Frequency Domain Data Transmission
using Reduced Computational Complexity algorithms

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Abstract

In this paper we describe a frequency domain data transmission method to be used for digital data transmission over analog telephone lines which exploits recently derived reduced computational complexity algorithms, such as the Winograd Fourier Transform, to achieve a significantly lower computational rate than comparable time domain QAM modems implemented digitally using signal processing techniques. In addition to the lower computational rate, the proposed method also allows for better channel bandwidth utilization by allowing optimal signal power allocation based on the channel’s signal to noise versus frequency characteristics. Experimental results on this method are presented, indicating that it may be possible to send over 10,000 BPS over an unconditioned telephone line while maintaining a $10^{-7}$ BER.

I. Introduction

We describe here a frequency domain data transmission scheme in which the modulation process is achieved by an IDFT and the demodulation process by a DFT, using reduced computational (RCC) algorithms such as the Winograd Fourier transform (WFT).

This type of system has been proposed before by Weinstein and Ebert [1]. However, at that time, it could only have been implemented in real-time thru the use of special purpose hardware. With the wide use of digital signal processing techniques, there has been some emphasis in using recently developed RCC algorithms for implementing signal processing algorithms in real-time in general purpose processors. The processors that we use to implement this system, although not restricted to work only with RCC algorithms, take advantage of them to perform with an attractive cost/performance ratio. A brief description of this processor and its environment can be found in Appendix A.

We will describe in this paper the basic theory behind the implementation of FDDT over an analog telephone channel. And we will also show how the data transmitted can be allocated according to the channel characteristics so as to maximize the usage of the channel.

II. Frequency Domain Data Transmission

A) Theory

The basic idea is to transmit thru the telephone channel, on a frame by frame basis, multiple low speed channels multiplexed in the frequency domain.

Let \( \{ A_n \}_{n=0}^{N/2-1} \) and \( \{ B_n \}_{n=0}^{N/2-1} \) be two data sequences, where \( A_n \) and \( B_n \) can take on any of a set of 2\(^k\) values or constellation points (where \( r_n \) is a function of \( n \)).

Let us also define the complex sequence \( \{ D_n \}_{n=0}^{N-1} \)

\[
D_n = \begin{cases} 
0 & n = 0 \\
A_n + jB_n & 0 < n \leq \frac{N}{2} - 1 \\
0 & n = \frac{N}{2} \\
A_{N-n} - jB_{N-n} & \frac{N}{2} < n \leq N - 1 
\end{cases}
\] (1)

This construction insures that the IDFT of \( \{ D_n \} \) is a real sequence (since \( \{ D_n \} = \{ D_{N-n} \} \)). Therefore in figure (1),

\[
x_t = \sum_{n=0}^{N-1} D_n e^{j2\pi nt/N} \] (2)

is real, and \( x(t) \) is obtained by passing \( \{ x_t \} \) through a D/A converter, at a rate of \( F_s \) Hz, and a low-pass filter,

\[
x(t) \cong \sum_{n=0}^{N-1} D_n e^{j2\pi F_s t / N} = \sum_{n=0}^{N-1} D_n e^{j2\pi f_n t} \] (3)

\[0 \leq t < NT, \quad T = \frac{1}{F_s}, \quad f_n = \frac{F_s}{N} n, \]

equivalently, \( x(t) \) is a sum of sinusoidal signals whose amplitudes are determined by the sets of data values \( \{ A_n \} \) and \( \{ B_n \} \), as depicted in figure (2). This interpretation makes the transmitter an implementation of a frequency division multiplexing (FDM) system [1].

If the signal \( x(t) \) is transmitted thru a linear channel with impulse response \( h(t) \) and frequency response \( H(w) \), then the signal at the output of the channel \( s(t) \) is determined by the convolution of \( x(t) \) and \( h(t) \), or in the frequency domain by \( S(w) = H(w)X(w) \).

It appears that at the receiver end, to recover \( \{ D_n \} \), we should compute \( S(n\Omega) \) (where \( \Omega = 2\pi F_s / N \)), that is the sampled version of \( S(w) \), and divide it by \( H(n\Omega) \), which can be obtained during an initialization procedure that will be described later. The idealized formulation above does not hold unless additional steps are taken to insure that certain conditions apply.

If we let \( \{ h_i \}_{i=0}^{N-1} \) be the impulse response coefficients that characterize the channel, then \( \{ x_t \} \) is given by

\[
x_t = \sum_{i=0}^{K-1} h_i x_{t-i-1} \] (4)

This formulation disregards any channel effects such as: phase jitter, amplitude modulation jitter, different types of noise, sampling jitter, and also differences in clock rates between the receiver and the transmitter. Although many of these effects will be present, we will ignore them for the moment and continue to describe the operation.

The box labeled "cyclic extension" performs the following map-
pings on \( x_f^k \) \( l = 0 \) (where \( k \) denotes the \( k \)th frame )

\[
  x_f^k = \begin{cases} 
    x_f^k & 0 \leq l \leq N - 1 \\
    x_f^{k+1} & N \leq l \leq L - 1
  \end{cases} \quad (5)
\]

Thus, each frame has a length of \( L = N + K - 1 \) because of the cyclic extension. This causes the sequence \( x_f^k \) \( l = 0 \) to appear periodic to a channel with memory of \( K \) samples (at the sampling rate \( F_T \) ), and therefore provides the required conditions for the discrete convolution theorem to hold.

In the receiver, the last \( K - 1 \) terms in each frame are ignored after synchronization is achieved. Thus

\[
  v_f^k = x_f^k \quad 0 \leq l \leq N - 1 \quad (6)
\]

Taking the DFT of \( \{ v_f \} \) to achieve demodulation we obtain \( \{ Q_p \} \)

\[
  \{ Q_p \} = \sum_{l=0}^{N-1} v_f^k e^{-j 2 \pi p \frac{l}{N}} \quad (7)
\]

From fig. (1), the equalizer \( \{ C_p \} \) multiplies \( \{ Q_p \} \) to obtain

\[
  \{ D_p \} = \{ Q_p \} \cdot \{ C_p \}
\]

where, it can be shown that \( \{ D_p \} = \{ D_p \} \cdot \{ C_p \} \), i.e. \( \{ D_p \} \) is the receiver's estimate of the transmitted data \( D_p \).

**B) Initialization**

The objectives of the initialization procedure are: synchronization, and initial equalizer setting. The transmitter sends two frames for initialization purposes. Each frame is \( L \) samples long, where \( K \) of these samples are the cyclic extension as in the mapping of equation (5), and \( L = K + N - 1 \) as before. Also, \( N \) is the size of the IDFT and DFT used in the modulation and demodulation processes respectively.

The first frame consists of \( M \) identical subsequences of length \( K = N/M \), where each subsequence is a pseudo-noise (PN) sequence whose power spectrum is approximately flat. The second frame consists of a length \( N \) PN sequence. The receiver has stored in its memory, as reference, the corresponding complex values of the \( K \) and \( N \) point DFT's of these two PN sequences.

To start the operation, the receiver monitors the channel for energy. Once a preset threshold is exceeded, it assumes that initialization has started and proceeds to take a \( K \) point DFT of the received first \( K \) point subsequence inside the first received frame. Since the subsequence frame boundary is out of synchronization, the receiver picks-up a time shifted version of the response of the channel to the repeated PN subsequence.

If \( \{ T_f \} \) is the original PN subsequence and \( \{ T_f \} \) \( l = 0 \) its DFT, then the \( K \) point DFT taken by the receiver \( \{ V_f \} \) \( l = 0 \) is also

\[
  V_f = T_f e^{-j \frac{2\pi m}{K} H_f}, \quad (8)
\]

where \( m \) is the time shift and \( \{ H_f \} \) is the DFT of the channel's discrete impulse response characteristic \( \{ h_f \} \) (at intervals \( F_T \)).

By dividing \( V_f \) by \( T_f \) we obtain \( O_f \) \( e^{-j \frac{2\pi m}{K} H_f} \).

And then taking the \( K \) point IDFT of \( \{ O_f \} \) we obtain \( \{ o_f \} \), where

\[
  \{ o_f \} \equiv \{ h_f \} \quad (ii)
\]

Now, since a physically realizable bandpass channel has a characteristic peak in the middle of its impulse response, we determine \( m \) (the shift) by simply looking for \( \text{MAX} \{ \text{ABS} \{ o_f \} \} \).

This has established frame synchronization. It should be noted here that any small error in synchronization (i.e., \( \pm \) a few samples) will be compensated by the equalizer. The receiver continues to take \( K \) point DFT's (with the new frame and subsequence boundaries) until we do not get \( \{ T_f \} \) anymore, at which time we know that the next frame has begun.

The receiver takes the \( N \) point DFT of the first \( N \) points inside this newly synchronized frame, to obtain \( \{ R_p \} \) \( l = 0 \), and proceeds to determine the initial equalizer setting by dividing as follows,

\[
  \{ C_p \} = \{ R_p \} / \{ Q_p \} \quad (9)
\]

where \( \{ R_p \} \) is the DFT of the \( N \) point PN sequence above, already stored as reference in the receiver. Having calculated \( \{ C_p \} \), the initial equalizer setting, we have completed the initialization procedure in only two frames duration.

**C) Equalizer Update**

The equalizer obtained in equation (9) has to be updated in every frame to account for the second order effects mentioned in section (IIa). However, our initial strategy for updating has originated from the fact that the predominant error occurring in the system is the clock differences between transmitter and receiver. Overall, for a net error in the sampling clocks, the received unequalized bins for the next frame are given by,

\[
  \{ Q_p^{k+1} \} = \{ Q_p^k \} e^{-j \frac{2\pi \beta p}{N}} \quad (10)
\]

where \( \beta \) is a constant dependent on frame length and clock error, and the angle \( (2\pi \beta p)/N \) is linearly related to \( p \) the frequency bin number.

Using a simple approach to the problem, the formulation for the equalizer update goes as follows. An error signal is formed for every frame, that is \( \{ e_p^k \} \) as in figure (1)

\[
  e_p^k = \widetilde{D}_p^k - \widetilde{D}_p^k \quad (11)
\]

Where \( \widetilde{D}_p^k \) denotes the received constellation points after equalization and \( \widetilde{D}_p^k \) denotes the decoded constellation points.

We also define the updated equalizer for the \( k + 1 \)th frame

\[
  C_p^{k+1} = D_p^k / Q_p^k \quad (12a)
\]

where \( Q_p^k \) denotes the received constellation points before equalization. Using eqs. (11) and (7), equation (12a) becomes

\[
  C_p^{k+1} = \frac{\widetilde{D}_p^k - e_p^k}{Q_p^k} = \frac{Q_p^k C_p^k - e_p^k}{Q_p^k} = C_p^k (1 - e_p^k) / \widetilde{D}_p^k \quad (12b)
\]

This equation has been empirically changed to contain the factor.
\( \alpha \ (0 < \alpha \leq 1.0) \), thus becoming

\[ C_{\rho}^{k+1} = C_{\rho}^{k} (1.0 - \alpha \frac{b}{D_{\rho}^{k}}) \]

(12c)

It can be shown that \( \alpha \) provides the best results when set to higher values for the first few frames and then a lowest steady state value later. This holds true even when the initial equalizer setting is obtained during reasonably noisy channel conditions. Additional work is required on equalizer updating to compensate for other channel impairments.

III. Communications aspects of the system

In this section we discuss varying the average power and number of different levels per bin to optimally utilize the channel characteristics.

Based on the spectral characteristics of the telephone channel, as obtained in the last survey on analog transmission performance of telephone lines [2], allocation of frequency bins can be made as shown in fig. (3). This allocation is based on \( f_{c} = 8 \text{kHz} \) and \( N = 240 \) the size of the IDFT and DFT. Accordingly, 120 bins are available, out of which, frequency region 0 to 234 Hz will not be used because of the inherent presence of 60 Hz and its first two harmonics, and also low S/N ratio characteristics. And frequency region 3400 to 4000 Hz will not be used because of insufficient S/N ratio characteristics for efficient equalization. These constraints translate into having bins 8 thru 102 available for transmission.

As we indicated in section (IIa), each \( D_{n} \) in eq. (1) can take on any of a set of \( 2^{n} \) complex values. This allocation can be made such that the number of complex levels and the power on each bin will follow the optimal power allocation strategy as required by the "water filling theorem" of communication theory, potentially being able to maximize the utilization of the channel bandwidth [5]. Although in our implementation \( r_{n} \) is an integer, it need not be that way, and in theory, we can come closer to using the maximum capacity of the channel much better if \( r_{n} \) is allowed to be non-integer also. In summary, we have the capability to allocate a different number of levels to the various spectral bins, thus having more in those \( D_{n} \)'s in the midband where the S/N ratio is highest, and less in those areas where the S/N ratio is lowest.

The above approach also allows the usage of more of the channel bandwidth while minimizing the noise enhancement problem associated with equalization in time domain data transmission schemes.

IV Experimental results, Conclusion

An FDDT system was first simulated in fixed-point PL/I on the IBM 370/168 to test all the algorithms, including the RCC algorithms. The 40 point and 240 point DFT's and IDFT's used for the simulation and the real-time implementation were WFTA's programmed as in reference [3]. Other RCC algorithms needed complex multiplication, complex division, and also scaling and normalization.

The real-time implementation was programmed on two RSP's, one RSP was programmed as a receiver and the other as a transmitter. Tests and preliminary evaluations of this system were made by running it on a telephone line impairment simulation instrument, and on short telephone line loops (< 50 miles) over the switched network. Results obtained for the 11,886 bps. modem as in fig. (3) are illustrated in fig. (4) which shows the corresponding photographs of various constellations as the modem was running over a short telephone switched line loop.

Preliminary results also indicate that we are able to run at a bit error rate of better than \(10^{-7}\) at about 23 db S/N ratio. For all of the above the processors are doing all the computations in about 60 percent of real-time while using about 65 percent of their data and instruction storage. The total number of operations is approximately 60,000 multiplies per second (MPS) and 140,000 additions per second (APS). This compares favorably with approximately 600,000 MPS and 600,000 APS for the digital implementation of the 9,600 BPS quadrature amplitude modulation (QAM) time-domain modem.

In summary, we have implemented a system that takes advantage of recently developed RCC algorithms in digital signal processing to transmit more data more efficiently (i.e. less computations) over an unconditioned analog telephone channel, while at the same time taking advantage of the channel's own S/N ratio characteristics to improve the bit error rate performance.

Appendix A

The research signal processor (RSP) is a Schottky-TTL implemented microprocessor specially designed for general signal processing applications. This processor has a 16 bit fixed point arithmetic unit, 64 machine instructions, and a 4 stage pipeline. Storage capabilities are 4K of data store and 5K of instruction store. An important characteristic is its capability to exploit reduced computational complexity algorithms to achieve high throughput and to efficiently decompose multiplications by constants into shifts and adds using canonical signed digit representation of numbers [4]. Furthermore, synchronous and asynchronous I/O are handled under program control.

The programs for the RSP are all written in the processor's own "high level" assembler language, which is facilitated by its single operand instruction format. The RSP runs under host control, mainly program loading and/or data initialization, after which the RSP can carry out a complete signal processing application without host intervention. Host control is not needed if programs are made to reside in read only instruction storage.

References

Figure 1. Frequency Domain Data Transmission system

Figure 2. Frequency division multiplexing interpretation of system

Figure 3. Telephone channel, allocation of frequency bins

Figure 4. Constellations for the 11,886 bps modem while running on a telephone line