6.1 Introduction

In this chapter, we present two discontinuous feedback controller design examples that are solved by a combination of off-line trajectory planning (OLTP), based on flatness, passivity-based control (PBC) and either pulse width modulation (PWM), or sliding modes (SM). The examples are presented in the context of two electrical systems. A permanent magnet (PM) stepper motor which is a weakly minimum phase multivariable nonlinear system and a dc-to-dc power converter of the “boost” type, which is a single input, non-minimum phase system. Both systems are switched systems, i.e., their control inputs take values on discrete sets.

The OLTP process for nonlinear differentially flat systems is not only natural but it is also quite flexible and powerful, as already demonstrated by many application examples and solid theoretical developments (see the work of Fliess and his colleagues [3][4] for interesting details and far reaching developments). The PM stepper motor and the “boost” converter, treated in this chapter, are differentially flat (see [6] and [9]).

Stabilization tasks for single input and for multivariable flat systems can be easily achieved, even in the case of non-minimum phase output requirements, thanks to the fact that the differential parameterization provided by flatness, degenerates, under equilibrium conditions, into a static...
parameterization which allows to reconcile the non-minimum phase output controlled maneuver objective with an equivalent objective for the flat output variable [5].

A more complex problem is that of having a non-minimum phase output follow a prespecified trajectory that leads to a desired, permanent, oscillation, as in the case of dc-to-ac power conversion. Part of the difficulty now arises from the fact that, in some cases, the differential parameterization is no longer helpful in directly establishing the corresponding signal to be tracked by the flat output or, alternatively, by a minimum phase output. The problem may be solved by resorting to an approximating sequence of finite differential parameterizations of the minimum phase output in terms of the non-minimum phase output. In the limit, this sequence may be interpreted to lead to an infinite order flatness. With the aid of digital computer simulations, we show that for the normalized model of the boost converter this sequence of parameterizations enjoys a rather fast convergence property and only one or two of its terms are required in order to obtain a tight solution to the tracking problem.

Section 6.2 presents a PM stepper motor controller design example solving a stabilization task requiring an equilibrium-to-equilibrium transfer via trajectory planning, exact linearization, and PWM control. Section 6.3 presents a boost converter design example. The general properties of flatness and passivity of the “boost” converter circuit are established and a general derivation is provided for a sliding mode solution (based on passivity and flatness), of a trajectory tracking task for the non-minimum phase output. The proposed solution is suitable for both the stabilization and trajectory tracking problems. The last section of this chapter presents some conclusions and suggestions for further research.

6.2 The permanent magnet stepper motor

Consider a nonlinear model of a permanent magnet (PM) stepper motor, taken from Bodson and Chiasson [1],

\[
\begin{align*}
\frac{di_a}{dt} &= \frac{1}{L} (v_a - Ri_a + K_m \omega \sin(N_r \theta)) \\
\frac{di_b}{dt} &= \frac{1}{L} (v_b - Ri_b - K_m \omega \cos(N_r \theta)) \\
\frac{d\omega}{dt} &= \frac{1}{J} (-K_m i_a \sin(N_r \theta) + K_m i_b \cos(N_r \theta) - B \omega - \tau) \\
\frac{d\theta}{dt} &= \omega
\end{align*}
\]  

(6.1)
where $i_a$ represents the current in phase $A$ of the motor, $i_b$ is the current in the phase $B$ of the motor, $\theta$ is the angular displacement of the shaft of the motor, and $v_a$ and $v_b$, are the voltage applied on the windings of the phase $A$ and phase $B$, respectively. The parameters $R$ and $L$, the resistance and self inductances in each of the phase windings, are constant and assumed to be known. Similarly the number of rotor teeth $N_r$, the torque constant of the motor $K_m$, the rotor load inertia $J$, and the viscous friction $B$ are assumed known and constant. The load torque perturbation, denoted by $\tau$, is, for analysis purposes, assumed to be zero.

The nature of the control inputs $v_a$ and $v_b$ is that of switched inputs respectively taking values in the discrete control sets, $U_a = \{+V_a, -V_a\}$ and $U_b = \{+V_b, -V_b\}$, as obtained from a diode-based PWM operated bridge inverter. However, our design developments treat $v_a$ and $v_b$ as if they were continuous valued inputs. The interpretation of this procedure is that the model (6.1) is being regarded as an infinite frequency PWM average model [8]. We shall consider the obtained feedback control law as a feedback duty ratio synthesizer and implement the derived control law through an actual switching law of the PWM type, taking values in $U_a$ and $U_b$. Trajectory planning is shown to naturally avoid the possible saturation of the computed duty ratio functions.

### 6.2.1 The simpler D-Q nonlinear model of the PM stepper motor

The so called $D-Q$ (direct-to-quadrature) transformation gets rid of all the trigonometric terms appearing in the motor model. This transformation is given by

\[
\begin{bmatrix}
    i_d \\
    i_q \\
    v_d \\
    v_q
\end{bmatrix} =
\begin{bmatrix}
    \cos(N_r\theta) & \sin(N_r\theta) \\
    -\sin(N_r\theta) & \cos(N_r\theta) \\
    \cos(N_r\theta) & \sin(N_r\theta) \\
    -\sin(N_r\theta) & \cos(N_r\theta)
\end{bmatrix} \begin{bmatrix}
    i_a \\
    i_b \\
    v_a \\
    v_b
\end{bmatrix}
\]

(6.2)

The current $i_d$ is the direct current and $i_q$ is the quadrature current. Also, $v_d$ and $v_q$ are addressed as the direct and quadrature voltages, respectively and act as the new control inputs to the system.

The transformed system is given by

\[
\begin{align*}
\frac{di_d}{dt} &= \frac{1}{L} (v_d - Ri_d + N_r\omega Li_q) \\
\frac{di_q}{dt} &= \frac{1}{L} (v_q - Ri_q - N_r\omega Li_d - K_m\omega)
\end{align*}
\]
6.2.2 The control problem

The control objective is to drive the system from a given initial equilibrium value towards a final equilibrium value achieving, as a result, a desired final value for the position variable $\theta$.

The equilibrium point $(\bar{i}_d, \bar{i}_q, \bar{\omega}, \bar{\theta})$ of the transformed system, for a given constant value of the direct voltage, $v_d = \bar{v}_d$, is given by

$$\bar{v}_q = 0, \quad \bar{i}_d = \frac{\bar{v}_d}{R}, \quad \bar{i}_q = 0, \quad \bar{\omega} = 0, \quad \bar{\theta} = \text{arbitrary constant}$$

We assume that the equilibrium value of $i_d$ is not zero. In fact, we will keep $i_d$ bounded away from zero throughout the equilibrium transfer maneuver. As will be shown, this is quite easy to guarantee.

Suppose, for a moment, that the vector relative degree $(1,1)$ outputs $i_d$ and $i_q$ are held constant at some value $(\bar{i}_d, \bar{i}_q) = (\bar{i}_d, 0)$. Then the zero dynamics corresponding to this set of values is given by the linear system

$$\frac{d\omega}{dt} = \omega, \quad \frac{d\theta}{dt} = -B\omega$$

which exhibits two eigenvalues; one located at the origin, and the other located in the left half portion of the complex plane, at the point $(-B, 0)$. The system outputs, $(i_d, i_q)$, are then weakly minimum phase and, according to the results of [2], they are passive outputs.

6.2.3 A passivity canonical model of the PM stepper motor

Consider the following positive definite (Lyapunov) energy storage function

$$H(i_d, i_q, \omega, \theta) = \frac{1}{2} [L (i_d^2 + i_q^2) + J\omega^2 + \gamma \theta^2]$$

The time derivative of the storage function, along the controlled motions of the system, satisfies

$$\dot{H} \leq \frac{1}{i_d} \left( v_d + \frac{\gamma \theta}{i_d} \right) + i_q v_q$$
This last expression, plus the weakly minimum phase character of the outputs $i_d$ and $i_q$, reveals that the system is a passive operator between the modified inputs $(\vartheta_d, \vartheta_q) = (v_d + \theta \omega / i_d, v_q)$ and the system outputs $(i_d, i_q)$. This justifies the following additional input coordinate transformation,

$$\vartheta_d = v_d + \theta \omega \frac{i_d}{d_d}; \quad \vartheta_q = v_q$$

(6.7)

We write the system, in matrix form, as

$$\begin{bmatrix}
L & 0 & 0 & 0 \\
0 & L & 0 & 0 \\
0 & 0 & J & 0 \\
0 & 0 & 0 & \gamma
\end{bmatrix}
\begin{bmatrix}
\frac{d i_d}{d t} \\
\frac{d i_q}{d t} \\
\frac{d \vartheta_d}{d t} \\
\frac{d \vartheta_q}{d t}
\end{bmatrix}
= \begin{bmatrix}
0 & N_T L \omega & 0 & -\gamma \omega i_d \\
-N_T L \omega & 0 & -K_m & 0 \\
0 & K_m & 0 & 0 \\
\gamma \omega \frac{i_d}{d_d} & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
i_d \\
i_q \\
\vartheta_d \\
\vartheta_q
\end{bmatrix}
+ \begin{bmatrix}
-R & 0 & 0 & 0 \\
0 & -R & 0 & 0 \\
0 & 0 & -B & 0 \\
0 & 0 & 0 & \omega
\end{bmatrix}
\begin{bmatrix}
i_d \\
i_q \\
\omega \\
\theta
\end{bmatrix}
+ \begin{bmatrix}
1 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\vartheta_d \\
\vartheta_q
\end{bmatrix}
$$

(6.8)

The obtained model, clearly exhibits the conservative and the dissipative structure of the system.

### 6.2.4 A controller based on “energy shaping plus damping injection”

The “energy shaping plus damping injection” dynamic feedback controller design method, extensively treated in [7], yields the following dynamical feedback controller specification,

$$\begin{align*}
\vartheta_d &= L \frac{d}{d t} i_d^*(t) - N_T L \omega i_d^*(t) + \gamma \frac{\omega}{i_d} \zeta_2 + R_i i_d^*(t) \\
\vartheta_q &= L \frac{d}{d t} i_q^*(t) + N_T L \omega i_q^*(t) + K_m \zeta_1 + R_i^* i_q^*(t)
\end{align*}$$

(6.9)

with $\zeta_1$ and $\zeta_2$ satisfying

$$\begin{align*}
J \dot{\zeta}_1 &= K_m i_q^*(t) - B \zeta_1 + R_B (\omega - \zeta_1) \\
\gamma \dot{\zeta}_2 &= \gamma \frac{\omega}{i_d} \dot{i}_d^*(t) + R_\theta (\theta - \zeta_2)
\end{align*}$$

(6.10)

with $R_B$ and $R_\theta$ positive design constants.
The transformed control inputs to the system are determined from the equalities

\[ v_d = \vartheta_d - \frac{\vartheta \omega}{i_d}; \quad v_q = \vartheta_q \quad (6.11) \]

The feedback controller, in terms of the original inputs \( \vartheta_a, \vartheta_b \) and the phase currents \( i_a, i_b \), is obtained from (6.2) and (6.10).

We state the tracking error stabilization properties of the feedback controller (6.9), (6.10), as follows.

**Proposition 56** The passivity-based dynamic feedback controller yields a state vector tracking error dynamics, described by the vector,

\[ e = [i_d - i_d^*(t), i_q - i_q^*(t), \omega - \zeta_1, \theta - \zeta_2] \]

which is globally exponentially asymptotically stable to zero.

**Proof**
Substituting the control input expressions, given in (6.9), into the D-Q system model (6.3), we obtain, using the following definitions of the state tracking error variables; \( e_1 = i_d - i_d^*(t) \) and \( e_2 = i_q - i_q^*(t) \), \( e_3 = \omega - \zeta_1 \) and \( e_4 = \theta - \zeta_2 \),

\[
\begin{bmatrix}
L & 0 & 0 & 0 \\
0 & L & 0 & 0 \\
0 & 0 & J & 0 \\
0 & 0 & 0 & \gamma
\end{bmatrix}
\begin{bmatrix}
\dot{e}_1 \\
\dot{e}_2 \\
\dot{e}_3 \\
\dot{e}_4
\end{bmatrix}
= \begin{bmatrix}
0 & N_e L \omega & 0 & 0 \\
-N_e L \omega & 0 & -K_m & 0 \\
0 & K_m & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
e_1 \\
e_2 \\
e_3 \\
e_4
\end{bmatrix}
+ \begin{bmatrix}
-R & 0 & 0 & 0 \\
0 & -R & 0 & 0 \\
0 & 0 & -B - R_B & 0 \\
0 & 0 & 0 & -R_\theta
\end{bmatrix}
\begin{bmatrix}
e_1 \\
e_2 \\
e_3 \\
e_4
\end{bmatrix}
\quad (6.12)
\]

Using the modified energy function \( H(e) = \frac{1}{2} (Le_1^2 + Le_2^2 + Je_3^2 + \gamma e_4^2) \), with \( \gamma \) being an arbitrary strictly positive constant, one establishes that, \( \dot{H}(e) \leq \alpha H(e) \), with \( \alpha < 0 \) as a constant dependent on the system parameters \( L, J, R, B, \gamma \), and the design parameters, \( R_B \) and \( R_\theta \). The tracking error vector \( e \) is globally exponentially asymptotically stable to zero, i.e.,

\[ i_d \to i_d^*(t), \quad i_q \to i_q^*(t), \quad \omega \to \zeta_1, \quad \theta \to \zeta_2 \quad (6.13) \]

In the absence of load torque perturbations, the desired current \( i_d^*(t) \) is made to converge to zero and, then, \( i_q \) also converges to zero. The planned flat current \( i_d^*(t) \) is made to converge to a nonzero constant. Then, \( i_d \)
Converges to the same value. The outputs \( i_d \) and \( i_q \) are passive, thus, \( \omega \) and \( \zeta_1 \) converge to zero. The angle \( \theta \), and \( \zeta_2 \), both converge to a constant to be established. The flatness property allows the final value of \( \theta \) to be completely determined at will, as will be shown in the following section.

### 6.2.5 Differential flatness of the system

The PM stepper motor is easily seen to be differentially flat, since all variables in the system can be completely parameterized in terms of differential functions of the independent variables constituted by the direct current \( i_d \) and the motor shaft angular position \( \theta \). (see [6] and [11] and the references therein). The flat outputs, denoted by \( F = (F_1, F_2) = (i_d, \theta) \), yield,

\[
\begin{align*}
    i_d &= F_1, \quad \theta = F_2, \quad \omega = \dot{F}_2, \quad i_q = \frac{J}{K_m} \ddot{F}_2 + \frac{B}{K_m} \dot{F}_2, \\
    v_d &= L \dddot{F}_1 + R F_1 - N_r L \ddot{F}_2 \left( \frac{J}{K_m} \dddot{F}_2 + \frac{B}{K_m} \dot{F}_2 \right), \\
    v_q &= \frac{L J}{K_m} F_2^{(3)} + \frac{L B}{K_m} \dddot{F}_2 + R \left( \frac{J}{K_m} \dddot{F}_2 + \frac{B}{K_m} \dot{F}_2 \right) + N_r L \ddot{F}_2 F_1 + K_m \dddot{F}_2 
\end{align*}
\]

(6.14)

All systems properties, in particular those concerning the ones needed for passivity-based controller design, are already reflected in the above complete differential parameterization, as it can be easily verified.

### 6.2.6 A dynamic passivity plus flatness based controller

The passivity-based controller (6.10) requires pre-specified passive outputs trajectories \( *_{i_d}(t) \) and \( *_{i_q}(t) \). Instead of directly specifying those trajectories, it was proposed to specify them in terms of the flat outputs, i.e., we take advantage of the fact that the passive outputs are differentially related to the flat outputs (which, incidentally, are devoid of zero dynamics). The (off-line) specification of such flat outputs already determines the rest of the system variables. The advantage of this approach resides in fact that the flat outputs are fundamental system outputs devoid of internal dynamics and correspond to the hidden linear controllability properties of the system.

The passivity-based controller, exploiting the flatness property of the
system, is then given by

\[ \dot{\theta}_d = L \ddot{F}_1^d(t) - N_r L \omega \left[ \frac{J}{K_m} \dddot{F}_2^d(t) + \frac{B}{K_m} \dot{F}_2^d(t) \right] - \gamma \frac{\omega}{i_d^2} \zeta_t + RF_1^d(t) \]

\[ \dot{\theta}_q = L \left[ \frac{J}{K_m} (F_2^d(t))^{(3)} + \frac{B}{K_m} \ddot{F}_2^d(t) \right] + N_r L \omega F_1^d(t) + K_m \zeta_2 \]

\[ + R \left[ \frac{J}{K_m} \dddot{F}_2^d(t) + \frac{B}{K_m} \dot{F}_2^d(t) \right] \]

(6.15)

with \( \zeta_1 \) and \( \zeta_2 \) satisfying

\[ J \dot{\zeta}_1 = K_m \left[ \frac{J}{K_m} \ddot{F}_2^d(t) + \frac{B}{K_m} \dot{F}_2^d(t) \right] - B \zeta_1 + R \beta (\omega - \zeta_1) \]

\[ \gamma \dot{\zeta}_2 = \gamma \frac{\omega}{i_d^2} F_1^d(t) + R \beta (\theta - \zeta_2) \]

(6.16)

### 6.2.7 Simulation results

We consider a PM stepper motor with the following parameters

\[ R = 8.4 \, \Omega \quad L = 0.010 \, \text{H} \quad K_m = 0.05 \, \text{Vs/rad} \quad J = 3.6 \times 10^{-6} \, \text{Nm}^2/\text{rad} \]

\[ B = 1 \times 10^{-4} \, \text{Nms/rad} \quad N_r = 50 \quad R_b = 0.05 \Omega \]

It is desired to transfer the angular position \( \theta \) from the initial value of \( \theta_0 = 0 \) rad, towards the final value \( \theta_F = 0.03 \) rad, following a trajectory specified by means of an interpolating time polynomial of the form \( \psi(t, t_0, t_f) \) satisfying

\[ \psi(t_0, t_0, t_f) = 0, \quad \psi(t_f, t_0, t_f) = 1 \]  

(6.17)

Thus,

\[ \theta^*(t) = \theta_0 + \psi(t, t_0, t_f) [\theta_F - \theta_0] \]  

(6.18)

One such possible expression, including a particular interpolating polynomial \( \psi(t, t_0, t_f) \), is given by

\[ \theta^*(t) = \theta_0 + \left( \frac{t - t_0}{t_f - t_0} \right)^5 \left[ r_1 - r_2 \left( \frac{t - t_0}{t_f - t_0} \right) + r_3 \left( \frac{t - t_0}{t_f - t_0} \right)^2 - \ldots \right. \]

\[ \left. \ldots - r_6 \left( \frac{t - t_0}{t_f - t_0} \right)^5 (\theta_F - \theta_0) \right] \]  

(6.19)

with

\[ r_1 = 252, \quad r_2 = 1050, \quad r_3 = 1800, \quad r_4 = 1575, \quad r_5 = 700, \quad r_6 = 126 \]
and $t_0 = 0.01\, \text{s}$ and $t_f = 0.02\, \text{s}$.

The flat output variable, $i_d$, is also made to follow a similar time trajectory $i_d^*(t)$, taking the d-current coordinate from the value $i_d(t_0) = i_{d0} = 0.3\, \text{A}$, towards the final value $i_d(t_f) = i_{df} = 0.5\, \text{A}$, during the same previous time interval $[t_0, t_f]$. In other words we specified $i_d^*(t)$ as

$$i_d^*(t) = i_{d0} + \psi(t, t_0, t_f) (i_{df} - i_{d0})$$

The passivity-based feedback controller, proposed in the previous section, is used with the passive output reference trajectories given by

$$i_d^*(t) = i_{d0} + \psi(t, t_0, t_f) [i_{df} - i_{d0}]$$

$$i_q^*(t) = \frac{J}{K_m} \dot{\theta}^*(t) + \frac{B}{K_m} \dot{\theta}^*(t),$$

The design constants $R_B$, $R_{\theta}$ and $\gamma$, are

$$R_B = 0.05, \quad R_\theta = 2, \quad \gamma = 1$$

Figure 6.1 shows the simulations of the closed loop performance of the stepper motor in original $a-b$ coordinates. The load torque was set to zero in these simulations.

Figure 6.1: PM Stepper motor closed-loop response to passivity plus flatness based controller ($a-b$ variables)
In order to account for unmodeled constant load torque perturbations, entering the angular velocity dynamics as \( \tau \), we use an outer loop proportional-integral-derivative (PID) controller, feeding back the dynamic controller angular velocity tracking error \( \epsilon(t) = \zeta_1 - \dot{\theta}^*(t) \). This controller guarantees that \( \zeta_1 \) actually tracks \( \dot{\theta}^*(t) \), in spite of the perturbation load torque. Since \( \omega \) is guaranteed to track \( \zeta_1 \), by the previous arguments, the net result is that \( \omega \) tracks \( \dot{\theta}^*(t) \) in spite of the unknown but constant perturbations. The integral action of the PID controller corrects the angular position deviations.

The modified controller is

\[
\begin{align*}
\dot{\theta}_d &= L \frac{d}{dt} i_d^*(t) - N_r L \omega i_q^*(t) + \frac{\omega}{i_d} \zeta_2 + R i_d^*(t) - k_p \dot{\theta}^* + k_i \epsilon + k_d \ddot{\theta} \\
\dot{\theta}_q &= L \frac{d}{dt} i_q^*(t) + N_r L \omega i_d^*(t) + K_m \zeta_1 + R i_q^*(t) + k_p \epsilon - k_i \dot{\theta} - k_d \dot{\theta} \\
\dot{\epsilon} &= \epsilon
\end{align*}
\]

(6.22)

Figure 6.2: PM stepper motor closed-loop response to passivity plus flatness based controller including perturbation torque

Figure 6.2 shows the performance of the modified passivity-based controller in the presence of constant but unknown load torque perturbations. We used \( k_{pd} = k_{pq} = 0.01 \), \( k_{ld} = k_{lq} = 60 \) and \( k_{pd} = k_{Dq} = 0.001 \). The load torque amplitude was taken to be \( 10^{-4} \) N-m.
6.2.8 A pulse width modulation implementation

The controller design, and the obtained simulation results, may be regarded as those corresponding to an average PWM model of a corresponding switched model of the PM stepper motor in which the input voltages, \( v_a \) and \( v_b \), are assumed to only take values, respectively, on the discrete sets \( \{-V_a, V_a\} \) and \( \{-V_b, V_b\} \) with \( V_a \) and \( V_b \) being constant values representing the maximum available input voltages.

Consider then the average PWM model of the PM stepper motor, obtained by simply substituting the control input voltages \( v_a \) and \( v_b \), respectively, by the expressions

\[
\begin{align*}
v_a &= \mu_a V_a; \quad v_b = \mu_b V_b \tag{6.23}
\end{align*}
\]

with \( \mu_a \) and \( \mu_b \) acting effectively as the average independent control inputs to the system, also known as duty ratios (see [8]). These control inputs are constrained to the open intervals \((-1, 1)\). \( V_a \) and \( V_b \) are positive constant values determined on the basis of specified maximum absolute values of the actual control input variables \( v_a \) and \( v_b \), respectively.

The average PWM model of the PM stepper motor is then given by

\[
\begin{align*}
\frac{di_a}{dt} &= \frac{1}{L} \left[ \mu_a V_a - R_i a + K_m \omega \sin(N_r \theta) \right] \\
\frac{di_b}{dt} &= \frac{1}{L} \left[ \mu_b V_b - R_i b - K_m \omega \cos(N_r \theta) \right] \\
\frac{d\omega}{dt} &= \frac{1}{J} \left[ -K_m i_a \sin(N_r \theta) + K_m i_b \cos(N_r \theta) - B \omega - \tau \right] \\
\frac{d\theta}{dt} &= \omega. \tag{6.24}
\end{align*}
\]

The actual switching control inputs \( v_a \) and \( v_b \) are specified according to a “PWM switching policy”, which entitles the sampling of the nonlinear system states at time instants \( t_k \), with a sampling period given by the fixed positive scalar \( T \). The pulsed control inputs \( v_d \) and \( v_q \) are then decided, at the beginning of each sampling interval, according to

\[
\begin{align*}
v_a(t) &= \begin{cases} V_a \text{sign}[\mu_a(t_k)] & \text{for } t_k < t \leq t_k + |\mu_a(t_k)| T \\ 0 & \text{for } t_k + |\mu_a(t_k)| T < t \leq t_k + T \end{cases} \\
v_b(t) &= \begin{cases} V_b \text{sign}[\mu_b(t_k)] & \text{for } t_k < t \leq t_k + |\mu_b(t_k)| T \\ 0 & \text{for } t_k + |\mu_b(t_k)| T < t \leq t_k + T \end{cases} \tag{6.25}
\end{align*}
\]

with the duty ratio values \( \mu_a(t_k) \) and \( \mu_b(t_k) \) obtained, in a feedback manner, as follows,

\[
\mu_a(t_k) = \left( \frac{v_a(t_k)}{V_a} \right) ; \quad \mu_b(t_k) = \left( \frac{v_b(t_k)}{V_b} \right) \tag{6.26}
\]
with $v_a(t_k)$ and $v_b(t_k)$ obtained by sampling the (average) feedback control laws, obtained in Section 6.3, from the controller design procedure, based on passivity and flatness. In the simulations, we used a sampling frequency of 5 KHz and complemented the dynamic feedback controller. The outer loop PID controller also managed to compensate for the constant errors arising from the finite frequency PWM sampling process.

**Figure 6.3:** PM stepper motor closed-loop response to passivity plus flatness based controller (PWM implementation)

**Figure 6.3** presents the actual switched responses of the system according to the previously described PWM control policy. In this instance we used $V_a = 7$ V. and $V_b = 5$ V. For simplicity, in these simulations, the load torque, $\tau$, was set to be zero.
6.3 The “boost” DC-to-DC power converter

Consider the “boost” converter circuit, shown in Figure 6.4. The system given by the following bilinear switched model

\[
\begin{align*}
\dot{x}_1 &= -u \frac{1}{L} x_2 + \frac{E}{L} \\
\dot{x}_2 &= u \frac{1}{C} x_1 - \frac{x_2}{RC}
\end{align*}
\]  

where \( u \in U \) is the control input, taking values on the discrete set \( U = \{0, 1\} \), \( x_1 \) is the inductor current, \( x_2 \) is the capacitor voltage, and the parameters \( R, L, C, \) and \( E, \) are known constants.

In order to simplify matters, we use, as in Zinober et al. [10], a per-unit normalized model of the above converter. This was achieved by setting the following state, and time variable, coordinates transformation, \( z_2 = x_2/E, \)

\[
z_1 = x_1/E \sqrt{C/L}, \quad \text{and} \quad \tau = t/\sqrt{LC}.
\]

This yields,

\[
\begin{align*}
\frac{d}{d\tau} z_1 &= -uz_2 + 1 \\
\frac{d}{d\tau} z_2 &= uz_1 - \frac{z_2}{Q}
\end{align*}
\]  

where \( Q \) is the circuit “quality”, given by \( Q = R \sqrt{C/L} \).
The “boost” converter is a dc voltage amplifier. This is translated into the fact that the normalized output capacitor voltage is greater than 1 under its normal “amplifying mode” operating condition. To see this, we give an heuristic argument. Suppose that we manage to hold \( z_2 \) ideally constant, at some value \( \bar{z}_2 \) (this will entitle infinite frequency switchings, of course). The average value of the control input \( u \), sustaining this condition, would be given by \( u = \mu = \bar{z}_2/(Qz_1) \). Since the actual \( u \) only takes values in the set \{0,1\}, this average value is necessarily bounded by the interval [0,1], given that neither of the extreme control values, \( u = 0 \), or \( u = 1 \), yields a prespecified constant value for the converter state variable \( z_2 \). The resulting differential equation for \( z_1 \) is given by \( \dot{z}_1 = -\bar{z}_2/(Qz_1) + 1 \), whose (unstable) equilibrium value is given by \( \bar{z}_1 = \bar{z}_2/Q \). The equilibrium value for \( \mu \) is then computed as \( \bar{\mu} = z_2/Q \bar{z}_1 = 1/\bar{z}_2 \). Since \( \bar{\mu} \in [0,1] \), \( \bar{z}_2 \) is necessarily larger than 1. An equivalent reasoning is achieved, starting with constant values of the normalized inductor current \( z_1 \).

6.3.1 Flatness of the “boost” converter

The total stored energy, given in this case by,

\[
F = \frac{1}{2} (z_1^2 + z_2^2)
\]

(6.29)

qualifies as a flat output, since all system variables can be obtained as differential functions of such an output. Indeed, derivation with respect to \( \tau \), denoted also by means of a “dot”, of the expression (6.29), yields

\[
\dot{F} = z_1 - \frac{z_2^2}{Q}
\]

(6.30)

From the set of Equations (6.27), (6.30) one can solve (uniquely up to physical considerations) for the state variables \( z_1 \) and \( z_2 \), in terms of \( F \) and \( \dot{F} \). One obtains

\[
\begin{align*}
  z_1 &= \frac{Q}{2} + \sqrt{\frac{Q^2}{4} + Q\dot{F} + 2F} \\
  z_2 &= \sqrt{-Q\dot{F} - \frac{Q^2}{2} + Q\sqrt{\frac{Q^2}{4} + Q\dot{F} + 2F}}
\end{align*}
\]

(6.31)

These equations point to the fact that the state variables are differential functions of the flat output \( F \). In the case of switched systems, the parameterization of the control input by differential functions of the flat output is to be understood only in an average sense as if representing the
duty ratio function of a PWM control scheme or an equivalent control. In this case we obtain,

$$u = \frac{Q}{(2z_1(F,F) + Q)z_2(F,F)} \left[ 1 + \frac{2}{Q^2} z_2^2(F,F) - \tilde{F} \right] \tag{6.32}$$

The differential parameterization (6.31) immediately allows for a convenient static parameterization of the state equilibria in terms of the flat output constant values. Indeed, letting $F = \tilde{F}$ be a constant, one obtains

$$\bar{x}_1 = -\frac{Q}{2} + \sqrt{\frac{Q^2}{4} + 2\tilde{F}}$$
$$\bar{x}_2 = \sqrt{-\frac{Q^2}{2} + Q\sqrt{\frac{Q^2}{4} + 2\tilde{F}}} \tag{6.33}$$

The parameterization (6.33), immediately leads to the following alternative parameterization of the normalized capacitor voltage equilibrium in terms of the normalized inductor current equilibrium,

$$\bar{x}_2 = \sqrt{\bar{x}_1 Q} \quad \rightarrow \quad \bar{x}_1 = \frac{\bar{x}_2}{Q} \tag{6.34}$$

6.3.2 Passivity properties through flatness

The system properties, especially those pertinent to passivity-based control, can be readily established from the differential parameterization (6.31) and (6.32).

Let $z_1 = \bar{x}_1$ be a constant in the first equation of (6.31). We obtain the following corresponding zero dynamics,

$$\dot{\tilde{F}} = -\frac{2}{Q} \left[ F - \frac{1}{2} (\bar{x}_1^2 + \bar{x}_1 Q) \right] = -\frac{2}{Q} \left[ F - \frac{1}{2} (\bar{x}_1^2 + \bar{x}_2^2) \right] \tag{6.35}$$

whose trajectories are exponentially asymptotically stable to the equilibrium value $1/2(\bar{x}_1^2 + \bar{x}_2^2)$. This establishes that the output $z_1$, the normalized inductor current, is a minimum phase output. Since it is also relative degree one, it is a passive output [2].

Let $z_2 = \bar{x}_2$ be a constant value. From (6.31), we obtain, after some algebraic manipulations, that the corresponding zero dynamics is given by the following implicit first-order differential equation,

$$\left( \tilde{F} + \frac{\bar{x}_2^2}{Q} \right)^2 = 2F - \bar{x}_2^2 \tag{6.36}$$
which, in the phase plane \((F, \dot{F})\), can be represented by a parabola, opening to the right, with the vertex located at the point \((0.5\bar{z}_2^2, -\bar{z}_2^2/Q)\). Incidentally, the vertex of the parabola is an impasse point since the differential equation degenerates into an algebraic equation. The equilibrium point for this differential equation is given by

\[
\bar{F} = \frac{1}{2} \left( \left( \frac{\bar{z}_2^2}{Q} \right)^2 + \bar{z}_2^2 \right) = \frac{1}{2} \left[ \bar{z}_1^2 + \bar{z}_2^2 \right]
\]  

A phase diagram of Equation (6.36) readily reveals that this equilibrium point is unstable. The normalized output capacitor voltage \(z_2\) is thus a non-minimum phase output.

The constant input equilibrium state detectability (see Byrnes et al. [2] for definitions and general results) of the output \(z_1\) also readily follows from the differential parameterization (6.31) and (6.32). This last fact implies that the system is also stabilizable by means of output feedback (see [2]).

6.3.3 A passivity-based sliding mode controller

The “energy shaping plus damping injection” controller design (see the recent book by Ortega et al. [7]) is based on the creation of a linear in-the-state, time-varying, “copy” of the plant, sharing the same control input as the given system. This reference model of the plant is provided with appropriate supplementary damping enhancing the corresponding “dissipation structure”. The virtual, or auxiliary, model of the system shares all the important properties of the original plant (such as passivity of the corresponding outputs and flatness over a larger ground field) has the property of “pulling” the systems state trajectories towards the desired prespecified trajectories. In our “boost” converter case, such an auxiliary model is given by

\[
\xi_1 = -ue_2 + 1 + Q_c(z_1 - \xi_1) \\
\xi_2 = ue_1 - \frac{\xi_2}{Q}
\]  

where \(Q_c\) is the added damping we impose on the auxiliary inductor current dynamics.

Notice that the reference model “tracking error” state \(e_1 = z_1 - \xi_1\), \(e_2 = z_2 - \xi_2\), satisfies the following controlled dynamics

\[
\dot{e}_1 = -ue_2 - Q_c e_1 \\
\dot{e}_2 = ue_1 - \frac{1}{Q} e_2
\]  

(6.39)
Along the trajectories of the system (6.39), the time derivative of the modified energy function \( V(e) = \frac{1}{2}(e_1^2 + e_2^2) \) satisfies
\[
\dot{V}(e) = -Q_c e_1^2 - \frac{1}{Q} e_2^2 \leq \alpha V(e) \tag{6.40}
\]
with \( \alpha = 2 \min\{Q_c, 1/Q\} \).

Thus, as stated, the tracking error state trajectories asymptotically exponentially converge to zero, independently of the control input. The system state trajectories asymptotically track the auxiliary system controlled trajectories. Thus, regulating the auxiliary system along a given desired trajectory results, in turn, in an effective regulation of the original plant. Notice that the auxiliary system initial states are entirely at our disposal.

Let \( z^*_1(\tau) \) be a desired trajectory for the auxiliary variable \( \xi_1 \). A sliding mode controller that forces the auxiliary state \( \xi_1 \) to track the specified trajectory is given by
\[
u = \frac{1}{2} (1 + \text{sign} (\xi_1 - z^*_1(\tau))) \tag{6.41}
\]

The existence of a sliding mode on the time-varying sliding surface
\[
S_0 = \{(\xi_1, \xi_2) \mid \sigma = \xi_1 - z^*_1(\tau) = 0\} \tag{6.42}
\]
is assessed in the following manner.

Suppose that the initial the auxiliary state \( \xi_1 \), is set to coincide with the plant’s state \( z_1 \). Thus, due to linearity of the model reference tracking error, the error state \( e = z - \xi \) remains constrained to zero. If such is not the case, this error, as it was already shown, exponentially decreases to zero. Suppose now that the quantity \( \xi_1 - z^*_1(\tau) \) is initially negative. According to (6.41), the control input \( u \) is initially set to zero, and it will remain clamped at this value until \( \xi_1 \) reaches the desired trajectory \( z^*_1(\tau) \). The time derivative of the “sliding surface” coordinate, \( \sigma = \xi_1 - z^*_1(\tau) \), throughout this phase is given by
\[
\dot{\sigma} = 1 + Q_c (z_1 - \xi_1) - \dot{z}^*_1(\tau) \tag{6.43}
\]
where the second term is either identically zero, or exponentially approaching zero, as already described. In order to reach the time-varying sliding surface, from below, i.e., \( \xi_1 < z^*_1(\tau) \), the time derivative of \( \sigma \) needs to be positive. This is achieved as long as the initial value of \( \xi_1 \) is chosen close to, or equal, that of \( z_1 \), and the desired trajectory \( z^*_1(\tau) \) has a normalized time derivative which is absolutely bounded above by 1, i.e., \( |\dot{z}^*_1(\tau)| < 1 \). Thus, we should specify the desired trajectory \( z^*_1(\tau) \) and initialize the auxiliary system state \( \xi_1 \) with these conditions in mind.
If \( \xi_1 \) is at certain time \( \tau_1 \) above the value of \( z_1^*(\tau) \), \( \sigma \) is positive. Then, the control input \( u \), according to (6.41), is set to adopt the value 1 and the time derivative of the sliding surface coordinate \( \sigma = \xi_1 - z_1^* \) is given by

\[
\dot{\sigma} = -\xi_2 + Qc(z_1 - \xi_1) - z_1^*(\tau) \quad (6.44)
\]

Notice that if we are already above the sliding surface, \( \xi_2 \) is necessarily larger than 1, due to the amplifying feature of the converter, and the tracking error \( (z_1 - \xi_1) \) may be considered to be negligible or already zero. This means, under the same previous assumption regarding the absolute value of the time derivative of the proposed trajectory, \( |\dot{z}_1^*(\tau)| < 1 \), that \( \dot{\sigma} \) will be locally negative and the sliding surface \( S_0 \) is guaranteed to be reached from above.

The sliding mode controller (6.41) achieves the convergence of \( \xi_1 \) towards the desired trajectory \( z_1^*(\tau) \). The equivalent control, defined as the virtual control action, \( u_{eq}(\tau) \), responsible for ideally maintaining the evolution of the sliding surface coordinate \( \sigma \) at the value zero, and is obtained from the invariance condition \( \dot{\sigma} = 0 \), which is evaluated at the ideal sliding condition \( \sigma = 0 \),

\[
u_{eq} = \frac{-\dot{z}_1^*(\tau) + 1 + Qc(z_1 - z_1^*(\tau))}{\xi_2} \quad (6.45)
\]

The ideal sliding dynamics, or remaining dynamics, is obtained by substituting the equivalent control expression \( (6.45) \) into the auxiliary capacitor voltage dynamic (6.38)

\[
\dot{\xi}_2 = \left( \frac{-\dot{z}_1^*(\tau) + 1 + Qc(z_1 - z_1^*(\tau))}{\xi_2} \right) z_1^*(\tau) - \frac{\xi_2}{Q} \quad (6.46)
\]

Since, \( z_1 \) also converges towards \( z_1^*(\tau) \), due to the fact that \( \xi_1 \) is forced to follow \( z_1^*(\tau) \) in finite time and, as we have seen, \( z_1 \) exponentially asymptotically converges towards \( \xi_1 \), the equivalent control and the ideal sliding dynamic asymptotically converge towards values given by the following expressions

\[
u_{eq} = \frac{1 - \dot{z}_1^*(\tau)}{\xi_2} \\
\dot{\xi}_2 = \left( \frac{1 - \dot{z}_1^*(\tau)}{\xi_2} \right) z_1^*(\tau) - \frac{\xi_2}{Q} \quad (6.47)
\]

Equations (6.45) and (6.46) can be regarded as a dynamic feedback equivalent controller, which only requires the feedback of the passive output \( z_1 \) from the plant.
6.3.4 Non-minimum phase output stabilization

The control objective is to perform an equilibrium-to-equilibrium transfer for the output capacitor voltage, $z_2$, of the converter. As will be shown, due to the non-minimum phase character of the output voltage, the corresponding regulation problem is not directly feasible. However, by embedding this problem into a corresponding equilibrium transfer for the flat output, the underlying internal stability problem is easily circumvented.

6.3.5 Trajectory planning

Suppose we specify a trajectory $z_1^*(\tau)$ for the auxiliary, minimum phase output variable $\xi_1$, in full accordance with the desired equilibrium to equilibrium transfer for the plant capacitor voltage. In other words, we assume that the capacitor voltage initial equilibrium, $\bar{z}_2$, is to be transferred, over a time period $\Delta T = \tau_f - \tau_0 > 0$, towards the final equilibrium value $z_2^F$. Due to the non-minimum phase character of $z_2$, this stabilization task needs to be reformulated in terms of a transfer defined on the corresponding equilibria for the minimum phase variable $z_1$. This is achieved by specifying a suitable trajectory $z_1^*(\tau)$ for the auxiliary variable $\xi_1$. If the auxiliary system state $\xi_1$ is forced to track the trajectory $z_1^*(\tau)$, as it has been previously demonstrated, the plant state trajectory $z_1(\tau)$ will follow suit. Thus, we specify

$$z_1^*(\tau) = \bar{z}_{10} + (\bar{z}_{1F} - \bar{z}_{10}) \psi(\tau, \tau_0, \tau_f) \quad (6.48)$$

with $\psi(\tau, \tau_0, \tau_f)$ being a time polynomial smoothly interpolating between 0 and 1, satisfying

$$\psi(\tau_0, \tau_0, \tau_f) = 0 \ ; \ \psi(\tau_f, \tau_0, \tau_f) = 1$$

and use this specification in the sliding mode controller expression (6.41).

This scheme yields desirable results and good quality of responses, for any suitably defined reference trajectory $z_1^*(\tau)$. However, we claim that the most natural choice for specifying this trajectory is to do so by resorting to the flatness property present in the original plant. Instead of directly specifying the trajectory for the auxiliary variable $\xi_1$ in terms of $z_1^*(\tau)$, we specify $z_1^*(t)$ through its relation with the flat output, as given by (6.33)

$$z_1^*(\tau) = -\frac{Q}{2} + \sqrt{\frac{Q^2}{4} + QF^*(\tau) + 2F^*(\tau)} \quad (6.49)$$

The described choice has a justification in terms of the simplicity of off-line trajectory planning tasks, when made in terms of the flat output,
and the induced response on the non-minimum phase normalized capacitor voltage variable $z_2$. The specification of a flat output reference trajectory enjoys a direct differentially parameterized relationship with the corresponding induced trajectory for $z_2$, as evidenced by (6.31). This, in a sense, is a static relationship, devoid of any dynamics, in which all features of the corresponding desired time response of $z_2$ can be assessed, predicted, and corrected without solving differential equations. On the other hand, the specification of $z_1$ undergoes an implicit dynamic relationship with the corresponding response trajectory of $z_2$, which needs to invoke not only the feedback expression for the equivalent control input but it also requires the solution of a differential equation. As a result, pre-specifying the flat output is not only more efficient from the design viewpoint but it also allows for simple off-line experimentation and evaluation that results in a better quality of response for the capacitor voltage variable. In fact, the typical “undershoot” response of the non-minimum phase variable $z_2$ can be effectively avoided by a reasonable off-line designed specification of the flat output trajectory. It is not clear how to achieve the same goal with a direct choice for $z_1(t)$.

6.3.6 Simulation results

Figure 6.5 shows the closed-loop responses of the state and input variables for a desired capacitor voltage equilibrium transfer on a typical dc-to-dc power converter of the “boost” type, with parameter values given by

\[ L = 0.02 \, \text{H}, \quad C = 1 \, \mu\text{F}, \quad R = 200 \, \Omega, \quad E = 15 \, \text{V} \]

A sliding mode passivity based controller was designed to increase the output capacitor voltage from the initial equilibrium value of 30 V, towards a final desired value of 60 V. The corresponding equilibrium values for the inductor current are 0.3 A, and 1.2 A, respectively. The flat output must be transferred from the initial value of 0.00135 towards the final value of 0.0162. In terms of the normalized state variables, the digital simulations are carried out more efficiently with corresponding simulation values.

\[ Q = \sqrt{2}, \quad z_{10} = 2\sqrt{2}, \quad z_{1F} = 8\sqrt{2}, \quad z_{20} = 2, \quad z_{2F} = 4, \]

\[ \tau = \frac{\sqrt{2}}{2} \times 10^4 \, t \]

The corresponding trajectory for the normalized flat output was designed using the expression (6.48) with the following Bézier polynomial,
smoothly interpolating between 0 and 1,

\[ \psi(\tau, \tau_0, \tau_f) = \left( \frac{\Delta \tau}{\Delta T} \right)^5 \left[ 21 - 35 \left( \frac{\Delta \tau}{\Delta T} \right) + 15 \left( \frac{\Delta \tau}{\Delta T} \right)^2 \right] \]

where \( \Delta T = \tau_f - \tau_0 \), and \( \Delta \tau = \tau - \tau_0 \) where \( \tau_0 = 200 \) and \( \tau_f = 1200 \), dimensionless units, which correspond, after the change of time scale, to \( t_1 = 0.028 \) s and \( t_2 = 0.144 \) s.

6.3.7 Dc-to-ac power conversion

A discontinuous feedback control law for \( u \) was desired, such that the normalized capacitor voltage, \( z_2 \), tracks a given desired voltage signal \( z_2^*(\tau) \), which never becomes constant. This signal was assumed to be bounded and sufficiently differentiable. In fact, we assumed that \( z_2^*(\tau) \) was smooth, i.e., infinitely differentiable. Specifically, we were interested in generating a
normalized output voltage of the form $z_2(t) = A + (B/2)\sin \omega t$ with $A > 0$ and $\omega > 0$ and $B$ being a constant of arbitrary sign.

The non-minimum phase properties of the output capacitor voltage, joined to the “control acquisition” structure of the converter equations and the discrete-valued nature of the control input $u$, made it especially difficult for the synthesis of a switching feedback control law that results in a stable ac capacitor voltage reference signal tracking scheme. It should be clear that the main task to be solved was to obtain a procedure by which an inductor current reference signal was approximately, or exactly, computed whose corresponding “remaining dynamics” trajectories are either given by the desired output capacitor voltage or by some reasonable approximation to it.

In order to obtain a suitable reference trajectory $z_1^r(\tau)$ for $z_1$, given that $z_2$ is of a particular form $z_2^r(\tau)$, one should proceed to eliminate the flat output $F^*$ from the set of relations

$$ F^* = \frac{1}{2} \left[ (z_1^r(\tau))^2 + (z_2^r(\tau))^2 \right] ; \quad \dot{F}^* = z_1^r(\tau) - \frac{(z_2^r(\tau))^2}{Q} \quad (6.50) $$

Such an elimination yields a differential relation between $z_1^r(\tau)$ and $z_2^r(\tau)$, i.e., one which, necessarily, involves an infinite number of time derivatives of $z_2^r(\tau)$. We will exploit this elimination idea in order to generate an approximating sequence of static differential algebraic relationships yielding the normalized input inductor current reference signal $z_1^r$, exclusively in terms of the output capacitor voltage reference trajectory $z_2^r$ and a finite number of its time derivatives. This finite differential parameterization of $z_1$ in terms of $z_2$ will, of course, allow for the indirect sliding mode generation of a large class of bounded ac output capacitor voltage profiles, which are sufficiently differentiable.

### 6.3.8 An iterative procedure for generating a suitable inductor current reference

In order to simplify the notation we will temporarily suppress the asterisks and the time argument in the developments of this subsection. Consider then the set of relations (6.50). Those relations can be alternatively viewed in the following manner:

$$ z_1 = \frac{z_2^2}{Q} + \dot{F} $$

$$ F = \frac{1}{2} (z_1^2 + z_2^2) \quad (6.51) $$
Evidently, one may “embed” the set of relations (6.51) as the outcome of a convergent iterative procedure, aimed at eliminating $F$, where the value of $z_1 = z_{1,\infty}$ was computed exclusively in terms of a given fixed function $z_\infty$ and, possibly, an infinite number of its time derivatives. In other words $z_1$, viewed as the outcome of such an iterative procedure, could be represented, after convergence, by

$$z_{1,\infty} = \frac{z_2^2}{Q} + F_\infty$$

(6.52)

$$F_\infty = \frac{1}{2} \left( z_{1,\infty}^2 + z_2^2 \right)$$

(6.53)

Equations (6.52) and (6.53) immediately suggest the consideration of the following iterative procedure,

$$z_{1,k} = \frac{z_2^2}{Q} + F_k$$

(6.54)

This algorithm sequentially yields an approximation of a static relationship between $z_1$ and $z_2$, which only involves polynomial expression of $z_2$ and of its time derivatives. The algorithm of course should be “initialized” by an arbitrary but reasonable trajectory $F_0(T)$ for the flat output $F$.

Starting from the equilibrium condition, $F_0(T) = constant$, one obtains the following sequence of approximating expressions for the normalized inductor current reference trajectory $z_1$,

$$z_{1,0} = \frac{z_2^2}{Q} \implies F_1 = \frac{1}{2} \frac{z_3^4}{Q^2} + \frac{1}{2} z_2^2$$

(6.55)

$$z_{1,1} = \frac{z_2^2}{Q} + z_2 \dot{z}_2 \left( 1 + \frac{2}{Q^2} z_2^2 \right)$$

$$\implies F_2 = \frac{1}{2} \left[ \frac{z_2^2}{Q} + z_2 \dot{z}_2 \left( 1 + \frac{2}{Q^2} z_2^2 \right) \right]^2 + \frac{1}{2} z_2^2$$

(6.56)

$$z_{1,2} = \frac{z_2^2}{Q} + \left( \frac{z_2^2}{Q} + z_2 \dot{z}_2 + \frac{2 z_3^3}{Q^2} \dot{z}_2 \right) \times$$

$$\left( \frac{2}{Q} z_2 \ddot{z}_2 + (\dot{z}_2)^2 + z_2 \ddot{z}_2 + \frac{6 z_3^3}{Q^2} (\dot{z}_2)^2 + \frac{2 z_3^3}{Q^2} \ddot{z}_2 \right) + z_2 \ddot{z}_2$$

(6.57)

$$\vdots$$

$$z_{1,\infty} = \psi(z_2, \dot{z}_2, z_2^{(2)}, \ldots, z_2^{(k)}, \ldots)$$
6.3.9 Simulation results

The generated normalized inductor current reference signals (6.55)-(6.57) were used in a passivity-based sliding mode control scheme, carried out in the same manner for the stabilization problem described in Section 6.2.

A typical “boost” converter was chosen, with circuit parameters \( L = 20 \) mH, \( C = 1 \) \( \mu \)F, \( R = 50 \) \( \Omega \), and \( E = 15 \) V. For the normalized “boost” converter dynamics, the dimensionless circuit quality turns out to be \( Q = 0.3535 \). We took the following candidates as sliding surfaces \( \sigma_k = z_1 - z_{1,k}(\tau) \) for \( k = 0, 1 \), with \( z_{1,0}(\tau) \) and \( z_{1,1}(\tau) \) as given by (6.55)-(6.56), respectively. As a desired output capacitor voltage signal, we chose \( z_2^*(\tau) = A + B/2 \sin\omega\tau \). The constants \( A > 0 \), \( B \) and \( \omega \) were set so that the sliding mode existence conditions were satisfied. The parameters of the desired normalized sinusoidal voltage reference signal, \( A + (B/2) \sin(\omega\tau) \), were set to be \( A = 1.5 \), \( B = 0.8 \), \( \omega = 0.02 \), i.e.,

\[
z_2^*(\tau) = 1.5 + 0.4\sin(0.02\tau)
\]
which corresponds with an denormalized sinusoidal voltage of the form

\[ x_2^*(t) = 22.5 + 6 \sin(\sqrt{2} \times 10^2 t) \text{ V} \]

The corresponding time basis for the adopted normalization was \( t_b = 0.1414 \text{ ms} \). The underlying sampling process for the simulation used normalized sampling periods of 0.1 time units, which corresponded to an actual sampling frequency of about 70.71 KHz.

Figure 6.6 shows the closed-loop output voltage response of the proposed sliding mode tracking controller for a sliding surface candidate of the form \( \sigma_k = z_1 - z_1^* k(\tau) \), with \( k = 0 \). The simulated output voltage response is shown, for comparison purposes, along with the desired output capacitor voltage signal \( z^*_c(\tau) \). These signals can hardly be distinguished from each other. Figure 6.6 also shows the trajectory of the off-line generated inductor current signal, along with the actual inductor current response. The equivalent control trajectory, also shown along with the denormalized voltage response, are bounded, after sliding starts, by the closed interval \([0, 1]\). The simulations corresponding to the sliding surface candidate obtained for \( k = 1 \), depicted in Figure 6.7, show that the agreement with the desired trajectories, obtained for \( k = 0 \), were not substantially improved.

![Figure 6.7: Closed-loop response for output capacitor voltage using \( z^*_1 \), as the normalized inductor current reference trajectory (normalized \( \omega = 0.02 \))](image-url)
6.4 Conclusions

In this chapter we proposed a flatness-based approach for the passivity-based stabilization and trajectory tracking of two switched electrical systems. One was weakly minimum phase while the other was a non-minimum phase system. The approach used a suitable combination of flatness-based trajectory tracking, passivity, and sliding mode control. The more complex problem of minimum phase output signal reference tracking, not leading to equilibrium, but sustaining a desired oscillatory behavior, required a new approach based on consideration, as reference trajectories candidates, those emerging from a sequence of finite-order differential parameterizations of the minimum-phase output in terms of the non-minimum phase output.

References


