

\[ H_k \approx \frac{\partial h(x, k)}{\partial x} \bigg|_{x=x_k^-} \]  

(11)

Computing the a priori covariance matrix:

\[ P_{k+1}^- = \Phi_k P_k \Phi_k^T + Q \alpha^{-2(k+1)} \]  

(12)

Re-writing (12) gives:

\[ P_{k+1}^{-} = \alpha^2 \Phi_k P_k \Phi_k^T + Q \]  

(13)

Computing the a posteriori covariance matrix gives:

\[ P_{k}^\alpha = (I - K_k H_k)P_{k}^{\alpha-} \]  

(14)

The initial condition is:

\[ P_{0}^{\alpha-} = P_0 \]

In equation (10), the term \( z_k - \hat{z}_k \) is called residuals or innovations. It reflects the degree to which the model fits the data.

### INS and GPS

The inertial navigation system (INS) consists of a sensor system, which includes accelerometers and gyros to measure accelerations and angular rates. By using these signals as input, the attitude angle and three-dimensional vectors of velocity and position are computed [7]. The errors in the measurements of force made by the accelerometers and the errors in the measurement of angular change in orientation with respect to inertial space made by gyroscopes are two fundamental error sources, which affect the error behavior of an inertial system. The inertial system error response, related to position, velocity, and orientation is divergent with time due to noise input [8]. There are biases associated with the accelerometers and gyros. In order to correct the errors of INS, the GPS measurements are used to estimate the inertial system errors, subtract them from the INS outputs, and then obtain the corrected INS outputs. There is number of errors in GPS, such as ephemeris errors, propagation errors, selective availability, multi-path, and receiver noise, etc. By using differential GPS (DGPS), most of the errors can be corrected, but the multi-path and receiver noise cannot be eliminated.

### Fuzzy Logic Adaptive System

It is assumed that both, the process noise \( w_k \) and the measurement noise \( v_k \) are zero-mean white sequences with known covariance \( Q \) and \( R \) in the Kalman filter. If the Kalman filter is based on a complete and perfectly tuned model, the residuals should be a zero-mean white noise process. If the residuals are not white noise, there is something wrong with the design and the filter is not performing optimally [4]. The Kalman filters will diverge or coverage to a large bound. In practice, parameters may be known only with some uncertainty or in order to reduce computation, we have to ignore some errors. Unfortunately, sometimes those unmodeled errors will become significant. One can readjust the assumed noise strengths in the filter's internal model, based on information obtained in real time from the measurements becoming available, so the filter is continually "tuned" as well as possible. The residuals can be used to adapt the filter. In fact, the residuals are the differences between actual measurements and best measurement predictions based on the filter's internal model. A well-tuned filter is that where the 95% of the autocorrelation function of innovation series should fall within the ± 2u boundary [9]. If the filter diverges, the residuals will not be zero mean and become larger.

There are a few papers on application of fuzzy logic to adapt the Kalman filter gain or parameters [5][10]. In [10], fuzzy logic is used to the on-line detection and correction of divergence in a single state Kalman filter. There were three inputs and two outputs to fuzzy logic controller (FLC), and 24 rules were used. The purpose of our fuzzy logic adaptive system...
(FLAS) is to detect the bias of measurements and prevent divergence of the extended Kalman filter. It has been applied in three axes — East (x), North (y), and Altitude (z) individually. There are two groups of fuzzy controllers. In those two fuzzy controllers, the covariance of the residuals and the mean of residuals are used as the inputs to both controllers for all three fuzzy inference engines. The exponential weighting $\alpha$ and the scales for three axes are the outputs. As an input to FLAS, the covariance of the residuals and mean values of residuals are used to decide the degree of divergence. The value of covariance $P_z$ relates to $R$. If a filter is performing optimally, the innovation is therefore a zero-mean white noise process. The equation for covariance of the residual is:

$$P_z = H_k P^{-1}_k H^T_k + R$$  \hspace{1cm} (15)$$

The first group, which output is $\alpha$ is used to detect the filter divergence. Generally, when the covariance is becoming large, and mean value is moving away from zero, the Kalman filter is becoming unstable. In this case, a large $\alpha$ will be applied. A large $\alpha$ means that process noises are added. It can ensure that in the model all states are sufficiently excited by the process noise. When the covariance is extremely large, there are some problems with the GPS measurements, so the filter cannot depend on these measurements anymore, and a smaller $\alpha$ will be used. By selecting appropriate $\alpha$, the fuzzy logic controller will adapt the Kalman filter optimally and try to keep the innovation sequence acting as zero-mean white noise.

The fuzzy logic controller uses 9 rules, such as:

- If the covariance of residuals is large and the mean value is zero Then $\alpha$ is zero.
- If the covariance of residuals is zero and the mean value is large Then $\alpha$ is small.

The second group, which output is scale is used to detect the change of measurement noise covariance $R$. From equation (15), the $R$ is related to the covariance of residual, the larger the covariance of residual, the more the measurement noise. When the fuzzy logic controller finds that the covariance of residual is larger than that expected, it applies a large scale to adjust the $\alpha$. A sample rule is:

- If the covariance of residuals is small and the mean values is small then the scale is large.

The fuzzy adaptive Kalman filtering has been used for guidance and navigation of mobile robots, especially for 3-D environment. The navigation of flying robots requires fast, and accurate on-line control algorithms. The "regular" Extended Kalman Filter requires high number of states for accurate navigation and positioning and is unable to monitor the parameters changing. The FLAC requires smaller number of states for the same accuracy and therefore it would need less computational effort. Alternatively, the same number of states (as in "regular" filter) would allow for more accurate navigation.
Table 1. Rule Table for $\alpha$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Z</th>
<th>S</th>
<th>L</th>
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</thead>
<tbody>
<tr>
<td>P</td>
<td>Z</td>
<td>Z</td>
<td>S</td>
</tr>
<tr>
<td>S</td>
<td>S</td>
<td>L</td>
<td>S</td>
</tr>
<tr>
<td>L</td>
<td>Z</td>
<td>NS</td>
<td>NS</td>
</tr>
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</table>

S --- Small; L --- Large; Z --- Zero; NS --- Negative Small

Table 2 Rule Table for Scale

<table>
<thead>
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<th>Scale</th>
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</tr>
<tr>
<td>L</td>
<td>L</td>
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Table 3 Comparison of theoretical and actual error variance (X-axis)

<table>
<thead>
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<th>Q</th>
<th>R</th>
<th>Theory</th>
<th>Actual</th>
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<tbody>
<tr>
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<td>5.3121</td>
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<tr>
<td>3Q</td>
<td>2R</td>
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<tr>
<td>5Q</td>
<td>4R</td>
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<td>8.3122</td>
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Table 4 Comparison of theoretical and actual error variance (Y-axis)

<table>
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<tr>
<td>5Q</td>
<td>4R</td>
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<td>7.7340</td>
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Table 5 Comparison of theoretical and actual error variance (Z-axis)

<table>
<thead>
<tr>
<th>Q</th>
<th>R</th>
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<td>3Q</td>
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<tr>
<td>5Q</td>
<td>4R</td>
<td>7.3417</td>
<td>2.5005</td>
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Simulation

The MATLAB codes developed by authors have been used to simulate and test the proposed method.

The state variables used in simulation are:

$$x_k = [x_k, \dot{x}_k, y_k, \dot{y}_k, z_k, \dot{z}_k, c\Delta t, \dot{c}\Delta t]$$  \hspace{1cm} (16)

The states are position, and velocity errors of the INS East, North, Altitude, GPS range bias and range drift. The designed covariance of GPS measurement $R$ is 5 [m$^2$]. It is assumed that the measurements of INS have some biases. In the first simulation (Fig. 5), the designed covariance $Q$ of INS are 0.0012 meter, 0.0012 meter, and 0.0012 meter for the East ($x$), North ($y$), and Altitude ($z$) respectively. The sample period is 1 second.

The simulations (Table 3, 4, 5 and Figure 5, 6) show that after the filter is stable, the actual error covariance of fuzzy logic controller EKF almost agrees with the theory error covariance. In the Table 3, 4, 5, the designed parameters are $Q$ and $R$. The $5Q$, $2R$ etc. mean that the real time parameters are 5 and 2 time large as the designed $Q$ and $R$. In figure 5, 6, The dashed lines are the covariance of theory EKF, and the solid lines are the covariance of fuzzy adaptive EKF.
Conclusions

In this paper, a fuzzy adaptive extended Kalman filter has been developed to detect and prevent the EKF from divergence. By monitoring the innovations sequences, the FLAS can evaluate the performance of an EKF. If the filter does not perform well, it would apply an appropriate weighting factor $\alpha$ to improve the accuracy of an EKF.

In the fuzzy logic controller, there are two group of fuzzy controllers and 18 rules therefore, little computational time is needed. When a designer lacks sufficient information to develop complete models or the parameters will slowly change with time, the fuzzy controller can be used to adjust the performance of EKF on-line, and it will remain sensitive to parameter variations by "remembering" most recent N date samples. It can be used to navigate and guide autonomous vehicles or robots and achieved a relatively accurate performance. The comparison between various design parameters of adaptive Kalman filter will help in proposing better and more accurate fuzzy logic rules and in general better and more stable FLC.

References


Figure 5 Actual and Theory Covariance for $5Q$ and $R$
Figure 6. Actual and Theory Covariance for 5Q and 4R