Abstract—Two methods for grating synthesis which have appeared in the literature recently are compared directly. In particular, we point out the similarity between the two; both algorithms are based on propagation of the fields through the structure with simultaneous evaluation of the coupling coefficient according to simple causality arguments (layer-peeling algorithms). The first published method (the discrete layer-peeling algorithm) is reformulated in a simpler, more efficient way, and it is shown that its implementation can be made exact. For mathematical comparison, a derivation of the second method (the continuous layer-peeling algorithm) is presented. The methods are compared both mathematically and numerically. We find that the discrete layer-peeling algorithm is significantly faster and can be more stable than its continuous counterpart, whereas the continuous algorithm offers some advantages in flexibility.

Index Terms—Differential Riccati equations, electromagnetic scattering inverse problems, gratings, optical fiber communication, optical fiber coupling.

I. INTRODUCTION

THE SYNTHESIS of fiber Bragg gratings has recently attracted many researchers in the field of fiber optics [1]–[6]. The problem amounts to finding the grating structure (grating amplitude and phase) from a specified, complex spectrum. Synthesis is useful both as a design tool, and for characterization of already fabricated gratings with complex profiles. It has been a common view that the synthesis problem is complicated, especially compared to the well-known direct problem of computing the reflection spectrum from a grating structure. In a recent article [2], Poladian points out that the synthesis problem actually is as simple as the direct problem; one can find the grating structure from the reflection spectrum simply by propagating the fields along the grating structure, while simultaneously evaluating the grating strength using a simple causality argument. In fact, this result has been appreciated in a number of other fields of physics, and this layer-peeling (sometimes called “differential inverse scattering”) approach has been used to analyze such systems as transmission lines, vibrating strings, layered acoustic media, and particle scattering in quantum mechanics [7], [8].

The purpose of this paper is to first simplify the discrete layer-peeling (DLP) synthesis method proposed by Feced et al. [1] to improve its clarity and efficiency, and then compare this method with the continuous method described by Poladian [2], referred to here continuous layer-peeling (CLP). We point out that the continuous method essentially makes use of the same principle as the discrete method. Both are based on the following procedure [8]: by causality, the coupling coefficient at the front end of the grating is determined only by the leading edge of the impulse response, since at the very beginning of the impulse response, light does not have time to propagate more deeply into the grating and, hence, “sees” only the first layer. After computing the value of the coupling coefficient in the first layer, the fields are propagated to the next layer of the grating. This propagation is essentially the direct (forward) calculation and can be accomplished by numerical integration of the coupled-mode equations or by the transfer matrix method described below. Now one is in the same situation as at the beginning, since the effect of the first layer is “peeled off.” The process is continued to the back of the grating, so that the entire grating structure is reconstructed. Note that the layer thickness must be chosen sufficiently small that the (complex) coupling coefficient can be approximated as constant throughout the layer.

While the principle behind the DLP and CLP methods is the same, the implementation of the two methods differs. The DLP model is based on a discretization of the grating model itself; i.e., the grating is approximated as a series of discrete, complex reflectors. Once this approximation is made, though, the reconstruction or synthesis algorithm can be made exact. In the CLP method, the coupling process is treated continuously, but discretization is introduced in the actual computation of the coupling coefficient and the propagation of the fields. This qualitative comparison is described more precisely and in mathematical terms below. Differences in the implementations of the two methods lead to differences in the numerical performance. As illustrated in the examples below, we find that the DLP algorithm is significantly faster and often more stable than its continuous counterpart, whereas the continuous algorithm can be more flexible. For example, in contrast to the DLP method, the CLP method allows the layer thickness to be varied for a given frequency bandwidth.

The remainder of this paper is organized as follows. In Section II, we reformulate the DLP algorithm reported in [1], showing that a “moving reference frame” simplifies the algorithm and improves its performance. In Section III, a derivation is provided for the CLP algorithm, since this has not been published yet, and it is necessary for a clear comparison between the two methods. In Section IV, we compare the two methods mathematically, showing that they are equivalent in certain limits and how they differ in others. Also, several numerical examples are provided to demonstrate the relative
performance of the two methods. Finally, in Section V, we summarize and draw conclusions.

II. DLP

In this section, we describe the layer-peeling method for synthesis of gratings from an inherently discrete model [1]. It was first developed by geophysicists like Goupillaud and Robinson, and was extended and generalized by Bruckstein et al. (see [7] and [8] for excellent reviews).

The starting point for the discretization is the well-known transfer matrix $T$ of a Bragg grating, which connects the fields at the point $z + \Delta$ with the fields at $z$, as in (1), shown at the bottom of the page. Here, $u(z, \delta)$ and $v(z, \delta)$ are the slowly varying amplitudes of the forward and backward propagating fields, respectively, and $\gamma^2 = |q|^2 - \delta^2$, where $q = q(z)$ is the coupling coefficient and $\delta = \beta - \beta_B$ is the wavenumber detuning compared to a Bragg design wavenumber $\beta_B$. The implicit harmonic time dependence is assumed to be $\exp(-i\omega t)$, and the spatial dependence $\exp(i\pm\beta z)$ has been removed from the fields. We refer to (1) as the piecewise uniform model, since the grating is considered uniform in the interval $[z, z+\Delta]$. A further discretization approximation involves replacement of the matrix $T$ by the product of two transfer matrices $T_\Delta T_p$ one of them ($T_p$) describing a discrete reflector, and the other ($T_\Delta$) describing the pure propagation of the fields [1], or

$$T_p = (1 - |\rho|^2)^{-1/2} \begin{bmatrix} 1 - \rho & -\rho^* \\ -\rho^* & 1 \end{bmatrix}$$

$$T_\Delta = \begin{bmatrix} \exp(i\delta\Delta) & 0 \\ 0 & \exp(-i\delta\Delta) \end{bmatrix}$$

The discrete, complex reflection coefficient is given by

$$\rho = -\tanh(|q|\Delta)|q|^\delta.$$  (4)

The transfer matrix $T_p$ can be obtained from $T$ by letting all coupling take place at a single point in (1); i.e., $|q| \to \infty$, while the product $q\Delta$ remains constant. It can also be written directly since it merely describes the standard transfer matrix of a discrete reflector with $\rho$ and $-\rho^*$ the reflection coefficients from the left and right, respectively, and $(1 - |\rho|^2)^{-1/2}$ the transmission coefficient for both directions.

The discrete model of the entire grating is, thus, a series of $N$ discrete, complex reflectors with a distance $\Delta$ between all reflectors. From a realizable complex reflection spectrum $r_1(\delta)$, we wish to reconstruct the complex reflector amplitudes $\rho_j$, $j = 1, 2, \ldots, N$. Define the forward- and backward-propagating fields before the $j$th section as $u_j(\delta)$ and $v_j(\delta)$, see Fig. 1. The fields before the first section are then given by

$$\begin{bmatrix} u_1(\delta) \\ v_1(\delta) \end{bmatrix} = \begin{bmatrix} 1 \\ r_1(\delta) \end{bmatrix}.$$  (5)

The causality argument illustrates how to find the complex amplitude of the first reflector. We note that the impulse response of the reflector stack for time $t = 0$ is independent of the reflectors $\rho_j$ for $j \geq 2$ because light does not have sufficient time to propagate to and from the second and higher reflectors. Thus, when looking at the impulse response of the stack for $t = 0$, we obtain the same response as if only the first reflector were present. Therefore, we can compute $\rho_1$ from the inverse Fourier transform of $r_1(\delta) = v_1(\delta)/u_1(\delta)$ evaluated at time $t = 0$. Since we then know $\rho_1$, we can use $T_\Delta T_p$ to transfer the fields to the next section. At this point, we find ourselves in the same situation as with the first reflector, so, in effect, the first layer is “peeled off.” This procedure can be repeated until the entire series of reflectors is determined. The coupling function $g(z)$ is then determined from (4).

Using (2) and (3), it is straightforward to show that the task of transferring the fields by the transfer matrix product $T_\Delta T_p$ can be described in terms of the local reflectivities as

$$r_2(\delta) = \exp(-i2\delta\Delta)\frac{r_1(\delta) - \rho_1}{1 - \rho_1^* r_2(\delta)}$$  (6)

where $r_j(\delta) = v_j(\delta)/u_j(\delta)$. It is interesting to note that (6) is equivalent to a recursion formula proposed by Schur for testing the boundedness of an analytic function outside the unit circle of the complex plane [8].

To obtain an explicit expression for the determination of $\rho_1$ by the inverse Fourier transform, we note that the spectrum $r_1(\delta)$ can be written as a discrete-time Fourier transform (Fourier series) of the impulse response $h_1(\tau)$.

$$r_1(\delta) = \sum_{\tau=-\infty}^{\infty} h_1(\tau) \exp(i\delta\tau 2\Delta)$$  (7)
because the impulse response is discrete with the sample period $2\Delta$, which is equal to the “round-trip” propagation length of one layer. We have defined $\tau = t/2\Delta$ as the discrete time variable with $t$ as the normalized time. Since the impulse response for $\tau = 0$ is the same as if only the first reflector were present, we can see that $\rho_1$ is simply the zeroth Fourier coefficient of the series (7), or

$$\rho_1 = h_1(0) = \frac{\Delta}{\pi} \int_{-\pi/2\Delta}^{\pi/2\Delta} r_1(\delta) \, d\delta.$$  \hspace{2cm} (8)

Thus far, the model is based on inherently discrete functions that describe the grating structure. For numerical implementation, the spectral dependence must also be discrete, and hence, the calculation of $\rho_1$ by the inverse Fourier transform of $r_1(\delta)$ can be achieved by the discrete Fourier transform

$$\rho_1 = \frac{1}{M} \sum_{m=1}^{M} r_1(m)$$ \hspace{2cm} (9)

where $r_1(m)$ denotes a discrete version of the spectrum $r_1(\delta)$ in the range $|\delta| \leq \pi/2\Delta$, and $M \geq N$ is the number of wavelengths in the spectrum. Note that (7)–(9) are valid for all layers $j$ by substituting $1 \rightarrow j$ in the subscripts, because the reference plane is transferred to the actual layer through (6).

The desired impulse response $h_1(\tau)$ must be nonzero for $\tau \geq 0$, and if necessary, it should therefore be apodized to reduce the Gibbs phenomenon, and shifted so that it starts at $\tau = 0$ [1]. The apodizing-windowing procedure reduces the undesirable oscillations in the spectrum due to the finite length of the desired impulse response that is represented in the computer. Note that $M$, the number of wavelengths in the discrete spectrum, which is equal to the number of points in the impulse response, should be greater than or equal to the number of layers $N$. The number $N$ is chosen from the desired grating length or detuning range, or is chosen such that we obtain a desired accuracy in the realized spectrum.

The DLP algorithm may be summarized in the following simple steps:

1) start with a physically realizable reflection coefficient $r_1(\delta)$ (see the Appendix);
2) compute $\rho_1$ from (9);
3) propagate the fields using the transfer matrices (2)–(4) or the equivalent expression (6).
4) repeat step 2) until the entire grating structure is determined.

A count of the number of operations shows that the running time is of the same order $O(MN^2)$ as the conventional approach for computing the direct problem, i.e., computing the spectrum $r_1(\delta)$ from the reflector amplitudes $\rho_j$ using the transfer-matrix. We also note that the inverse problem is actually as simple as the forward problem.

A difference between the approach described here and that of Feced [1] is that we move the reference plane as we propagate through the structure (or peel off the layers). The reference plane in [1] is always located at $z = 0$, and, consequently, it is necessary to take into account the propagation of the fields back and forth from $z = 0$ to the layer being reconstructed. While this approach certainly works, it unnecessarily increases the computation time and the clarity of the presentation.

We conclude this section by discussing the implications of choosing the frequency domain vs. the time domain for implementation of the layer-peeling algorithm. In a similar fashion to that chosen by Feced [1], we implement the algorithm in the frequency domain. The main disadvantage with this choice is that, strictly speaking, the synthesis algorithm is not exact unless we have perfect spectral resolution ($M \rightarrow \infty$). However, this is not a fundamental limitation of the DLP method, because the algorithm can be made exact by transforming it into the time domain [7]. By an inverse Fourier transform of the discrete transfer-matrix relation, we find

$$\begin{bmatrix}
    u_{j+1}(t + \Delta) \\
    u_{j+1}(t - \Delta)
\end{bmatrix} = \begin{bmatrix}
    1 - |\rho_j|^2 & |\rho_j|^2 \\
    -|\rho_j| & 1
\end{bmatrix} \begin{bmatrix}
    u_j(t) \\
    v_j(t)
\end{bmatrix}$$ \hspace{2cm} (10)

where $u_j(t)$ and $v_j(t)$ are the inverse Fourier transforms of the fields $u_j(\delta)$ and $v_j(\delta)$, respectively. Since the reference plane for the fields $u_j(t)$ and $v_j(t)$ is fixed at $z = 0$, the local reflectivities are simply

$$\rho_j = \frac{v_j(z=0)}{u_j(z=0)}.$$ \hspace{2cm} (11)

The vector $u_1(t)$ is initialized to the unit impulse function, and $u_1(t)$ is set equal to the desired impulse response in agreement with (5). From (10) and (11), we can then determine the local reflectivities exactly from the impulse response (by $O(N^2)$ operations), and we, therefore, expect better accuracy than with the frequency-domain DLP implementation. However, for the frequency-domain approach, if the number of spectral points $M$ is set significantly larger than $N$, for example $M = 2N$, the performance in most practical situations is about the same as for the time-domain implementation. Finally, we note that only the first $N$ points of the impulse response are needed to reconstruct the grating up to layer $N$. In principle, this requirement applies frequency-domain DLP also, but since errors associated with the utilization of band-limited reflection spectra are propagated and thereby exacerbated by (6), the accuracy of the frequency-domain DLP procedure improves when we increase the number of points in the impulse response (i.e., increase the spectral resolution).

III. CLP

The CLP method was first developed by Bruckstein et al. and Corones et al. [8], [9]. In this section, we summarize the main principles of the method for synthesis of Bragg gratings, particularly as they relate to the DLP method. The synthesis relation resulting from the causality argument is derived, and it is shown that the method introduced in [2] is clearly a version of the CLP method.

The continuous model for coupling of counter-propagating modes in a grating is described by the familiar coupled-mode equations

$$\frac{d}{dz} u(z, \delta) = + i \delta u(z, \delta) + q(z) v(z, \delta)$$
$$\frac{d}{dz} v(z, \delta) = - i \delta v(z, \delta) + q^*(z) u(z, \delta)$$ \hspace{2cm} (12)
where all of the parameters are defined in the previous section. The two coupled equations can be combined into a single Riccati equation for the local reflectivity \(r(z, \delta) = r(z, \delta)/u(z, \delta)\) by computing \(dr(z, \delta)/dz\) and substituting the derivatives from (12)

\[
\frac{d}{dz} r(z, \delta) = -2i\delta r(z, \delta) - q(z)r(z, \delta)^2 + q^4(z). \tag{13}
\]

Equation (13) can be solved numerically by Runge–Kutta integration methods. Thus, the model itself is not inherently discrete (like a series of localized reflectors); nevertheless, some form of discretization must be introduced in order to numerically solve the coupled-mode or Riccati equations to propagate the fields.

As indicated previously, inverse scattering relies heavily on causality, so in order to find a relation analogous to (8), we must also consider the time domain in the description of propagation in the grating, or

\[
\frac{\partial u(z, t)}{\partial t} = -\frac{\partial u(z, t)}{\partial t} + q(z)u(z, t)
\]

\[
\frac{\partial v(z, t)}{\partial t} = +\frac{\partial v(z, t)}{\partial t} + q^4(z)u(z, t) \tag{14}
\]

where (14) is obtained by inverse Fourier transform of (12). Note that the time \(t\) here is normalized such that the velocity of the waves equals unity. Now, we imagine that the grating is probed by a wave with a leading impulse. The grating is assumed to be dark at \(t = 0\), so by causality, the fields must vanish for \(t < z\) and have the form [8]

\[
u(z, t) = \delta(t - z) + H(t - z)\tilde{u}(z, t)
\]

\[
u(z, t) = H(t - z)\tilde{v}(z, t) \tag{15}
\]

where \(\tilde{u}, \tilde{v}\) functions describing the fields in the grating;

\(H(t - z)\) unit step function that is 0 for \(t < z\) and 1 for \(t \geq z\);

\(\delta(t - z)\) Dirac delta function.

By substituting (15) into the second part of (14), and equating the coefficients of \(\delta(t - z)\) on both sides, we obtain

\[
q^4(z) = -2\tilde{v}(z, t = z^+) = -2\tilde{v}(z, t = z^+). \tag{16}
\]

By the initial value theorem for the unilateral Fourier transform (Laplace transform) [10], we have \(v(z, t) = 0^+ = -\lim_{\delta \to -\infty} i\delta v(z, \delta)\), and therefore \(\nu(z, t = z^+) = -\lim_{\delta \to -\infty} i\delta v(z, \delta) \exp(-i\delta z)\) from the shift property of the Fourier transform. Hence, we can write

\[
q^*(z) = \lim_{\delta \to -\infty} 2i\delta v(z, \delta) \exp(-i\delta z) = \lim_{\delta \to -\infty} 2i\delta r(z, \delta) \tag{17}
\]

where the last equality results from the fact that \(u(z, \delta) \to \exp(i\delta z)\) as \(\delta \to -\infty\) according to (15). By using the initial value theorem once again, we can see that the complex conjugate of the coupling coefficient is equal to the leading edge of the impulse response multiplied by a factor of \(-2\), or

\[
q^*(z) = -1/\pi \int_{-\infty}^{\infty} r(z, \delta) \exp(-i\delta z) d\delta. \tag{18}
\]

In [2], this relation is incorrectly written as \(q^*(z) = -1/\pi \int_{-\infty}^{\infty} r(z, \delta) d\delta\); after translation to our convention for the coupled-mode equations. However, because the impulse response is discontinuous at \(t = 0\), the Fourier integral evaluated at \(t = 0\) is given by the average of the impulse response at \(t = 0^-\) and \(t = 0^+\). Therefore, one should either multiply the synthesis relation in [2] by a factor of 2, or be careful to write the relation explicitly in the form (18). This problem does not occur in the DLP algorithm because there we choose the spectrum \(r_j(\delta)\) to be the discrete-time Fourier transform of the discrete impulse response (see discussion in Section IV).

The CLP algorithm is, thus, performed by following these simple steps:

1) start with a physically realizable reflection coefficient \(r(0, \delta)\) (see the Appendix);
2) compute \(q(z)\) for the present \(z\) value from (18) or

\[
q^*(z) = -1/\pi \int_{-\infty}^{\infty} r(z, \delta) d\delta, \tag{19}
\]

(the integral may be evaluated using standard numerical integration routines, such as Simpson’s rule);
3) propagate the fields using the transfer matrix relation (1), or by integration of (13) using a backward difference scheme;
4) repeat step 2) until the entire grating structure is determined.

The primary sources of error in the CLP procedure are the evaluation of the synthesis integral (19) and the propagation of the fields.

IV. COMPARISON AND NUMERICAL PROPERTIES

In this section, we first consider the similarities and differences between the two methods from a fundamental, mathematical standpoint. Then we compare the numerical performance using several realistic examples.

First, we expect that the two methods should be equivalent in the limit where the discretization step (layer thickness for matrix multiplication) approaches zero. By letting \(\Delta \to 0\) in (4), we obtain \(\tilde{\rho} \to -q^*\Delta\). Substituting this result into (8) gives

\[
qu^*(j\Delta) = -\frac{1}{\pi} \int_{-\pi/2\Delta}^{\pi/2\Delta} r_j(\delta) d\delta \tag{20}
\]

for the \(j\)th layer. In the limit of small \(\Delta\), we obtain the relation (19) (divided by \(2\)). However, we must be careful at this point, because the spectra \(r_j(\delta)\) and \(r(j\Delta, \delta)\) are not exactly equivalent to one another. For a fixed position given by \(z = j\Delta\), the spectrum \(r(j\Delta, \delta)\) is the Fourier transform of a continuous impulse response, whereas \(r_j(\delta)\) is a Fourier series with a discrete impulse response as its coefficients. While the determination of the impulse response using \(r(j\Delta, \delta)\) requires a factor of 2 to fix the “wrong” convergence of the inverse Fourier integral at \(t = 0\), the determination of the Fourier coefficient (8) from \(r_j(\delta)\) does not. With this subtle difference taken into account, relations (19) and (20) are consequently equivalent in the limit \(\Delta \to 0\).

Owing to the nonequivalence between \(r_j(\delta)\) and \(r(j\Delta, \delta)\), the target spectra \(r_j(\delta)\) and \(r(0, \delta)\), the starting physical realizable spectrum for DLP and CLP, respectively, should ideally be specified in different ways. The discrete model spectrum \(r_k(\delta)\) should equal a discrete-time Fourier transform of a causal, discrete impulse response, whereas the continuous model spectrum \(r(0, \delta)\) should equal the Fourier transform of a causal, continuous impulse response. Therefore, if one starts with a true
grating spectrum in both DLP and CLP (reconstruction problems), the DLP will give wrong result for the coupling coefficient of the first section, whereas the CLP will give the right answer. Similarly, the situation will be reversed in synthesis problems where one starts with a spectrum that is a discrete Fourier transform of a discrete impulse response. Usually, this does not impose any problems as long as the grating starts smoothly at the front end. If this is not the case, one could fix the algorithm at the first layer by including an extra factor of 2 in (9) for reconstruction problems, and exclude the extra factor of 2 in (19) for synthesis problems.

To make the equivalence between the two methods clearer, it is also instructive to derive the synthesis relation (8) from a more physical standpoint than that of the derivation in Section II. As above, we assume the grating is comprised of a series of N discrete reflectors separated by equal distance of \( \Delta \). Also, the scattering is assumed to happen at the beginning of each layer. At the beginning of the \((j+1)\)th layer, \( r \) is the corresponding local reflectivity associated with the grating beyond that layer. Recall that the reference plane is shifted from \( z = 0 \) to \( z = j \Delta \), and the associated definition of zero time is also shifted to the layer of interest.

The reflection of a single layer between \( z \) and \( z+\Delta \) is simply that of a discrete reflector at \( z = j \Delta \) followed by a free propagation of distance of \( \Delta \). Thus, according to (2), the reflection from this layer only is \( \rho_j \). Note that this single-layer reflection is independent of frequency, which is a direct result of the discretization of the coupling process. Based on causality, at time \( t = 0^+ \) the impulse response contains a contribution only from the reflector at \( z = j \Delta \). Therefore, we can write

\[
\int_{-\pi/\Delta}^{\pi/\Delta} \rho_j \exp(-i\delta \tau) \, d\delta |_{\tau=0^+} = \int_{-\pi/\Delta}^{\pi/\Delta} r(z, \delta) \exp(-i\delta \tau) \, d\delta |_{\tau=0^+}.
\]

The limits on the integral arise because the discrete grating model produces a periodic spectrum with periodicity of \( \delta = \pi/\Delta \). Consequently, only a single period of the spectrum should be considered.

Next, we note that since \( \rho_j \) is independent of the detuning \( \delta \), the integral on the left may be evaluated so that (21) becomes

\[
\frac{\pi \rho_j}{\Delta} = \int_{-\pi/\Delta}^{\pi/\Delta} r(z, \delta) \exp(-i\delta \tau) \, d\delta |_{\tau=0^+}.
\]

According to the definition of \( \rho_j \) in (4), this expression may be written in terms of the coupling coefficient as

\[
-\frac{q'(z)}{q(z)} \tanh[q(z) \Delta] \frac{\pi}{\Delta} = \int_{-\pi/\Delta}^{\pi/\Delta} r(z, \delta) \exp(-i\delta \tau) \, d\delta |_{\tau=0^+}.
\]

Equation (23) is the integral form of the synthesis relation for the DLP method, analogous to (8). By taking the limit as \( \Delta \to 0 \), on both sides of (23), the synthesis relation (18) for the CLP method is readily obtained

\[
q(z) = -\frac{1}{\pi} \int r(z, \delta) \exp(-i\delta 0^+) \, d\delta.
\]

Although simple, this analysis makes it clear that the CLP method is essentially identical to the DLP method in the limiting case of infinitesimally thick layers, or \( \Delta \to 0 \).

For the first numerical comparison example, we consider a dispersionless bandpass filter. We take the ideal spectrum to be a flat-top, nearly rectangular passband filter described by the “super-Gaussian” function

\[
r(\delta) = \sqrt{R} \exp[-(\delta/\delta_{FB})^{20}]
\]

where the maximum reflectivity is \( R \approx 0.90 \) and the width is determined by \( \delta_{FB} = 19.2 \, \text{cm}^{-1} \), which corresponds to a passband full-width at half-maximum (FWHM) of 37.84 cm\(^{-1}\) in wavenumber, or 1 nm in wavelength at a center wavelength of 1550 nm. We choose the detuning window (frequency range) to be 157 cm\(^{-1}\), which corresponds to a wavelength window of about 4 nm. This choice determines the layer thickness, since for the DLP method the layer thickness \( \Delta_{DLP} \) and detuning window \( \delta_W \) must be related by

\[
\Delta_{DLP} = \pi/\delta_{FB}.
\]

Thus, \( \Delta_{DLP} = 0.02 \, \text{cm} \). Furthermore, we choose the length of the grating to be \( L = 10 \, \text{cm} \), which, therefore, determines the number of layers \( N = L/\Delta_{DLP} \); in this case, \( N \approx 500 \). Finally, we choose the number of wavelengths \( M = 1000 \). For both methods, the actual target spectrum is obtained as the discrete Fourier transform of the impulse response associated with (25), windowed to be causal and apodized by a Hanning function. Ideally, for comparison, we would like to choose the same parameters for the CLP method. However, for this choice of parameters, we find that the CLP method does not converge on a reasonable solution. In fact, this behavior appears to be a general result (at least for some structures)—the CLP method does not yield a reasonable structure for layer thickness roughly greater than or equal to that dictated by the DLP method [through (26)]. Therefore, for the CLP simulation, we must choose a smaller layer thickness. We pick 0.01 cm, and keep all other parameters equal. We note that this ability is actually an inherent advantage of the CLP method—the layer thickness can be adjusted independently of \( \delta_W \), as long as it is not made too large. Unfortu-
nately, the smaller layer thickness does not enable a totally fair comparison, so we also include for a third case a DLP simulation with the same layer thickness (0.01 cm), but because of the constraint in (26) we must increase the frequency range to 314 cm$^{-1}$, and we double the number of wavelengths ($M = 2000$) in order to keep the frequency sampling resolution constant. To summarize, the three cases considered for the first example are denoted (by layer thickness) as “DLP 0.02 cm,” “CLP 0.01 cm,” and “DLP 0.01 cm.”

The three calculations for this example had associated processor run times of 0.66, 25.26, and 2.25 s for the “DLP 0.02 cm,” “CLP 0.01 cm,” and “DLP 0.01 cm” cases, respectively. These were implemented using the program MATLAB on a 550-MHz Pentium III computer. The DLP method is at least an order of magnitude faster than the CLP method. In Fig. 2, we plot the coupling coefficient (which is real for this example) determined for each of the three cases described above. On this plot it is very difficult to distinguish the three curves. The main difference is a slight lateral shift in the position of the profile with respect to the spatial window. In Fig. 3(a), we show the calculated reflectivity spectra for the three cases. Note that all spectra in this figure are computed using the “exact” transfer matrix method described by (1). Fig. 3(b) shows the group delay curves associated with the power spectra in Fig. 3(a). Again, all are nearly indistinguishable, except for a slight difference in the absolute delay, which is expected given the lateral shifts seen in Fig. 2. From the power reflectivity curves, we observe that the layer thickness is important in determining how well the spectrum of the reconstructed grating approximates the target spectrum. For the direct comparison of the DLP and CLP methods with the same layer thickness (0.01 cm), the DLP method synthesizes an excellent approximation to the target spectrum for reflectivities down to about $-40$ dB, whereas the CLP method is able to match the spectrum down to about $-50$ dB. However, sidelobes produced by the CLP method do not fall off quite as rapidly as those produced by the DLP method.

In Section II, we make the claim that except for the approximation resulting from the grating coupling model itself (the discretized transfer matrix model), the DLP method can be made exact. This claim is now verified numerically, although we use the frequency domain implementation of the DLP, so here it is “exact” only within the error resulting from the finite spectral resolution limitation discussed in Section II. In Fig. 4, we plot.
Fig. 5. Plots of the: (a) magnitude of the coupling coefficient and (b) relative chirp of the grating period with respect to a nominal period (arbitrarily defined to be at the middle of the grating), for gratings reconstructed from a constant-dispersion, Gaussian-shaped reflection spectrum using both DLP and CLP methods and different sets of parameters.

Fig. 6. Plots of the: (a) reflectivity spectra and (b) group delay curves associated with the reconstructed gratings in Fig. 5. All spectra are computed using the “exact” transfer matrix described by (1).
the reflection spectrum for the example above, but only for the case “DLP 0.02 cm.” The dashed line is from Fig. 3(a), and the solid line is computed from the same coupling coefficient, but for the forward problem calculation we use the discretized matrix (2)–(4) instead of the “exact” matrix (1). Here we obtain an excellent approximation to the target spectrum for reflectivities down to about −170 dB. Of course, this result is not necessarily physically significant, since for strong gratings, the actual spectrum is usually more accurately predicted by the piecewise uniform (“exact”) matrix calculation, but it shows that the method is exceptionally self-consistent; it does an almost perfect job of reproducing the desired spectrum within the limits of the discretized-matrix approximation.

For a second example, we consider for the target reflection spectrum a simple Gaussian function with a quadratic phase (nonzero dispersion) of the form

\[ r(\delta) = \sqrt{R} \exp\left[-(\delta/\Delta_{FB})^2\right] \exp\left[-i\beta_2 L_F(\delta/\alpha)^2/2\right] \]  

(27)

where the maximum reflectivity is \( R = 0.95 \) and the width is determined by \( \Delta_{FB} = 12.3 \text{ cm}^{-1} \), which corresponds to a passband FWHM of 20.4 cm\(^{-1}\) in wavenumber, or 0.54 nm in wavelength at a center wavelength of 1550 nm. The dispersion is described by the product of the \( \beta_2 \) parameter (second derivative of the propagation constant with respect to frequency) and a characteristic length \( L_F \) for an optical fiber. Here, we assume \( \beta_2 = 21.7 \text{ ps}^2/\text{km} \) \( (D = 17 \text{ ps/nm-km}) \) and \( L_F = 30 \text{ km} \), so that the grating is capable of compensating the dispersion of a standard (nondispersion-shifted) fiber over a length of 30 km. The detuning window and layer thicknesses are the same as in the first example. The length of the grating is chosen to be \( L = 12 \text{ cm} \), which determines the number of layers \( N = 600 \). For this example, the number of wavelengths is \( M = 1200 \). As in the example above, to present the fairest comparison, we consider three cases: DLP with equal layer thickness to CLP, DLP with equal detuning window to CLP, and the CLP method.

The three calculations for the second example had associated processor run times of 0.93, 35.92, and 3.29 s, for the “DLP 0.02 cm,” “CLP 0.01 cm,” and “DLP 0.01 cm” cases, respectively. The relative run times among the three cases for this example are almost identical to those for the first example. The coupling coefficient profiles are plotted in Fig. 5(a) and (b), where Fig. 5(a) shows the magnitude of the coupling coefficient and Fig. 5(b) shows the relative chirp of the grating period with respect to a nominal period (arbitrarily defined to be at the middle of the grating). Again, the curves are nearly indistinguishable. In Fig. 6, we plot the spectra associated with the reconstructed gratings. For this example, there is almost no difference among the spectra, indicating that for relatively smooth functions both methods work equally well for practical applications.

V. CONCLUSION

We have reformulated the DLP method for the synthesis of fiber gratings and showed how the discrete grating model can be reconstructed exactly from its spectrum, since the only approximation in this method is that resulting from the discretization of the mode coupling itself. The CLP method for grating reconstruction has been derived and has been compared both mathematically and numerically to the discrete method. We show that the methods are based on the same principle, and that the continuous method is essentially equivalent to the discrete method in the limit of infinitesimally thin layers.

A numerical comparison reveals several interesting facts and apparent trends. First, we find that the DLP method is significantly faster than the CLP method, due in large part to the absence of any calculation of special (hyperbolic) functions for wave propagation and to the sufficiency of simple summations instead of more sophisticated numerical integration routines. Both of these simplifications result from the discretized approximation to the transfer matrix, and they come at a price. The DLP method is fundamentally limited by the degree of approximation associated with the simplified matrix calculation, while the CLP method has the advantage of allowing for a variable layer thickness for a given choice of the spectral window. However, we find empirically that the method is not able to produce reasonable results for layer thicknesses similar to or greater than the thickness required for the DLP method. Thus, CLP seems to inherently require smaller layer thickness than the DLP method, a characteristic which, while compounding its computation-time disadvantage, does allow it to yield somewhat better results for difficult structures (like those with rapidly varying coupling coefficients). When both methods are implemented with equal layer thicknesses (but a significantly larger frequency window and identical frequency resolution used for the DLP method), we still find about an order of magnitude improvement in the computation time for the DLP method over the CLP method. In this case, the accuracy is similar for the two methods. Decreasing the layer thickness in the CLP method so that it is much smaller than the thickness dictated by the DLP method (i.e., for thicknesses down to even just a couple of grating periods), does not appear to provide much improvement in the accuracy with which the reproduced spectrum matches the target spectrum.

APPENDIX

SPECTRUM REALIZABILITY

When using the layer-peeling algorithm for synthesis of actual filters, the desired reflection spectrum is not necessarily realizable for a series of \( N \) reflectors or a grating of length \( L \). To obtain a realizable reflection spectrum, we can use the (apodizing-)windowing procedure that is common in digital finite-impulse-response (FIR) filter design. This procedure was used by Feced for the synthesis of practical fiber gratings using the layer-peeling algorithm [1]. It consists of forcing the impulse response to be zero outside of a certain window, apodizing the windowed impulse response to cause the tails to approach zero more smoothly, and then shifting the whole response so that it starts at \( t = 0 \). This new impulse response is referred to as the “target impulse response,” and we note that it has a finite duration. Strictly speaking, the (complex) reflection spectrum of any real fiber grating structure contains poles, and consequently the associated impulse response has infinite duration. In other words, even the target impulse response is not exactly realizable—the impulse response of the realized grating contains tails that extend beyond the windowed target impulse response, and these tails have nothing to do with the tails that were windowed. The realized tails cause undesirable fluctuations in the realized.
reflection spectrum, but as the examples in Feced’s paper indicate [1], the influence of the tails is small for most practical filters with $N \gg 1$ and $|\rho_s| \ll 1$.

This windowing procedure is not necessarily optimal for the synthesis of finite length gratings, as the following example demonstrates. Suppose one desires to synthesize a grating of length $L$ with reflection spectrum $r(\delta)$, and assume that $r(\delta)$ happens to be identical to the reflection spectrum of a certain shorter grating, but of course this coincidence is not known in advance. Then, by using the windowing procedure, one gets a target spectrum $r_1(\delta)$ that is approximately realizable for a grating of length $L$. By applying the layer-peeling algorithm to this spectrum, one ends up with a grating of length $L$ with a spectrum that approximates the desired function $r(\delta)$. Since the spectrum could have been realized exactly using a shorter grating, we would have been better off shortening the starting grating length than attempting to window the starting impulse response. Thus, it should be clear that adjusting the windowing is not necessarily the best way in all cases to achieve the simplest structure to match a desired reflection spectrum. Nevertheless, it is a straightforward approach and often yields good results in practice.

It should also be noted that for the case of a discrete grating model, the realizable filter functions can be identified exactly. For $N$ reflectors, the class of realizable filters is $N$th-order rational functions of $z^{-1} = \exp(i2\delta \Delta)$ with rather strict conditions on the coefficients [11]. However, these conditions are difficult to handle, and it is not a trivial problem to find the allowed coefficients that give the best fit to a desired response.

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