CONTROL ENGINEERING
A Series of Reference Books and Textbooks

Editor

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Additional Volumes in Preparation
Series Introduction

Many textbooks have been written on control engineering, describing new techniques for controlling systems, or new and better ways of mathematically formulating existing methods to solve the ever-increasing complex problems faced by practicing engineers. However, few of these books fully address the applications aspects of control engineering. It is the intention of this new series to redress this situation.

The series will stress applications issues, and not just the mathematics of control engineering. It will provide texts that present not only both new and well-established techniques, but also detailed examples of the application of these methods to the solution of real-world problems. The authors will be drawn from both the academic world and the relevant applications sectors.

There are already many exciting examples of the application of control techniques in the established fields of electrical, mechanical (including aerospace), and chemical engineering. We have only to look around in today's highly automated society to see the use of advanced robotics techniques in the manufacturing industries; the use of automated control and navigation systems in air and surface transport systems; the increasing use of intelligent control systems in the many artifacts available to the domestic consumer market; and the reliable supply of water, gas, and electrical power to the domestic consumer and to industry. However, there are currently many challenging problems that could benefit from wider exposure to the applicability of control methodologies, and the systematic systems-oriented basis inherent in the application of control techniques.

This series presents books that draw on expertise from both the academic world and the applications domains, and will be useful not only as academically recommended course texts but also as handbooks for practitioners in many applications domains. Sliding Mode Control
in Engineering is another outstanding entry to Dekker's Control Engineering series.

Neil Munro
Preface

Many physical systems naturally require the use of discontinuous terms in their dynamics. This is, for instance, the case of mechanical systems with friction. This fact was recognized and advantageously exploited since the very beginning of the 20th century for the regulation of a large variety of dynamical systems. The keystone of this new approach was the theory of differential equations with discontinuous right-hand sides pioneered by academic groups of the former Soviet Union.

On this basis, discontinuous feedback control strategies appeared in the middle of the 20th century under the name of theory of variable-structure systems. Within this viewpoint, the control inputs typically take values from a discrete set, such as the extreme limits of a relay, or from a limited collection of prespecified feedback control functions. The switching logic is designed in such a way that a contracting property dominates the closed-loop dynamics of the system thus leading to a stabilization on a switching manifold, which induces desirable trajectories. Based on these principles, one of the most popular techniques was created, developed since the 1950s and popularized by the seminal paper by Utkin (see [30] in chapter 7): the sliding mode control. The essential feature of this technique is the choice of a switching surface of the state space according to the desired dynamical specifications of the closed-loop system. The switching logic, and thus the control law, are designed so that the state trajectories reach the surface and remain on it.

The main advantages of this method are:

- its robustness against a large class of perturbations or model uncertainties
- the need for a reduced amount of information in comparison to classical control techniques
- the possibility of stabilizing some nonlinear systems which are not stabilizable by continuous state feedback laws
The first implementations had an important drawback: the actuators had to cope with the high frequency bang-bang type of control actions that could produce premature wear, or even breaking. This phenomenon was the main obstacle to the success of these techniques in the industrial community. However, this main disadvantage, called chattering, could be reduced, or even suppressed, using techniques such as nonlinear gains, dynamic extensions, or by using more recent strategies, such as higher-order sliding mode control (see Chapter 3).

Once the constraint sliding function (CSF) was chosen according to some design specifications (stabilizing dynamics or tracking), then two difficulties may appear:

1. the CSF should be of relative degree one (differentiating once for this function with respect to time: the control should appear) in order to provide the existence of a sliding motion; and
2. the CSF may depend on the whole state (and not only on the measured outputs).

To circumvent D1 one may use a new CSF of relative degree one (see the introduction of Chapter 3 and the choice of the CSF in subsection 13.3.1). Another promising alternative to this difficulty is based on higher-order sliding mode controller design (see Chapter 3). Concerning D2 when the CSF depends on other variables than the measured outputs, a natural solution is provided by observer design. This approach has one advantage which concerns the natural filtering of the measurements (see Chapter 4 p. 121). But the drawback is that the class of admissible perturbations is reduced, since the perturbation should match two conditions: one for the control (see Chapter 1 p. 20) and the other for the observer (see Section 4.5).

We are currently living in an important time for these types of techniques. Now they may become more popular in the industrial community: they are relatively simple to implement, they show a great robustness, and they are also applicable to complex problems. Finally, many applications have been developed (see the Table of Contents):

- Control of electrical motors, DTC
- Observers and signal reconstruction
- Mechanical systems
- Control of robots and manipulators
- Magnetic bearings
Based on these facts, several active researchers in this field combined their efforts, thanks to the support of many French institutions\textsuperscript{1}, to present new trends in sliding mode control.

In order to clearly present new trends, it is necessary to first give an historical overview of classical sliding mode (Chapter 1).

In the same manner of thinking, it is important to recall and introduce, from a very clear educational standpoint, a mathematical background for discontinuous differential equations, which is done in Chapter 2.

Next, a new concept in variable structure systems is introduced in Chapter 3: the higher-order sliding mode. Such control design is naturally motivated by the limits of classical sliding mode (see Chapter 1) and completely validated by the mathematical background (see Chapter 2).

On the basis of these chapters, some control domains and methods are discussed with a sliding mode point of view:

- **Chapter 4** deals with observer design for a large class of nonlinear systems.
- **Chapter 5** presents a complementary point of view concerning the design of dynamical output controllers, instead of observer and state controllers.
- **Chapter 6** presents the link between three of the most popular nonlinear control methods (i.e., sliding mode, passivity, and flatness) illustrated through power converter examples.
- **Chapter 7** is dedicated to stability and stabilization. The domain of sliding mode motion is particularly investigated and the usefulness of the regular form is pointed out.
- **Chapter 8** recalls some problems due to the discretization of the sliding mode controller. Some solutions are recalled and the usefulness under sampling of the higher-order sliding mode is highlighted.
- **Chapter 9** deals with adaptive control design. Here, some basic features of control algorithms derived from a suitable combination of sliding mode and adaptive control theory are presented.
- **Chapters 10 and 11** are dedicated to time delay effects. They deal, respectively, with relay control systems and with changes of behavior due to the delay presence.

\textsuperscript{1}CNRS, GdR Automatique, GRAISyHM, LAIL-UPRESA CNRS 8021, ECE-ENSEA and Ecole Centrale de Lille.
• Chapter 12 is dedicated to the control of infinite-dimensional systems. A disturbance rejection for such systems is particularly presented.

In order to increase interest in the proposed methods, the book ends with two application fields. Chapter 13 is dedicated to robotic applications and Chapter 14 deals with sliding mode control for induction motors.

Wilfrid PERRUQUETTI
Jean-Pierre BARBOT
FRANCE
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