A Fast Convergence Multiuser Detection Scheme for Uplink SCMA Systems

Yang Du, Binhong Dong, Zhi Chen, Member, IEEE, Jun Fang, Member, IEEE, and Xianjun Wang

Abstract—Sparse code multiple access (SCMA) is a competitive non-orthogonal multiple access technique for fifth generation (5G) wireless communication networks which can accommodate massive connectivity. Among the existing multiuser detection schemes for uplink (UP) SCMA systems, the asynchronous message passing algorithm (ASYN-MPA) scheme, where messages are updated sequentially, generally converges faster than the synchronous variant, where all messages are updated in parallel manner. Nevertheless, the specific pre-defined order of the ASYN-MPA scheme is not always the best option. In this letter, we tackle the problem of how to schedule message for the ASYN-MPA scheme that results in the best convergence rate. Specifically, a novel residual-aided message propagation approach for scheduling messages in the ASYN-MPA scheme, termed residual-aided MPA (R-aided MPA), is proposed for UP SCMA systems, which exploits the dynamic selection of the message which has the largest residual. Simulations show that the proposed scheme with 2 iterations provides very similar bit error rate (BER) performance to the existing multiuser detection schemes with 6 iterations (only less than 0.05dB degradation), whose complexity is reduced substantially.

Index Terms—Fifth generation (5G), sparse code multiple access (SCMA), multiuser detection, message passing algorithm (MPA), residual-aided message propagation.

I. INTRODUCTION

The increasing demand of mobile Internet and the Internet of Things poses challenging requirements for fifth generation (5G) wireless communications, such as high massive connections and spectral efficiency [1]. To address these challenges, non-orthogonal multiple access schemes have received more and more attention for 5G [2].

During the past decade, non-orthogonal approach of code division multiple access (CDMA) has been deeply investigated by a quite large number of scholars. A familiar example of non-orthogonal CDMA is sparse spread CDMA, including low density spreading CDMA (LDS-CDMA) and sparse code multiple access (SCMA) [2]-[8]. In SCMA systems, the QAM mapper and LDS spreader are merged together to directly map incoming data streams to multi-dimensional complex domain codewords selected from a predefined cookbook set. Consequently, SCMA generalizes the idea of LDS, which better accommodates the demand of massive connectivity in 5G [5]-[8]. However, in order to make SCMA as a competitive candidate for 5G air interfaces, two vital issues, excellent SCMA codebook design and efficient multiuser detection, need to be further addressed. In [8], a systematic approach toward the design of SCMA codebooks is proposed and evaluated. In this letter, we focus on the efficient multiuser detection.

By making full use of the sparsity of LDS structure in SCMA spreading signature, several iterative multiuser detection schemes for uplink (UP) SCMA systems [3], [9], [10], which are based on message passing algorithm (MPA) [11], were proposed to efficiently approximate the optimal maximum a posteriori (MAP) scheme. Scheduling strategy plays an important role in multiuser detection schemes of UP SCMA systems. Among them, the synchronous scheduling strategy is the basic one, where in each iteration all resource nodes (RNs), and subsequently all the user nodes (UNs), pass new messages to their neighbors [3], [9]. The asynchronous strategy is another one, in which messages are updated sequentially according to a fixed update order [10]. In [10], simulations show that the asynchronous MPA (ASYN-MPA) scheme converges twice as fast as the synchronous MPA (SYN-MPA) scheme. Nevertheless, the update of messages in the ASYN-MPA scheme follows a pre-defined order, which is not always the best option since the convergence rate can still be further accelerated. Consequently, the ASYN-MPA scheme raises the problem of constructing an effective message scheduling order that results in the better BER and convergence rate.

To tackle this problem, for UP SCMA systems, we propose a novel residual-aided (the residual is defined as the difference between the values of a UN-to-RN message value before and after an update) message propagation approach for scheduling messages in ASYN-MPA scheme, termed residual-aided MPA (R-aided MPA), which exploits the dynamic selection of the message which has the largest residual, whenever messages are updated sequentially. Simulation results demonstrated that the proposed R-aided scheme significantly improves the convergence rate in terms of number of iterations. More specifically, compared to the BER performance of the existing MPA schemes with 6 iterations, the proposed R-aided scheme with 2 iterations only has less than 0.05dB degradation, whose complexity is reduced substantially. As a result, the proposed R-aided scheme offers a better BER-complexity trade-off.

II. SYSTEM MODEL

A. UP SCMA Model

We consider uplink SCMA systems with $J$ users and $K$ resources. In order to accommodate the demand of massive connectivity in 5G, the overloading factor is defined as $\lambda =$
$J/K$, which is usually larger than 1 ($J > K$). Incoming bits of each user are mapped from $\log_2(M)$ to a $K$-dimensional complex codeword based on each individual codebook, where $M$ represents the SCMA codebook size. The codewords of each codebook are sparse vectors.

At the receiver side, the received signal is represented by

$$\mathbf{y} = \sum_{j=1}^{J} \text{diag} (\mathbf{h}_j) \mathbf{x}_j + \mathbf{n},$$

where $\mathbf{x}_j = (x_{1,j}, \cdots, x_{K,j})^T$ is the vector of SCMA codeword of user $j$, $\mathbf{h}_j = (h_{1,j}, \cdots, h_{K,j})^T$ is channel vector of user $j$, and $\mathbf{n} \sim \mathcal{C}\mathcal{N}(0, \sigma^2 \mathbf{I})$ denotes the Gaussian noise.

Given the received signal $\mathbf{y}$ and the channel knowledge $\{\mathbf{h}_j\}_{j=1}^{J}$, the joint optimum MAP detection of user codewords $\mathbf{X} = (\mathbf{x}_1, \cdots, \mathbf{x}_J)$ can be given by

$$\hat{\mathbf{X}} = \arg \max_{\mathbf{x} \in \{\mathbf{x}_j\}_{j=1}^{J}} P(\mathbf{X} | \mathbf{y}),$$

where $\chi_j$ is the codeword set of user $j$.

**B. MPA-based Multiuser Detection Schemes for UP SCMA**

Leveraging the sparsity property of LDS structure in SCMA, the iterative SCMA multiuser detection scheme based on MPA with acceptable complexity and its generalizations have been proposed with the underlying factor graph. The procedure of MPA can be explained by the factor graph, which is a bipartite graph including UNs and RNs. In general, MPA consists of the exchange of messages between the nodes of a factor graph. Obviously, there are two approaches for scheduling messages in MPA, i.e., synchronous and asynchronous schemes.

Let $M_{r_k \rightarrow u_j}(\mathbf{x}_j)$ and $M_{u_j \rightarrow r_k}(\mathbf{x}_j)$ be the message sent along edge $e_{k,j}$ from RN $r_k$ and UN $u_j$, respectively. Specifically, the SYN-MPA scheme can be depicted as follows. During each iteration, messages are first sent from UNs to RNs; Each RN then computes extrinsic messages and sends back to the UNs based on the previously received information. These UN-to-RN messages will then be used to calculate the new RN-to-UN messages in the next iteration. Thus, the two message generating functions for the SYN-MPA scheme [3], [9], are defined as follows:

$$M_{r_k \rightarrow u_j}^{t}(\mathbf{x}_j) = \sum_{\mathbf{x}_j} \left\{ \frac{1}{\sqrt{2\pi \sigma}} \exp \left( -\frac{1}{2\sigma^2} \left\| \mathbf{y}_k - \sum_{m \in \xi_k} h_{k,m,}\mathbf{x}_{k,m} \right\|^2 \right) \times \prod_{l \in \xi_k \setminus \{j\}} M_{u_l \rightarrow r_k}^{t-1}(\mathbf{x}_l) \right\},$$

$$M_{u_j \rightarrow r_k}^{t}(\mathbf{x}_j) = \prod_{m \in \zeta_j \setminus \{k\}} M_{r_m \rightarrow u_j}^{t}(\mathbf{x}_j),$$

where $t$ is the iteration index, $\xi_k$ and $\zeta_j$ are the set of non-zero’s position in the $k$th row and $j$th column, respectively.

In the ASYN-MPA scheme, an iteration consists on the sequential update of all the messages $M_{u_j \rightarrow r_k}(\mathbf{x}_j)$ in the ascending ordinal of UNs. Specifically, for the ASYN-MPA scheme, the messages are updated from one UN to another, with the RNs and UNs updated together [10]. For $1 \leq j \leq J$ and each $k \in \zeta_j$, process RN and UN message update steps jointly. Thus, (3) can be modified as

$$M_{r_k \rightarrow u_j}^{t}(\mathbf{x}_j) = \sum_{\mathbf{x}_j} \left\{ \frac{1}{\sqrt{2\pi \sigma}} \exp \left( -\frac{1}{2\sigma^2} \left\| \mathbf{y}_k - \sum_{m \in \xi_k} h_{k,m,}\mathbf{x}_{k,m} \right\|^2 \right) \times \prod_{l \in \xi_k \setminus \{j\}} M_{u_l \rightarrow r_k}^{t-1}(\mathbf{x}_l) \prod_{l \in \xi_k \setminus \{j\}} M_{u_l \rightarrow r_k}^{t-1}(\mathbf{x}_l) \right\},$$

**III. PROPOSED DETECTION SCHEME FOR UP SCMA**

To the best of our knowledge, the ASYN-MPA scheme, where messages are updated sequentially, generally converges faster than the SYN-MPA scheme, where all messages are updated in parallel. However, the fixed update sequence of the ASYN-MPA scheme is not always the best option since the BER performance can still be further improved.

We now address the task of constructing an effective message scheduling strategy (an order for message propagation) that achieves better convergence properties than conventional synchronous and asynchronous message update.

**A. Redidual-aided Multiuser Detection Scheme**

Empirically, when running MPA, we see that some nodes of the underlying factor graph converge very quickly, whereas others take more iteration times to reach reasonable values. In the face of the phenomenon, the inspiration of the proposed R-aided MPA scheme is that not all messages are equally useful towards achieving convergence. Sending a message whose current value is quite similar to its value in the previous iteration is not very efficient, while sending a message that is very different from its previous value is likely to be more informative which leads to more rapid transfer of message throughout the factor graph.

Motivated by the analysis mentioned above, we can construct a dynamic message scheduling strategy which reasonably choose the message update order. Unlike the ASYN-MPA scheme, our proposed dynamic scheduling strategy does not have a pre-defined order. In fact, before the next update step starting, the dynamic selection strategy is first processed to determine which edge will be updated. In this way, the proposed R-aided MPA scheme updates the least reliable message preferentially.

The proposed R-aided MPA scheme takes the residual as the basis of the dynamic selection criterion, where the residual is defined as the difference between the values of a UN-to-RN message value before and after an update. For a message $M_{u_j \rightarrow r_k}(\mathbf{x}_j)$, the residual is defined as:

$$R [M_{u_j \rightarrow r_k}(\mathbf{x}_j)] = \frac{1}{|\chi_j|} \sum_{\mathbf{x}_j \in \chi_j} \left| M_{r_k \rightarrow u_j}^{\text{new}}(\mathbf{x}_j) - M_{r_k \rightarrow u_j}^{\text{old}}(\mathbf{x}_j) \right|.$$
where the superscript of “old” and “new” denotes the message before and after $M_{u_j \rightarrow r_k}^t(x_j)$ changes, respectively.

The intuitive justification of the proposed R-aided MPA scheme is that the differences between the messages before and after an update go to zero as MPA converges. Hence, if a message has a large residual, it means that the message is located in a part of the factor graph that has not converged yet. By that reason, propagating the message having the largest residual first should accelerate the process. In the proposed R-aided MPA scheme, for each step, all residuals are calculated and the message with the largest residual is propagated, i.e.,

$$M_{u_j \rightarrow r_k}^t(x_j) = \text{argmax}_{(j,k)} \left\{ R \left[ M_{u_j \rightarrow r_k}^t(x_j) \right] \right\}, \quad (7)$$

In the proposed R-aided MPA scheme, the update order of message is formulated from the viewpoint of UNs, which is that the least reliable UN has the highest update priority.

Now, the proposed R-aided MPA scheme can be decomposed into two key steps as shown in Fig. 1. Without loss of generality, $M_{u_j \rightarrow r_k}^t(x_j)$ is the message having the largest residual $R \left[ M_{u_j \rightarrow r_k}^t(x_j) \right]$.

**Step 1:** if the edge corresponding to $M_{u_j \rightarrow r_k}^t(x_j)$ is chosen, it sets $R \left[ M_{u_j \rightarrow r_k}^t(x_j) \right] = 0$, and then updates $M_{r_k \rightarrow u_a}(x_a)$ for all $u_a \in (\xi_k \setminus \{x_j\})$.

**Step 2:** the messages $M_{u_a \rightarrow r_k}(x_a)$ are updated, and then calculate the corresponding new residuals $R \left[ M_{u_a \rightarrow r_k}(x_a) \right]$, for all $r_k \in (\xi_a \setminus \{x_j\})$.

The detailed procedures of the proposed R-aided MPA scheme can be described in Table I.

### B. Computational Complexity Analysis

For convenient comparison, the computational complexities of different detection schemes are presented in Table II. It is clear that the main computational complexities of the detection schemes are due to the operations at the RNs, which are dominated by $t_{max}$, $M$ and $d_r$. For the same $M$ and $d_r$, the effect of $t_{max}$ on the computational complexity is quite different for different detection schemes. Obviously, the smaller the value of $t_{max}$, the lower the computational complexity. Thus, reducing the number of iterations is significant for complexity reduction. Due to the residual belief propagation, the proposed scheme incurs two extra processes: residual computation and ordering of the residuals, which requires additional complexities. However, these additional complexities are not counted, due to the negligible demand compared to the message update complexity.

#### IV. Numerical Results

This section presents the AWGN performance of the proposed R-aided MPA, SYN-MPA and ASYN-MPA schemes for UP SCMA systems ($K = 4$, $J = 6$, $M = 4$, $d_r = 3$, $d_u = 2$).

Fig. 2 shows the BER performance comparison of the proposed R-aided MPA, SYN-MPA and ASYN-MPA schemes. From Fig. 2, it is clearly shown that the proposed R-aided MPA scheme obtains the best BER performance for a small number of iterations among the schemes of interest. Specifically, for the target BER of $1.0 \times 10^{-5}$, compared to the SYN-MPA and ASYN-MPA schemes at 2 iterations, the proposed R-aided MPA scheme with 2 iterations gains more than 1.7dB and 0.25dB, respectively. And more importantly, the proposed scheme with 2 iterations can achieve the very similar performance of the SYN-MPA and ASYN-MPA schemes with 6 iterations (only less than 0.05dB degradation). This can be explained that the proposed R-aided MPA scheme is able to utilize more effective messages to accelerate the convergence rate by dynamically adjusting the order of message updates.

The complexity comparison of the SYN-MPA scheme with 6 iterations, ASYN-MPA scheme with 2 iterations and proposed R-aided MPA scheme with 2 iterations is shown in Fig. 3. It can be observed from Fig. 2 and Fig. 3 that, at 2 iterations, the proposed R-aided MPA scheme has better BER performance than the ASYN-MPA scheme, which almost has no complexity increases. Meanwhile, compared to the SYN-MPA scheme with 6 iterations, the proposed scheme with 2 iterations saves more than 66.67% multiplications with negligible performance loss. Additionally, the average runtimes

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**TABLE I**

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Step 2</th>
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<tbody>
<tr>
<td>1: Input: $y$, $H$, $\sigma^2$, $t_{max}$</td>
<td>2: Output: $Q(x_j)$</td>
</tr>
<tr>
<td>3: Initialization:</td>
<td>4: for all $k = 1, \cdots, K$ and $j = 1, \cdots, J$ do</td>
</tr>
<tr>
<td>5: $M_{u_j \rightarrow r_k}^0(x_j) = \frac{1}{x_j}$ for all $x_j \in \chi_j$</td>
<td>6: Iterative Update of Messages:</td>
</tr>
<tr>
<td>7: Perform the proposed dynamic selection strategy and select the largest $R \left[ M_{u_j \rightarrow r_k}^t(x_j) \right]$, set $R \left[ M_{u_j \rightarrow r_k}^t(x_j) \right] = 0$</td>
<td>8: for all $u_a \in (\xi_k \setminus {x_j})$ do</td>
</tr>
<tr>
<td>9: Generate and propagate $M_{r_k \rightarrow u_a}(x_a)$ using (3)</td>
<td>10: end for</td>
</tr>
<tr>
<td>11: for all $r_k \in (\xi_a \setminus {x_j})$ do</td>
<td>12: Generate and propagate $M_{u_a \rightarrow r_k}(x_a)$ using (4)</td>
</tr>
<tr>
<td>13: Compute $R \left[ M_{u_a \rightarrow r_k}(x_a) \right]$</td>
<td>14: end for</td>
</tr>
<tr>
<td>15: if stopping rule is not satisfied then</td>
<td>16: Go back to line 7</td>
</tr>
<tr>
<td>17: end if</td>
<td>18: Calculate the LLR for each bit</td>
</tr>
<tr>
<td>19: for all $j$ do</td>
<td>20: $Q(x_j) = \prod_{k \in \xi_j} M_{r_k \rightarrow u_j}(x_j)$</td>
</tr>
<tr>
<td>21: end for</td>
<td></td>
</tr>
</tbody>
</table>

Note: $t_{max}$ is the maximum number of iterations.
The convergence behavior comparison of the considered detection schemes is presented in Fig. 4. It is observed that the proposed R-aided MPA scheme exhibits a near-optimum BER performance with reduced complexity.

V. CONCLUSION

This letter designed residual-aided message propagation to formulate the message update order. Hence, a new dynamic scheduling strategy, the R-aided MPA scheme, was proposed for UP SCMA systems. In contrast to the existing schemes, the proposed R-aided MPA scheme exhibits a near-optimum BER performance with reduced complexity.

TABLE II
COMPUTATION COMPLEXITY OF DIFFERENT DETECTION SCHEMES

<table>
<thead>
<tr>
<th></th>
<th>R-aided MPA</th>
<th>SYN-MPA and ASYN-MPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplications</td>
<td>$(t_{\text{max}} - 1)(d_u - 1) Jd_u M (d_u - 2) + Jd_u M (d_u - 2) + t_{\text{max}} Jd_u M^{d_r} (2d_r + 1)$</td>
<td>$t_{\text{max}} Kd_r M^{d_r} (d_r + 1) + t_{\text{max}} Jd_u M (d_u - 2)$</td>
</tr>
<tr>
<td>Additions</td>
<td>$t_{\text{max}} Kd_r (d_r + 1) M^{d_r} - M + (t_{\text{max}} - 1)(d_u - 1)(2M - 1) + Jd_u (2M - 1)$</td>
<td>$t_{\text{max}} Kd_r (d_r + 1) M^{d_r} - M$</td>
</tr>
<tr>
<td>Exponents</td>
<td>$t_{\text{max}} Kd_r M^{d_r}$</td>
<td>$t_{\text{max}} Kd_r M^{d_r}$</td>
</tr>
</tbody>
</table>

Note: $d_r$ and $d_u$ denote the average degrees of RNs and UNs, respectively.

REFERENCES