Power frequency harmonic measurement using integer periodic extension method

Shun-Li Lu, Chin E. Lin *, Ching-Lien Huang
Department of Electrical Engineering, National Cheng Kung University, Tainan 701, Taiwan, ROC
Received 13 June 1997

Abstract

Reduced frequency resolution and spectral leakage are the two major defects on the Fourier spectrum resulting from a data window function and fast Fourier transform (FFT) in power system harmonic measurement. In this paper a novel method, termed the integer periodic extension (IPE) method, is proposed to overcome the noted problems. The proposed method simplifies measurement process and increases measurement accuracy. Finally, five cases from practical demand-side power systems are demonstrated with improved performance. © 1998 Elsevier Science S.A.

Keywords: Power frequency harmonic measurement; Integer periodic extension method; Fast Fourier transform

1. Introduction

Power electronic components are widely used in power distribution system. Due to its nonlinear load characteristics, harmonic pollution is getting worse in distribution systems. Therefore, harmonic problems have been studied in recent years [1,2]. The research scope includes harmonic source analysis and effects, measurement technology and elimination strategy, harmonic model and control, etc.

Many different modern digital signal analysis techniques including fast Fourier transform (FFT), fast Hartley transform (FHT), least-square method, Kalman filter, neural network and others [2,3], have been used to measure and estimate power system harmonics. The FFT algorithm is an effective method to analyze and assess power system harmonic levels. It is usually discussed in literature or used to solve power system harmonic problems [4–8]. The FFT algorithm may produce aliasing, spectral leakage and picket fence effect [6,9,10] which devalue its direct application to this field. Methods to reduce these effects are important study tasks. However, FFT has been applied to many useful applications in power system phasor measurements, and harmonic and harmonic power flow analysis [4,5,8]. Many different windows were applied to harmonic measurement, and the problems that occur are well researched [10,11]. Moo et al. [12] studied the Parseval’s theorem application by FFT for time varying transient harmonics. It may be a useful algorithm to estimate transient or non-stationary signal harmonic component in each cycle.

The problem of harmonic measurement lies in the time varying dynamic characteristics due to load variations. Most harmonic patterns are not constantly fixed within a small limit. The conventional measurement method has inherent defects to handle harmonic signals in dynamic variation. The primary purpose to using windows in the time-dependent Fourier transform is to limit the extent of the finite-length data sequence to the transformed so that the Fourier spectrum may be reasonably stationary over the duration of the window function. But the frequency resolution is primarily influenced by the width of the main-lobe in Fourier spectrum of windows function. Besides, the degree of leakage depends on the relative amplitude of the main-lobe and side-lobe in Fourier spectrum of windows function. The proposed integer periodic extension (IPE) method measures harmonic signals under dynamic conditions for a few cycles and extends one typical cycle signal into any integer cycle. This method can effectively increase the harmonic measurement with good accuracy. Statistically, several cycles of samples can be
tested to obtain a better Fourier spectrum picture of the load harmonic following the proposed IPE method. Case studies from different load characteristics are measured and discussed by comparison. The results present the effectiveness of the proposed IPE method.

2. Measurement theory

2.1. FFT algorithm and windows

For an \( N \) point data sequence \( x[n] \), and its discrete Fourier transform (DFT) is defined as:

\[
X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}, \quad k = 0, 1, 2, \ldots, N-1, \tag{1}
\]

\[
x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] W_N^{-kn}, \quad n = 0, 1, 2, \ldots, N-1. \tag{2}
\]

where \( W_N = \exp(-j2\pi/N) \), \( k \) is a frequency index, and \( x[n] \) is the inverse DFT of \( X[k] \).

For finite-length and time-dependent discrete signals, the DFT provides frequency-domain samples of the discrete-time Fourier transformer. But for a large value of \( N \) point sequence \( x[n] \), the computing time excusing \( N^2 \) complex multiplication and \( N(N-1) \) complex additions of the DFT may become prohibitive in computer calculation process. The FFT instead of DFT, takes advantage of the similarity of many elements in the matrix \( W_N^{kn} \) and produces the same frequency components using only \( (N/2) \log_2 N \) multiplication to execute Eq. (1) [2,9].

The spectral leakage of the FFT algorithm implementation process is encountered in the calculation. When \( x[n] \) is an infinite sequence, it must be truncated to finite \( N \) point sequence in order to implement the discrete signal process. This process is the same as \( x[n] \) multiplying by a rectangle window. Therefore, the spectral leakage is produced by the window effect. It is a useful method in that the choice of proper window function may reduce leakage effect. There are many different window functions cited in the literature [10,11], including Rectangle, Triangle, Hanning, Blackman, Papuolis, etc. However, both Rectangle and Hanning are the most important windows, and are described as follows [10].

The Rectangle window is defined by:

\[
w[n] = 1, \quad n = 0, 1, 2, 3, \ldots, N-1, \tag{3}
\]

and its Fourier transform is

\[
W_R(\omega) = \exp\left(j \frac{N-1}{2} \omega \right) \frac{\sin \frac{N\omega}{2}}{\sin \frac{\omega}{2}}. \tag{4}
\]

It has the narrowest main-lobe for a given length \( N \), but has the largest side-lobe of all commonly used windows. It should yield the sharpest transition side-lobe. Note that the main-lobe or rectangle window is inversely proportional to the length \( N \).

The Hanning window was defined by:

\[
w[n] = 0.5 \left[ 1 - \cos \frac{2\pi n}{N} \right], \quad n = 0, 1, 2, 3, \ldots, N-1, \tag{5}
\]

and its Fourier transform is defined by:

\[
W_H(\omega) = 0.5 W_R(\omega)
\]

\[+ 0.25 \left[ W_R\left(\omega - \frac{2\pi}{N} \right) + W_R\left(\omega + \frac{2\pi}{N} \right) \right]. \tag{6}
\]

This Fourier spectrum of the Hanning window becomes smoother than the Rectangle window, and it reflects this increased smoothness in decreased side-lobe level and faster fall-off but with an increased main-lobe width. Therefore, its frequency resolution is less than the Rectangle window. However, it has phase shift property, and does not suit the harmonic active and reactive power contents calculation.

The Rectangle window is the simplest method in application. However, the Hanning window is one of the superior windows, and is widely used as a digital spectrum analysis instrument in our laboratory. Many commercial instruments use it as well.

The normalized equivalent noise bandwidth (ENBW) of window is defined as:

\[
\text{ENBW} = \frac{\sum_{n=0}^{N-1} w^2[n]}{\left[ \sum_{n=0}^{N-1} w[n] \right]^2}. \tag{7}
\]

The ENBW of window is a conventional measure of bandwidth in a Fourier spectrum. It is the width of a rectangle filter with the same peak power gain in a Fourier spectrum. Note that the higher ENBW permits additional noise contribution to a spectral estimate.

The process gain (PG) of window is defined as:

\[
\text{PG} = \frac{\left[ \sum_{n=0}^{N-1} w[n] \right]^2}{\sum_{n=0}^{N-1} w^2[n]}. \tag{8}
\]

Note that the PG is the reciprocal of the normalized ENBW, it implies a higher ENBW to cause a degraded PG.

The \( N \) points FFT may be considered as \( N \) narrow band-pass filter bank. When the spectrum is between two FFT frequency bins, it can not be detected. This effect is termed the scalloping loss or the picket fence effect. The scalloping loss (SL) can be defined as:
\( S_L = \left| \sum_{n=0}^{N-1} w[n] \exp \left( -j \frac{\pi n}{N} \right) \right| = \left| \frac{W(0)}{W(0)} \right| \) \tag{9}

The SL implicates the maximum reduction of PG due to signal frequency. Then, the worst case process loss (WCPL) is defined as:

\[ WCPL = S_L + 10 \log |PG| \] \tag{10}

The WCPL of window represents a figure of extent. If the WCPL exceeds 3.8 dB, the window is poor and is not used.

2.2. Parameter selection

An incorrect spectrum analysis might occur when the sampled data was blindly chosen. In the power distribution system, the voltage and current signal frequency can hardly be a constant. Therefore, the data length selection must include at least one cycle to implement FFT.

The power system harmonic component is usually analyzed to 50-th harmonic components, that is 3000 Hz in a 60 Hz power frequency system. Therefore, it is very important work to choose a proper sampling rate. The sampling frequency is defined by:

\[ f_s = N \times f_1, \] \tag{11}

where \( f_1 \) is the basic frequency component, \( N \) is sampling data point per cycle and \( n \)-th power of two. When \( N \) is 256, the harmonic component can be analyzed up to 128-th harmonic.

According to the Nyquist sampling theorem, \( f_s \geq 2f_n \) must be satisfied to prevent aliasing effect. In 60 Hz fundamental power frequency, the sampling rate of 15.36( = 256 \times 60) kHz is necessary to analyze up to 50-th harmonic components, because \( f_s \) is selected 4\( f_n \) or higher in practical use.

An \( N \) point sequence \( x[n] \), whose sampling rate is \( f_s \), has resolution frequency \( F \) after FFT process may be defined by:

\[ F = \frac{f_s}{N} = \frac{1}{T} \] \tag{12}

where \( T \) is data length.

According to Eq. (12), it is important to increase frequency resolution by increasing \( f_s \) under a fixed data length.

### Table 1
Comparison of operation parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>The zero padding method</th>
<th>Proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main lobe width</td>
<td>( 2f_s/N )</td>
<td>( 4f_s/N )</td>
</tr>
<tr>
<td>Main lobe 3-dB width</td>
<td>( 0.89f_s/N )</td>
<td>( 1.44f_s/N )</td>
</tr>
<tr>
<td>Highest side-lobe level</td>
<td>( 1.21f_s/N )</td>
<td>( 2f_s/N )</td>
</tr>
<tr>
<td>Side-lobe fall off (dB/oct)</td>
<td>-6 dB</td>
<td>-18 dB</td>
</tr>
<tr>
<td>Coherent gain</td>
<td>1.4</td>
<td>1.0</td>
</tr>
<tr>
<td>ENBW</td>
<td>( f_s/N )</td>
<td>( 1.5f_s/N )</td>
</tr>
<tr>
<td>PG (dB)</td>
<td>0</td>
<td>1.76</td>
</tr>
<tr>
<td>SL (dB)</td>
<td>3.92</td>
<td>1.42</td>
</tr>
<tr>
<td>WCPL (dB)</td>
<td>3.92</td>
<td>3.89</td>
</tr>
<tr>
<td>Frequency resolution</td>
<td>( f_s/N )</td>
<td>( 2f_s/N )</td>
</tr>
</tbody>
</table>

\( m = 64 \).

### Table 2
The measured THD values using different methods in case studies

<table>
<thead>
<tr>
<th>Case</th>
<th>THD (%)</th>
<th>Padding zeros with window</th>
<th>Proposed methods</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Rectangle window 1-Cycle</td>
<td>Hanning window 1-Cycle</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2-Cycle</td>
<td>1-Cycle</td>
</tr>
<tr>
<td>First</td>
<td>THDv</td>
<td>3.893</td>
<td>3.728</td>
</tr>
<tr>
<td></td>
<td>THDi</td>
<td>17.673</td>
<td>15.009</td>
</tr>
<tr>
<td>Second</td>
<td>THDv</td>
<td>2.978</td>
<td>2.817</td>
</tr>
<tr>
<td></td>
<td>THDi</td>
<td>50.789</td>
<td>48.749</td>
</tr>
<tr>
<td>Third</td>
<td>THDv</td>
<td>4.817</td>
<td>4.959</td>
</tr>
<tr>
<td>Fourth</td>
<td>THDv</td>
<td>1.975</td>
<td>1.957</td>
</tr>
<tr>
<td></td>
<td>THDi</td>
<td>5.962</td>
<td>5.916</td>
</tr>
<tr>
<td>Fifth</td>
<td>THDv</td>
<td>62.049</td>
<td>59.896</td>
</tr>
<tr>
<td></td>
<td>THDi</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. The proposed method

The zero padding method is used to increase frequency resolution of a FFT processor. This method with a window can be expressed as:

\[ X[k] = \sum_{n=0}^{mN-1} x[n]w[n]W_{mn}^{kn}, \]  

(13)

where, \( W_{mn} = \exp(j2\pi/mN), \) \( m \) is the integer number,

\[ x[n] = \begin{cases} \text{data sequence,} & 0 \leq n \leq N-1, \\ 0, & N \leq n \leq mN-1. \end{cases} \]  

(14)

and

\[ w[n] = \begin{cases} \text{data window function,} & 0 \leq n \leq N-1, \\ 0, & N \leq n \leq mN-1. \end{cases} \]  

(15)

For example, a 60 Hz harmonic pollution power system voltage signal is measured with a sampling rate of 15.36 kHz. The results of FFT, which sampled the data sequence has 256 points per cycle, 60 Hz frequency resolution and analyze up to 128-th harmonic components. By padding \( 63 \times 256 \) zeros, those data may become \( 64 \times 256 \) points and the frequency resolution after FFT can increase to 0.9375 Hz by the linear interpolation method. Using a similar approach as the zero padding method, the full length of sampled period data is copied and extended for increasing frequency resolution. The proposed formula can be presented as following:

\[ X[k] = \sum_{n=0}^{mN-1} x'[n]W_{mn}^{kn}, \]  

(16)

where

\[ x'[n] = \sum_{r=0}^{m} x[n + rN], \]  

(17)

\( r \) and \( m \) are integer numbers.

The difference between Eq. (13) and Eq. (16) are illustrated as follows. For a complete representation of a data sequence of length \( N \) (at least including one cycle finite-length data), the \( N \)-point FFT is sufficient since the original sequence can be completely recovered from it.

However, a simple examination of the \( N \)-point FFT can result in misleading interpretation on sub-harmonic frequency. For this reason, it is common to use zero padding in the time-domain so that the Fourier spectrum is sufficiently oversampled and important features are therefore readily apparent [9].

In fact, the use of higher degrees of the time-domain zero padding method is interpolation (e.g. linear inter-
polation) between the FFT values. However, it cannot provide a reasonably accurate picture of the Fourier spectrum. It cannot be used to estimate the location and amplitude of spectral peaks. But the use of time-domain padding of original data will overcome the noted drawback therefore the IPE method arises here.

For example, in the 60 Hz harmonic power system, the signal is measured by a sampling rate of 15.36 kHz. By extending 63 periodic data, the data sequence becomes 64 periods. According to Eq. (12), the frequency resolution can also increase to 0.9375 Hz. These results are improved from the zero padding method because the proposed method uses real values. This is why the proposed method is termed an IPE method. This is a very useful method to analyze power system stationary sinusoidal waveforms signal harmonic components in off-line operation.

Using the proposed IPE method, an important study to detect and assess the power system fundamental frequency should be carried out before harmonic measurements. The sampling rate selection is necessary in the proposed method to ensure data accuracy. Comparison of the operation parameter of the zero padding method with the Rectangle window and the Hanning window to the proposed IPE method are listed in Table 1 [10].

4. Implementation results

4.1. Measurement

Data acquisition is implemented by a portable PC instrument with 16-bits ADC interface and it includes these following main steps:
1. Measure system power frequency and then set instrument sampling rate,
2. Measurement,
3. Save data.

The data analysis software program includes the zero padding method and the proposed IPE method. The software program of the zero padding method includes the following main steps:
1. Load data,
2. De-average value,
3. Multiply window,
4. Padding zeros,
5. Fast Fourier transform,
6. Calculate THD and plot analysis results,
and the proposed IPE method:
1. Load data,
2. De-average value,
3. Extension periodic,
4. Fast Fourier transform,
5. Calculate THD and plot analysis results.

The former method is a conventional algorithm for harmonic analysis. To reduce the FFT error, the de-average value of signal process is necessary. The application of window function can reduce the spectral leakage. The proposed IPE method is based on a fixed frequency system signal. In practical power systems, frequency variation occurs if the load is nonstationary. It is necessary beforehand to use a frequency meter to measure the actual power frequency, the exact integer periodic length of sampling data is necessary in order to reduce picket fence effect and spectral leakage.

5. Case studies

There are five different case studies from different plants, using different methods to analyze spectrum presented in this paper. These tests facilitate include: (1) a DC motor driver; (2) a mill of armour plate of iron-work; (3) a variable speed coupling (or namely eddy current coupling) motor; (4) a main power transformer of distribution system; and (5) an uninterruptible power supply (UPS).

If the fundamental power frequency is 60 Hz, the sampling rate of three-phase voltage and current signals in these test facilities is adjusted to 15.36 kHz per channel so that these sampling points have 256 points per cycle. The data are analyzed as follows:
1. One cycle raw data (256 point) multiplies Rectangle and Hanning windows, respectively and pads 16128 zeros, then 16384 points FFT are implemented.
2. Two cycles raw data (512 points) multiplies Rectangle and Hanning windows, respectively and pads 15872 zeros, then 16384 points FFT are implemented.
3. One cycle (256 points) and two cycle (512 points) raw data are extended to 64 cycles, then 16384 points FFT are implemented, respectively.

Due to paper length limitations, the analyzed results in steps 2–3 are shown as the following and all voltage spectrum analyzed diagrams will be omitted in this paper. Moreover, all THD values are listed in Table 2.

5.1. Case one results

The first facility of the distribution system under measurement is the DC motor driver, which consists of two 750 V/1042 kW DC loads, on a three-phase 11.4 kV bus. These motor drivers are 12-plus AC–DC con-
verters. The measurement point is located on 11.4 kV side, and fundamental power frequency is 60.1 Hz. So, the sampling rates of all three-phase voltage and current signals are adjusted on 15.386 kHz. Fig. 1(a) shows one-phase component voltage and current waveforms, and the time resolution is 0.065 ms corresponding to this sampling rate. Fig. 1(b) and 1(c) show the Fourier spectra corresponding to the current waveform in the Fig. 1(a) with zero padding method by Rectangle and Hanning windows, respectively. The frequency resolution of the Fourier spectra picture in Fig. 1(b) and 1(c) are 60.1 and 120.2 Hz, respectively. Additionally, the highest side-lobe level to Fig. 1(b) and 1(c) are −13 and −32 dB respectively. Fig. 1(d) shows the Fourier spectrum with the IPE method, and its frequency resolution is 0.9391 Hz. The highest side-lobe effect on each power frequency harmonic component is 43 dB. Finally, the THD values of current and voltage corresponding to these methods are measured and listed as Table 2.

5.2. Case two results

The second facility of the distribution system under measurement is an armour plate mill. On the 11.4 kV side, single-phase current and voltage waveforms of mill driver are shown in Fig. 2(a). The mill driver also has a 12-pulse AC–DC converter. The testing point is also located on the 11.4 kV side, and the fundamental power frequency is 60 Hz. So, the sampling rate of all three-phase voltage and current signals are adjusted to 15.36 kHz. Fig. 2(a) shows one-phase component voltage and current waveforms, and the time resolution is 0.065 ms corresponding to this sampling rate. Fig. 2(b) and 2(c) show the Fourier spectra corresponding to the current waveform in Fig. 2(a) with the zero padding method by Rectangle and Hanning windows, respectively. The frequency resolution of the Fourier spectrum picture in Fig. 2(b) and 2(c) is 60 and 120 Hz, respectively. Additionally, the highest side-lobe level of Fig. 2(b) and 2(c) is −13 and −32 dB, respectively. Fig. 2(d) shows the Fourier spectrum with the IPE method, and its frequency resolution is 0.9375 Hz. The highest side-lobe effect in power frequency harmonic component is 43 dB. Finally, the THD values of current and voltage corresponding to these methods are measured and listed in Table 2.

5.3. Case three results

The third facility is a variable speed coupling (VSC) motor in dye cloth works. Fig. 3(a) shows waveforms of one-phase load on a three-phase 380 V bus. The fundamental power frequency is this load is also 60 Hz. So,
Fig. 5. The case five measurement results for: (a) single phase voltage and current waveforms; (b) and (c) analyzed current spectrum by zero padding method with Rectangle and Hanning windows, respectively; and (d) with proposed method.

5.4. Case four results

The fourth facility is a three-phase main power transformer (69/11.4 kV, 12/15 MVA) in textile plant. Single-phase component voltage and current waveforms on the 11.4 kV side are shown as Fig. 4(a). The fundamental power frequency of this load is also 60 Hz. So, the sampling rate of all three-phase voltage and current signals are adjusted to 15.36 kHz. Therefore, these voltage and current waveforms time resolutions are 0.065 ms corresponding to the sampling rate. Fig. 3(b) and 3(c) show the Fourier spectra corresponding to the current waveform in Fig. 3(a) with the zero padding method by Rectangle and Hanning windows, respectively. The frequency resolution of the Fourier spectra pictures in Fig. 3(b) and 3(c) are 60 and 120 Hz, respectively. Additionally, the highest side-lobe level of Fig. 3(b) and 3(c) are −13 and −32 dB, respectively. Fig. 3(d) shows the Fourier spectrum with the IPE method, and its frequency resolution of 0.9375 Hz. And, the highest side-lobe effect in the power frequency harmonic component is −43 dB. Finally, the THD values of current and voltage corresponding to these methods are measured and listed as Table 2.

5.5. Case five results

The fifth facility is a UPS under 50% loading. It is used for supplying critical computer load for accounting systems in supermarkets. For supplying power to the inverter and keeping the battery bank charged, UPS has a 12-pulse AC–DC converter type of rectifier. Fig. 5(a) shows the waveforms of one phase component load current on a 380 V bus. The fundamental power fre-
quency of this load is also 60 Hz. So, the sampling rate of all three-phase voltage and current signals are adjusted to 15.36 kHz. Therefore, these voltage and current waveform corresponding time resolutions are 0.065 ms. Fig. 5(b) and 5(c) show the Fourier spectra corresponding to the current waveform in Fig. 5(a) with zero padding method by Rectangle and Hanning windows, respectively. The frequency resolutions of the Fourier spectra picture in Fig. 5(b) and 5(c) are 60 and 120 Hz, respectively. Additionally, the highest side-lobe level of Fig. 5(b) and 5(c) are $-13$ and $-32$ dB, respectively. Fig. 5(d) shows the Fourier spectrum with the IPE method, and its frequency resolution is 0.9375 Hz. The highest side-lobe effect in power frequency harmonic components is $-43$ dB. Finally, the THD values of current and voltage corresponding to these methods are measured and listed in Table 2. To illustrate the advantage of the IPE method further, referring to Figs. 1–5, and corresponding FFT values shown Table 1, the proposed IPE method has resulted in a finer sampling spectrum. Comparing Figs. 1–5 and Table 1, the proposed IPE method would be much more effective for previously locating the peak of the windowed Fourier Transform.

6. Conclusions

From Table 1, we have examined the Rectangle window, Hanning window and the proposed IPE method which satisfy some optimum criteria. The maximum dynamic range of spectra measurement requires the transform of the window to show a highly concentrated main-lobe with very-low side structure. We have proved that the IPE method satisfies this criterion with varying degrees of success.

Using the FFT for spectra analysis in power distribution systems, the window employed has a considerable effect. From Table 2, the THD values have little error between 1-cycle and 2-cycle raw data with Rectangle window and the IPE method because, in general, the power system signals are unstable magnitudes of sinusoidal waveforms. There are large value errors between 1-cycle and 2-cycle raw data with the Hanning window because the frequency resolution of this window is $2f_s/N$ instead of $f_s/N$. Furthermore, the subharmonic contents are neglected in the THD value analysis of power system harmonics, therefore, the used 1-cycle or 2-cycle original data can be sufficient in power frequency.

From Tables 1 and 2, and actual analyzed Fourier spectra diagram of Figs. 1–5 of the proposed IPE method clearly has superior performance than the zero padding method. These advantages can be summarized as follows:

From Table 1 using the proposed IPE method to analyze harmonics, the main lobe width of the spectrum is the narrowest. Moreover, the others performance such as ENBW, SL and WCPL are less and have the highest frequency resolutions.

Five different facility load current harmonics, shown in Figs. 1–5, have resulted in a confusing spectra diagram using the zero padding method, but have resulted in a very good performance with the proposed IPE method.

According to practical measurements, the proposed IPE method is the superior method and is a very suitable and useful method to measure power distribution harmonics.

References