7.1. Clutter Spectrum

The power spectrum of stationary clutter (zero Doppler) can be represented by a delta function. However, clutter is not always stationary; it actually exhibits some Doppler frequency spread because of wind speed and motion of the radar scanning antenna. In general, the clutter spectrum is concentrated around \( f = 0 \) and integer multiples of the radar PRF \( f_r \), and may exhibit a small amount of spreading.

The clutter power spectrum can be written as the sum of fixed (stationary) and random (due to frequency spreading) components. For most cases, the random component is Gaussian. If we denote the stationary-to-random power ratio by \( W^2 \), then we can write the clutter spectrum as

\[
S_c(\omega) = \frac{\delta(\omega)}{1 + W^2} + \frac{\sigma_0}{(1 + W^2)} \exp\left(\frac{-(\omega - \omega_0)^2}{2\sigma_0^2}\right) (7.1)
\]

where \( \omega_0 = 2\pi f_0 \) is the radar operating frequency in radians per second, \( \sigma_0 \) is the rms frequency spread component (determines the Doppler frequency spread), and \( \delta(\omega) \) is the Weibull parameter.

The first term of the right-hand side of Eq. (7.1) represents the PSD for stationary clutter, while the second term accounts for the frequency spreading. Nevertheless, since most of the clutter power is concentrated around zero Doppler with some spreading (typically less than 100 Hz), it is customary to model clutter using a Gaussian-shaped power spectrum (which is easier to analyze than Eq. (7.1)). More precisely,
where $P_c$ is the total clutter power; $\sigma^2_\omega$ and $\omega_0$ were defined earlier. Fig. 7.1 shows a typical PSD sketch of radar returns when both target and clutter are present. Note that the clutter power is concentrated around DC and integer multiples of the PRF.

\[
S_c(\omega) = \frac{P_c}{\sqrt{2\pi\sigma^2_\omega}} \exp\left(\frac{-(\omega - \omega_0)^2}{2\sigma^2_\omega}\right)
\]

(7.2)

7.2. Moving Target Indicator (MTI)

The clutter spectrum is normally concentrated around DC ($f = 0$) and multiple integers of the radar PRF $f_r$, as illustrated in Fig. 7.2a. In CW radars, clutter is avoided or suppressed by ignoring the receiver output around DC, since most of the clutter power is concentrated about the zero frequency band. Pulsed radar systems may utilize special filters that can distinguish between slowly moving or stationary targets and fast moving ones. This class of filter is known as the Moving Target Indicator (MTI). In simple words, the purpose of an MTI filter is to suppress target-like returns produced by clutter, and allow returns from moving targets to pass through with little or no degradation. In order to effectively suppress clutter returns, an MTI filter needs to have a deep stop-band at DC and at integer multiples of the PRF. Fig. 7.2b shows a typical sketch of an MTI filter response, while Fig. 7.2c shows its output when the PSD shown in Fig. 7.2a is the input.

MTI filters can be implemented using delay line cancelers. As we will show later in this chapter, the frequency response of this class of MTI filter is periodic, with nulls at integer multiples of the PRF. Thus, targets with Doppler fre-
quencies equal to \( nf_r \) are severely attenuated. Since Doppler is proportional to target velocity \( (f_d = 2v/\lambda) \), target speeds that produce Doppler frequencies equal to integer multiples of \( f_r \) are known as blind speeds. More precisely,

\[
v_{\text{blind}} = \frac{n f_r}{2}; \quad n \geq 0
\]  

(7.3)

Radar systems can minimize the occurrence of blind speeds by either employing multiple PRF schemes (PRF staggering) or by using high PRFs where in this case the radar may become range ambiguous. The main difference between PRF staggering and PRF agility is that the pulse repetition interval (within an integration interval) can be changed between consecutive pulses for the case of PRF staggering.

![Figure 7.2. (a) Typical radar return PSD when clutter and target are present. (b) MTI filter frequency response. (c) Output from an MTI filter.](image)
**Fig. 7.3** shows a block diagram of a coherent MTI radar. Coherent transmission is controlled by the STAble Local Oscillator (STALO). The outputs of the STALO, \( f_{LO} \), and the COHerent Oscillator (COHO), \( f_C \), are mixed to produce the transmission frequency, \( f_{LO} + f_C \). The Intermediate Frequency (IF), \( f_C \pm f_d \), is produced by mixing the received signal with \( f_{LO} \). After the IF amplifier, the signal is passed through a phase detector and is converted into a base band. Finally, the video signal is inputted into an MTI filter.

**Figure 7.3.** Coherent MTI radar block diagram.

### 7.3. Single Delay Line Canceler

A single delay line canceler can be implemented as shown in **Fig. 7.4**. The canceler’s impulse response is denoted as \( h(t) \). The output \( y(t) \) is equal to the convolution between the impulse response \( h(t) \) and the input \( x(t) \). The single delay canceler is often called a “two-pulse canceler” since it requires two distinct input pulses before an output can be read.

The delay \( T \) is equal to the PRI of the radar \( (1/f_r) \). The output signal \( y(t) \) is

\[
y(t) = x(t) - x(t - T)
\]  \hspace{1cm} (7.4)

The impulse response of the canceler is given by

\[
h(t) = \delta(t) - \delta(t - T)
\]  \hspace{1cm} (7.5)
where \( \delta(t) \) is the delta function. It follows that the Fourier transform (FT) of \( h(t) \) is

\[
H(\omega) = 1 - e^{-j\omega T}
\]  \hspace{1cm} (7.6)

where \( \omega = 2\pi f \).

In the z-domain, the single delay line canceler response is

\[
H(z) = 1 - z^{-1}
\]  \hspace{1cm} (7.7)

The power gain for the single delay line canceler is given by

\[
|H(\omega)|^2 = H(\omega)H^*(\omega) = (1 - e^{-j\omega T})(1 - e^{j\omega T})
\]  \hspace{1cm} (7.8)

It follows that

\[
|H(\omega)|^2 = 1 + 1 - (e^{j\omega T} + e^{-j\omega T}) = 2(1 - \cos \omega T)
\]  \hspace{1cm} (7.9)

and using the trigonometric identity \((2 - 2\cos 2\theta) = 4(\sin\theta)^2\) yields

\[
|H(\omega)|^2 = 4(\sin(\omega T/2))^2
\]  \hspace{1cm} (7.10)

**MATLAB Function “single_canceler.m”**

The function “single_canceler.m” computes and plots (as a function of \( f/f_r \)) the amplitude response for a single delay line canceler. It is given in Listing 7.1 in Section 7.11. The syntax is as follows:

\[
[\text{resp}] = \text{single-canceler}(f/\text{fr})
\]

where \( f/\text{fr} \) is the number of periods desired. Typical output of the function “single_canceler.m” is shown in Fig. 7.5. Clearly, the frequency response of a
single canceler is periodic with a period equal to \( f_c \). The peaks occur at \( f = (2n + 1)/(2f_c) \), and the nulls are at \( f = nf_c \), where \( n \geq 0 \).

In most radar applications the response of a single canceler is not acceptable since it does not have a wide notch in the stop-band. A double delay line canceler has better response in both the stop- and pass-bands, and thus it is more frequently used than a single canceler. In this book, we will use the names “single delay line canceler” and “single canceler” interchangeably.

**7.4. Double Delay Line Canceler**

Two basic configurations of a double delay line canceler are shown in Fig. 7.6. Double cancelers are often called “three-pulse cancelers” since they require three distinct input pulses before an output can be read. The double line canceler impulse response is given by

\[
h(t) = \delta(t) - 2\delta(t - T) + \delta(t - 2T)
\]  

(7.11)

Again, the names “double delay line” canceler and “double canceler” will be used interchangeably. The power gain for the double delay line canceler is

![Figure 7.5. Single canceler frequency response.](image)
\[ |H(\omega)|^2 = |H_1(\omega)|^2 |H_1(\omega)|^2 \]  \hspace{1cm} (7.12)

And in the z-domain, we have

\[ H(z) = \left(1 - z^{-1}\right)^2 = 1 - 2z^{-1} + z^{-2} \]  \hspace{1cm} (7.14)

MATLAB Function “double_canceler.m”

The function “double_canceler.m” computes and plots (as a function of \( f/f_r \)) the amplitude response for a double delay line canceler. It is given in Listing 7.2 in Section 7.11. The syntax is as follows:

\[ [\text{resp}] = \text{double-canceler} \left( fofr \right) \]

where \( fofr \) is the number of periods desired.

Fig. 7.7 shows typical output from this function. Note that the double canceler has a better response than the single canceler (deeper notch and flatter pass-band response).
7.5. Delay Lines with Feedback (Recursive Filters)

Delay line cancelers with feedback loops are known as recursive filters. The advantage of a recursive filter is that through a feedback loop we will be able to shape the frequency response of the filter. As an example, consider the single canceler shown in Fig. 7.8. From the figure we can write

\[
y(t) = x(t) - (1 - K)w(t) \tag{7.15}
\]

\[
v(t) = y(t) + w(t) \tag{7.16}
\]

\[
w(t) = v(t - T) \tag{7.17}
\]

Applying the z-transform to the above three equations yields

\[
Y(z) = X(z) - (1 - K)W(z) \tag{7.18}
\]

\[
V(z) = Y(z) + W(z) \tag{7.19}
\]

\[
W(z) = z^{-1}V(z) \tag{7.20}
\]
Solving for the transfer function \( H(z) = Y(z)/X(z) \) yields

\[
H(z) = \frac{1 - z^{-1}}{1 - Kz^{-1}} \quad (7.21)
\]

The modulus square of \( H(z) \) is then equal to

\[
|H(z)|^2 = \frac{(1 - z^{-1})(1 - z)}{(1 - Kz^{-1})(1 - Kz)} = \frac{2 - (z + z^{-1})}{(1 + K^2) - K(z + z^{-1})} \quad (7.22)
\]

Using the transformation \( z = e^{j\omega T} \) yields

\[
z + z^{-1} = 2 \cos \omega T \quad (7.23)
\]

Thus, Eq. (7.22) can now be rewritten as

\[
|H(e^{j\omega T})|^2 = \frac{2(1 - \cos \omega T)}{(1 + K^2) - 2K \cos(\omega T)} \quad (7.24)
\]

Note that when \( K = 0 \), Eq. (7.24) collapses to Eq. (7.10) (single line canceler). Fig. 7.9 shows a plot of Eq. (7.24) for \( K = 0.25, 0.7, 0.9 \). Clearly, by changing the gain factor \( K \) one can control the filter response.

In order to avoid oscillation due to the positive feedback, the value of \( K \) should be less than unity. The value \((1 - K)^{-1}\) is normally equal to the number of pulses received from the target. For example, \( K = 0.9 \) corresponds to ten pulses, while \( K = 0.98 \) corresponds to about fifty pulses.
7.6. PRF Staggering

Target velocities that correspond to multiple integers of the PRF are referred to as blind speeds. This terminology is used since an MTI filter response is equal to zero at these values (see Fig. 7.7). Blind speeds can pose serious limitations on the performance of MTI radars and their ability to perform adequate target detection. Using PRF agility by changing the pulse repetition interval between consecutive pulses can extend the first blind speed to tolerable values.

In order to show how PRF staggering can alleviate the problem of blind speeds, let us first assume that two radars with distinct PRFs are utilized for detection. Since blind speeds are proportional to the PRF, the blind speeds of the two radars would be different. However, using two radars to alleviate the problem of blind speeds is a very costly option. A more practical solution is to use a single radar with two or more different PRFs.

For example, consider a radar system with two interpulse periods $T_1$ and $T_2$, such that

![Frequency response corresponding to Eq. (7.24). This plot can be reproduced using MATLAB program “fig7_9.m” given in Listing 7.3 in Section 7.11.](image)
where \( n_1 \) and \( n_2 \) are integers. The first true blind speed occurs when

\[
\frac{T_1}{T_2} = \frac{n_1}{n_2} \tag{7.25}
\]

This is illustrated in Fig. 7.10 for \( n_1 = 4 \) and \( n_2 = 5 \). Note that if \( n_2 = n_1 + 1 \), then the process of PRF staggering is similar to that discussed in Chapter 3. The ratio

\[
k_s = \frac{n_1}{n_2} \tag{7.27}
\]

is known as the stagger ratio. Using staggering ratios closer to unity pushes the first true blind speed farther out. However, the dip in the vicinity of \( 1/T_1 \) becomes deeper, as illustrated in Fig. 7.11 for stagger ratio \( k_s = 63/64 \). In general, if there are \( N \) PRFs related by

\[
\frac{n_1}{T_1} = \frac{n_2}{T_2} = \ldots = \frac{n_N}{T_N} \tag{7.28}
\]

and if the first blind speed to occur for any of the individual PRFs is \( v_{\text{blind}_1} \), then the first true blind speed for the staggered waveform is

\[
v_{\text{blind}} = \frac{n_1 + n_2 + \ldots + n_N}{N} v_{\text{blind}_1} \tag{7.29}
\]

### 7.7. MTI Improvement Factor

In this section two quantities that are normally used to define the performance of MTI systems are introduced. They are “Clutter Attenuation (CA)” and the MTI “Improvement Factor.” The MTI CA is defined as the ratio between the MTI filter input clutter power \( C_i \) to the output clutter power \( C_o \),

\[
CA = \frac{C_i}{C_o} \tag{7.30}
\]

The MTI improvement factor is defined as the ratio of the Signal to Clutter (SCR) at the output to the SCR at the input,

\[
I = \left( \frac{S_o}{C_o} \right) / \left( \frac{S_i}{C_i} \right) \tag{7.31}
\]

which can be rewritten as
Figure 7.10. Frequency responses of a single canceler. Top plot corresponds to $T_1$, middle plot corresponds to $T_2$, bottom plot corresponds to stagger ratio $T_1/T_2 = 4/3$. This plot can be reproduced using MATLAB program “fig7_10.m” given in Listing 7.4 in Section 7.11.
The ratio $I$ is the average power gain of the MTI filter, and it is equal to \( |H(\omega)|^2 \). In this section, a closed form expression for the improvement factor using a Gaussian-shaped power spectrum is developed. A Gaussian-shaped clutter power spectrum is given by

$$ I = \frac{S_o}{S_i} CA $$  \hspace{1cm} (7.32)

**Figure 7.11.** MTI responses, staggering ratio 63/64. This plot can be reproduced using MATLAB program “fig7_11.m” given in Listing 7.5 in Section 7.11.
where $P_c$ is the clutter power (constant), and $\sigma_t$ is the clutter rms frequency (which describes the clutter spectrum spread in the frequency domain). It is given by

$$\sigma_t = \frac{\sqrt{\sigma_v^2 + \sigma_s^2 + \sigma_w^2}}{\lambda}$$

(7.34)

$\sigma_v$ is the standard deviation for the clutter spectrum spread due to wind velocity; $\sigma_s$ is the standard deviation for the clutter spectrum spread due to antenna scanning; and $\sigma_w$ is the standard deviation for the clutter spectrum spread due to the radar platform motion (if applicable). It can be shown that$^1$

$$\sigma_v = \frac{2\sigma_w}{\lambda}$$

(7.35)

$$\sigma_s = 0.265 \left( \frac{2\pi}{\Theta_a T_{scan}} \right)$$

(7.36)

$$\sigma_s \approx \frac{v}{\lambda} \sin \theta$$

(7.37)

where $\lambda$ is the wavelength and $\sigma_w$ is the wind rms velocity; $\Theta_a$ is the antenna 3-db azimuth beamwidth (in radians); $T_{scan}$ is the antenna scan time; $v$ is the platform velocity; and $\theta$ is the azimuth angle (in radians) relative to the direction of motion.

The clutter power at the input of an MTI filter is

$$C_i = \int_{-\infty}^{\infty} \frac{P_c}{\sqrt{2\pi} \sigma_t} \exp \left( -\frac{f^2}{2\sigma_t^2} \right) df$$

(7.38)

Factoring out the constant $P_c$ yields

$$C_i = P_c \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi} \sigma_t} \exp \left( -\frac{f^2}{2\sigma_t^2} \right) df$$

(7.39)

It follows that

---

The clutter power at the output of an MTI is

\[ C_i = P_c \quad (7.40) \]

The frequency response for a single delay line canceler is given by Eq. (7.6). The single canceler power gain is given in Eq. (7.10), which will be repeated here, in terms of \( f \) rather than \( \omega \), as Eq. (7.42),

\[ |H(f)|^2 = 4 \left( \sin \left( \frac{\pi f}{f_r} \right) \right)^2 \quad (7.42) \]

It follows that

\[ C_o = \int_{-\infty}^{\infty} \frac{P_c}{\sqrt{2\pi} \sigma_i} \exp \left( -\frac{f^2}{2\sigma_i^2} \right) 4 \left( \sin \left( \frac{\pi f}{f_r} \right) \right)^2 df \quad (7.43) \]

Now, since clutter power will only be significant for small \( f \), then the ratio \( f/f_r \) is very small (i.e., \( \sigma_i \ll f_r \)). Consequently, by using the small angle approximation, Eq. (7.43) is approximated by

\[ C_o \approx \int_{-\infty}^{\infty} \frac{P_c}{\sqrt{2\pi} \sigma_i} \exp \left( -\frac{f^2}{2\sigma_i^2} \right) 4 \left( \frac{\pi f}{f_r} \right)^2 \frac{1}{f^2} \exp \left( -\frac{f^2}{2\sigma_i^2} \right) f^2 df \quad (7.44) \]

which can be rewritten as

\[ C_o = \frac{4P_c\pi^2}{f_r^2} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi} \sigma_i^2} \exp \left( -\frac{f^2}{2\sigma_i^2} \right) f^2 df \quad (7.45) \]

The integral part in Eq. (7.45) is the second moment of a zero mean Gaussian distribution with variance \( \sigma_i^2 \). Replacing the integral in Eq. (7.45) by \( \sigma_i^2 \) yields

\[ C_o = \frac{4P_c\pi^2}{f_r^2} \sigma_i^2 \quad (7.46) \]

Substituting Eqs. (7.46) and (7.40) into Eq. (7.30) produces
\[ CA = \frac{C_i}{C_o} = \left( \frac{f_r}{2\pi\sigma_f} \right)^2 \]  
(7.47)

It follows that the improvement factor for a single canceler is

\[ I = \left( \frac{f_r}{2\pi\sigma_f} \right)^2 \frac{S_o}{S_i} \]  
(7.48)

The power gain ratio for a single canceler is (remember that \(|H(f)|\) is periodic with period \(f_r\))

\[ \frac{S_o}{S_i} = |H(f)|^2 = \frac{1}{f_r} \int_{-f_r/2}^{f_r/2} 4 \left( \sin \frac{\pi f}{f_r} \right)^2 df \]  
(7.49)

Using the trigonometric identity \((2 - 2\cos 2\vartheta) = 4(\sin \vartheta)^2\) yields

\[ |H(f)|^2 = \frac{1}{f_r} \int_{-f_r/2}^{f_r/2} \left( 2 - 2\cos \frac{2\pi f}{f_r} \right) df = 2 \]  
(7.50)

It follows that

\[ I = 2 \left( \frac{f_r}{2\pi\sigma_f} \right)^2 \]  
(7.51)

The expression given in Eq. (7.51) is an approximation valid only for \(\sigma_f \ll f_r\). When the condition \(\sigma_f \ll f_r\) is not true, then the autocorrelation function needs to be used in order to develop an exact expression for the improvement factor.

Example:

A certain radar has \(f_r = 800\text{Hz}\). If the clutter rms is \(\sigma_v = 6.4\text{Hz}\) (wooded hills with \(\sigma_w = 1.16311 \text{Km/hr}\)), find the improvement factor when a single delay line canceler is used.

Solution:

In this case \(\sigma_f = \sigma_v\). It follows that the clutter attenuation CA is

\[ CA = \left( \frac{f_r}{2\pi\sigma_f} \right)^2 = \left( \frac{800}{(2\pi)(6.4)} \right)^2 = 395.771 = 25.974\text{dB} \]

and since \(S_o/S_i = 2 = 3\text{dB}\) we get

\[ I_{dB} = (CA + S_o/S_i)_{dB} = 3 + 25.97 = 28.974\text{dB}. \]
7.7.2. The General Case

A general expression for the improvement factor for the n-pulse MTI (shown for a 2-pulse MTI in Eq. (7.51)) is given by

\[ I = \frac{1}{Q^2(2(n-1)-1)!! \left(\frac{f_r}{2\pi\sigma_f}\right)^{2(n-1)}} \] (7.52)

where the double factorial notation is defined by

\[ (2n-1)!! = 1 \times 3 \times 5 \times \ldots \times (2n-1) \] (7.53)

\[ (2n)!! = 2 \times 4 \times \ldots \times 2n \] (7.54)

Of course 0!! = 1; \( Q \) is defined by

\[ Q^2 = \frac{1}{n} \sum_{i=1}^{\infty} A_i^2 \] (7.55)

where \( A_i \) are the Binomial coefficients for the MTI filter. It follows that \( Q^2 \) for a 2-pulse, 3-pulse, and 4-pulse MTI are respectively

\[ \left\{ \begin{array}{ccc} 1 & 1 & 1 \\ 2 & 20 & 70 \end{array} \right\} \] (7.56)

Using this notation, then the improvement factor for a 3-pulse and 4-pulse MTI are respectively given by

\[ I_{3-pulse} = 2\left(\frac{f_r}{2\pi\sigma_f}\right)^4 \] (7.57)

\[ I_{4-pulse} = 4\left(\frac{f_r}{3\pi\sigma_f}\right)^6 \] (7.58)

7.8. “MyRadar” Design Case Study - Visit 7

7.8.1. Problem Statement

The impact of surface clutter on the “MyRadar” design case study was analyzed. Assume that the wind rms velocity \( \sigma_w = 0.45 \text{m/s} \). Propose a clutter mitigation process utilizing a 2-pulse and a 3-pulse MTI. All other parameters are as calculated in the previous chapters.
7.8.2. A Design

In earlier chapters we determined that the wavelength is \( \lambda = 0.1 \, m \), the PRF is \( f_r = 1 \, KHz \), the scan rate is \( T_{scan} = 2 \, s \), and the antenna azimuth 3-db beamwidth is \( \Theta_a = 1.3^\circ \). It follows that

\[
\sigma_v = \frac{2\sigma_w}{\lambda} = \frac{2 \times 0.45}{0.1} = 9 \, Hz
\]  
\( \text{(7.59)} \)

\[
\sigma_s = 0.265 \left( \frac{2\pi}{\Theta_a T_{scan}} \right) = 0.265 \times \frac{2 \times \pi}{1.32 \times \frac{\pi}{180} \times 2} = 36.136 \, Hz
\]  
\( \text{(7.60)} \)

Thus, the total clutter rms spectrum spread is

\[
\sigma_t = \sqrt{\sigma_v^2 + \sigma_s^2} = \sqrt{81 + 1305.810} = \sqrt{1386.810} = 37.24 \, Hz
\]  
\( \text{(7.61)} \)

The expected clutter attenuation using a 2-pulse and a 3-pulse MTI are respectively given by

\[
I_{2\text{pulse}} = 2 \left( \frac{f_r}{2\pi\sigma_r} \right)^2 = 2 \times \left( \frac{1000}{2 \times \pi \times 37.24} \right)^2 = 36.531 \frac{W}{W} \Rightarrow 15.63 \, dB
\]  
\( \text{(7.62)} \)

\[
I_{3\text{pulse}} = 2 \left( \frac{f_r}{2\pi\sigma_r} \right)^4 = 2 \times \left( \frac{1000}{2 \times \pi \times 37.24} \right)^4 = 667.247 \frac{W}{W} \Rightarrow 28.24 \, dB
\]  
\( \text{(7.63)} \)

To demonstrate the effect of a 2-pulse and 3-pulse MTI on “MyRadar” design case study, the MATLAB program “myradar_visit7.m” has been developed. It is given in Listing 7.6 in Section 7.5. This program utilizes the radar equation with pulse compression. In this case, the peak power was established in Chapter 5 as \( P_t \leq 10 \, KW \). Figs. 7.12 and 7.13 show the desired SNR and the calculated SIR using a 2-pulse and a 3-pulse MTI filter respectively, for the missile case. Figs. 7.14 and 7.15 show similar output for the aircraft case.

One may argue, depending on the tracking scheme adopted by the radar, that for a tracking radar

\[
\sigma_r = \sigma_v = 9 \, Hz
\]  
\( \text{(7.64)} \)

since \( \sigma_s = 0 \) for a radar that employs a monopulse tracking option. In this design, we will assume a Kalman filter tracker. For more details the reader is advised to visit Chapter 9.
Figure 7.12. SIR for the missile case using a 2-pulse MTI filter.

Figure 7.13. SIR for the missile case using a 3-pulse MTI filter.
Figure 7.14. SIR for the aircraft case using a 2-pulse MTI filter.

Figure 7.15. SIR for the aircraft case using a 3-pulse MTI filter.
As clearly indicated by the previous four figures, a 3-pulse MTI filter would provide adequate clutter rejection for both target types. However, if we assume that targets are detected at maximum range (90 Km for aircraft and 55 Km for missile) and then are tracked for the rest of the flight, then 2-pulse MTI may be adequate. This is true since the SNR would be expected to be larger during track than it is during detection, especially when pulse compression is used. Nonetheless, in this design a 3-pulse MTI filter is adopted.

### 7.9. MATLAB Program and Function Listings

This section contains listings of all MATLAB programs and functions used in this chapter. Users are encouraged to rerun this code with different inputs in order to enhance their understanding of the theory.

#### Listing 7.1. MATLAB Function “single_canceler.m”

```matlab
function [resp] = single_canceler (fofr1)
    eps = 0.00001;
    fofr = 0:0.01:fofr1;
    arg1 = pi .* fofr;
    resp = 4.0 .* (sin(arg1)).^2;
    max1 = max(resp);
    resp = resp ./ max1;
    subplot(2,1,1)
    plot(fofr,resp,'k')
    xlabel ('Normalized frequency - f/fr')
    ylabel ('Amplitude response - Volts')
    grid
    subplot(2,1,2)
    resp=10.*log10(resp+eps);
    plot(fofr,resp,'k');
    axis tight
    grid
    xlabel ('Normalized frequency - f/fr')
    ylabel ('Amplitude response - dB')
```

#### Listing 7.2. MATLAB Function “double_canceler.m”

```matlab
function [resp] = double_canceler(fofr1)
    eps = 0.00001;
    fofr = 0:0.01:fofr1;
    arg1 = pi .* fofr;
    resp = 4.0 .*((sin(arg1)).^2);
    max1 = max(resp);
    resp = resp ./ max1;
    subplot(2,1,1)
    plot(fofr,resp,'k')
    xlabel ('Normalized frequency - f/fr')
    ylabel ('Amplitude response - Volts')
    grid
    subplot(2,1,2)
    resp=10.*log10(resp+eps);
    plot(fofr,resp,'k');
    axis tight
    grid
    xlabel ('Normalized frequency - f/fr')
    ylabel ('Amplitude response - dB')
```

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resp = 4.0 .* ((sin(arg1)).^2);
max1 = max(resp);
resp = resp ./ max1;
resp2 = resp .* resp;
subplot(2,1,1);
plot(fofr,resp,'k--',fofr, resp2,'k');
ylabel ('Amplitude response - Volts')
resp2 = 20. .* log10(resp2+eps);
resp1 = 20. .* log10(resp+eps);
subplot(2,1,2)
plot(fofr,resp1,'k--',fofr,resp2,'k');
legend ('single canceler','double canceler')
xlabel ('Normalized frequency f/fr')
ylabel ('Amplitude response - dB')

Listing 7.3. MATLAB Program “fig7_9.m”
clear all
fofr = 0:0.001:1;
arg = 2.*pi.*fofr;
nume = 2.*(1.-cos(arg));
den11 = (1. + 0.25 * 0.25);
den12 = (2. * 0.25) .* cos(arg);
den1 = den11 - den12;
den21 = 1.0 + 0.7 * 0.7;
den22 = (2. * 0.7) .* cos(arg);
den2 = den21 - den22;
den31 = (1.0 + 0.9 * 0.9);
den32 = ((2. * 0.9) .* cos(arg));
den3 = den31 - den32;
resp1 = nume ./ den1;
resp2 = nume ./ den2;
resp3 = nume ./ den3;
plot(fofr,resp1,'k',fofr,resp2,'k-.',fofr,resp3,'k--');
xlabel('Normalized frequency')
ylabel('Amplitude response')
legend('K=0.25','K=0.7','K=0.9')
grid
axis tight

Listing 7.4. MATLAB Program “fig7_10.m”
clear all
fofr = 0:0.001:1;
\begin{verbatim}
\texttt{clear all}
\texttt{fofr = 0.01:0.001:32;}
\texttt{a = 63.0 / 64.0;}
\texttt{term1 = (1. - 2.0 .* cos(a*2*pi*fofr) + cos(4*pi*fofr)).^2;}
\texttt{term2 = (-2. .* sin(a*2*pi*fofr) + sin(4*pi*fofr)).^2;}
\texttt{resp = 0.25 .* sqrt(term1 + term2);}
\texttt{resp = 10. .* log(resp);}
\texttt{plot(fofr,resp);}
\texttt{axis([0 32 -40 0]);}
\texttt{grid}
\end{verbatim}

Listing 7.5. MATLAB Program “fig7_11.m”

\begin{verbatim}
\texttt{clear all}
\texttt{fofr = 0.01:0.001:32;}
\texttt{a = 63.0 / 64.0;}
\texttt{f1 = 4.0 .* fofr;}
\texttt{f2 = 5.0 .* fofr;}
\texttt{arg1 = pi .* f1;}
\texttt{arg2 = pi .* f2;}
\texttt{resp1 = abs(sin(arg1));}
\texttt{resp2 = abs(sin(arg2));}
\texttt{resp = resp1+resp2;}
\texttt{max1 = max(resp);}
\texttt{resp = resp./max1;}
\texttt{plot(fofr,resp1,fofr,resp2,fofr,resp);}
\texttt{xlabel('Normalized frequency f/fr')}
\texttt{ylabel('Filter response')}
\end{verbatim}

Listing 7.6. MATLAB Program “myradar_visit7.m”

\begin{verbatim}
\texttt{clear all}
\texttt{close all}
\texttt{clutter_attenuation = 28.24;}
\texttt{thetaA= 1.33; \% antenna azimuth beamwidth in degrees}
\texttt{thetaE = 11; \% antenna elevation beamwidth in degrees}
\texttt{hr = 5.; \% radar height to center of antenna (phase reference) in meters}
\texttt{htm = 2000.; \% target (missile) height in meters}
\texttt{hta = 10000.; \% target (aircraft) height in meters}
\texttt{SL = -20; \% radar rms sidelobes in dB}
\texttt{sigma0 = -15; \% clutter backscatter coefficient in dB}
\texttt{b = 1.0e6; \%1-MHz bandwidth}
\texttt{t0 = 290; \% noise temperature 290 degrees Kelvin}
\texttt{f0 = 3e9; \% 3 GHz center frequency}
\texttt{pt = 114.6; \% radar peak power in KW}
\texttt{f = 6; \% 6 dB noise figure}
\texttt{l = 8; \% 8 dB radar losses}
\end{verbatim}
range = linspace(25,120,500); % radar slant range 25 to 120 Km, 500 points
% calculate the clutter RCS and the associated CNR for both targets
[sigmaCa,CNRa] = clutter_rcs(sigma0, thetaE, thetaA, SL, range, hr, hta, pt, f0, b, t0, f, l, 2);
[sigmaCm,CNRm] = clutter_rcs(sigma0, thetaE, thetaA, SL, range, hr, htm, pt, f0, b, t0, f, l, 2);
close all

np = 4;
pfa = 1e-7;
pdm = 0.99945;
pda = 0.99812;
% calculate the improvement factor
Im = improv_fac(np, pfa, pdm);
Ia = improv_fac(np, pfa, pda);
% caculate the integration loss
Lm = 10*log10(np) - Im;
La = 10*log10(np) - Ia;
pt = pt * 1000; % peak power in watts
range_m = 1000 .* range; % range in meters
g = 34.5139; % antenna gain in dB
sigmam = 0.5; % missile RCS m squared
sigmaa = 4; % aircraft RCS m squared
nf = f; %noise figure in dB
loss = l; % radar losses in dB
losstm = loss + Lm; % total loss for missile
lossta = loss + La; % total loss for aircraft
% modify pt by np*pt to account for pulse integration
SNRm = radar_eq(np*pt, f0, g, sigmam, t0, b, nf, losstm, range_m);
SNRa = radar_eq(np*pt, f0, g, sigmaa, t0, b, nf, lossta, range_m);
snrm = 10.^(SNRm./10);
snra = 10.^(SNRa./10);
CNRm = CNRm - clutter_attenuation;
CNRa = CNRa - clutter_attenuation;
cnrm = 10.^(CNRm./10);
cnra = 10.^(CNRa./10);
SIRm = 10*log10(snrm ./ (1+cnrm));
SIRA = 10*log10(snra ./ (1+cnra));
figure(3)
plot(range, SNRm,'k', range, CNRm,'k :', range,SIRm,'k -.'
grid
legend('Desired SNR; from Chapter 5', 'CNR', 'SIR with 3-pulse', 'MTI filter')
xlabel('Slant Range in Km')
ylabel('dB')
title('Missile case; 21-frame cumulative detection')
figure(4)
plot(range, SNRa,'k', range, CNRa,'k :', range, SIRa,'k -.' )
grid
legend('Desired SNR; from Chapter 5', 'CNR', 'SIR with 3-pulse', 'MTI filter')
xlabel('Slant Range in Km')
ylabel('dB')
title('Aircraft case; 21-frame cumulative detection')