Nonlinear Control of a Permanent Magnet Synchronous Motor with Disturbance Torque Estimation

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Abstract—This paper introduces a sensorless nonlinear control scheme for controlling the speed of a permanent magnet synchronous motor (PMSM) driving an unknown load torque. The states of the motor and disturbance torque are estimated via an extended nonlinear observer avoiding the use of mechanical sensors. The control strategy is an exact feedback linearization law, with trajectory tracking evaluated on estimated values of the PMSM states and the disturbance torque. The system performance is evaluated by simulations.

I. INTRODUCTION

Electrical drives using permanent magnet synchronous motors (PMSM’s) are often used in many applications. The performance of adjustable speed drives containing PMSM’s can be improved implementing nonlinear control strategies. Among others, feedback linearization has emerged as a very useful control law for electrical drives [1], [2]. It consists in exactly linearizing the PMSM by feedback and transformation, so that well-known linear control strategies can be used on the whole state space. The implementation of feedback linearization, as well as the other strategies, requires an optical/mechanical sensor to obtain position and speed as part of the state to be fed back. However, mechanical sensors can be avoided when sensorless control strategies are designed. In such cases rotor position and speed must be estimated and these estimated values used to compute the control law.

State observers can be used to estimate the rotor position and speed of PMSM’s. Several approaches to obtain PMSM state observers have been proposed, nonlinear full order observers based on linearization, extended Kalman filter (EKF), viz. nonlinear observers, nonlinear reduced order observers (see [3] and references therein). In [4], [5] and [6] observer-based speed controllers have been proposed. In these papers certain assumptions have to be introduced to design the observer-based controller. In [5], a known load torque has been considered, while in [6] the value of inductance is assumed to be zero to design the controller. In [4], the authors assume that machine speed is approximately constant during a short time interval. Nevertheless, when higher performance is required the mismatches caused by an unknown load torque, a nonzero inductance and variable speed have to be compensated. In this paper we use the extended nonlinear observer presented in [7] to estimate the PMSM state vector and the disturbance torque. Then, we construct a sensorless speed control strategy capable of tracking speed references when the PMSM drives unknown load torques.

The paper is organized as follows. In section II the equations describing the PMSM model are reviewed. In section III feedback linearization applied to PMSM is obtained. The nonlinear extended observer used to estimate PMSM’s states and unknown load torque is briefly described in section IV. The proposed sensorless nonlinear speed control is introduced in section V. System performance is evaluated in section VI. Finally, conclusions are drawn in section VII.

II. PMSM MODEL

The PMSM model is described in stationary two-axes reference frame by the following equations:

\[
\frac{d\theta}{dt} = \omega
\]

\[
\frac{d\omega}{dt} = \frac{T_d}{J} - \frac{B}{J} \omega - \frac{T_l}{J}
\]

\[
\frac{di_x}{dt} = -\frac{R}{L} i_x + \frac{\phi}{L} \omega \sin \theta + \frac{v_{cx}}{L}
\]

\[
\frac{di_\beta}{dt} = -\frac{R}{L} i_\beta - \frac{\phi}{L} \omega \cos \theta + \frac{v_{\beta}}{L}
\]

with

\[
T_d = \frac{3}{2} \phi (i_x \sin \theta + i_\beta \cos \theta)
\]

where \(i_x\), \(i_\beta\), \(v_{cx}\), \(v_{\beta}\), \(R\), \(L\), and \(\phi\) are currents and voltages in the stationary two-axes reference frame, resistance and inductance and permanent magnet flux linkage, respectively; and \(\theta\), \(\omega\), \(B\), \(J\), and \(T_d\) are rotor position, rotor speed, viscosity, inertia and load torque, respectively. The model current and voltages...
are related to the actual physical quantities by a simple linear transformation given by:

\[ i_\alpha = \frac{2}{3} \left( i_a - \frac{i_b + i_c}{2} \right) \]  
\[ i_\beta = \frac{i_b - i_c}{\sqrt{3}} \]  
\[ v_\alpha = \frac{2}{3} \left( v_a - \frac{v_b + v_c}{2} \right) \]  
\[ v_\beta = \frac{v_b - v_c}{\sqrt{3}} \]  

III. CONTROL LAW

The aim of feedback linearization is to transform the nonlinear system into a linear one using possibly a transformation and a nonlinear feedback law [9]. Consider the PMSM described by (1)–(5); an adequate coordinate transformation and a nonlinear feedback law can be found to obtain a linear description of the PMSM. In [10] a proof is given using geometric control theory. We present the PMSM feedback linearization in a different way, more oriented to the reader familiar with vector control of AC drives.

Voltages and currents in the stationary reference frame are projected onto those in a frame that is synchronous with the rotor, via the transformation:

\[ v_d = \frac{1}{L} v_d - \frac{R}{L} i_d + \omega i_q \]  
\[ v_q = \frac{1}{L} v_q - \frac{R}{L} i_q - \omega i_d - \frac{\phi}{L} \omega \]  

Replacing (16) and (17) into (14) and (15) the following description is obtained:

\[ \frac{d\theta}{dt} = \omega \]  
\[ \frac{d\omega}{dt} = \frac{3}{2} \frac{\phi}{J} i_q - \frac{B}{J} \omega - \frac{T_d}{J} \]  

IV. EXTENDED NONLINEAR OBSERVER

An extended nonlinear observer is used to estimate the PMSM states and the unknown load torque. The observer copies the PMSM model adding a correction term that works as a driving input. A new state \( i_d \) is included to track a slowly varying load torque [8]. In this case

\[ \frac{dT_d}{dt} \approx 0 \]  

We assume that the approximation

\[ \frac{dT_d}{dt} = 0 \]  

models the unknown load torque in a satisfactory way. The torque due to viscosity is incorporated to the disturbance torque \( T_d \), so the \( B_\omega \) term does not appear explicitly in the observer and the controller.

The derivation of the extended nonlinear observer can briefly be described as follows. First, a description in new coordinates is obtained [3] by means of a nonlinear transformation. Then,
in order to satisfy convergence conditions, the correction term and the adaptation laws are derived using Lyapunov’s theory [7]. The extended observer is given by:

\[
\begin{bmatrix}
\frac{d\hat{\theta}}{dt} \\
\frac{d\hat{\omega}}{dt} \\
\frac{d\hat{\alpha}}{dt} \\
\frac{d\hat{\beta}}{dt}
\end{bmatrix}
= \begin{bmatrix}
\frac{3}{2} \frac{\phi}{J} \left( -\hat{\gamma}_\alpha \sin \hat{\theta} + \hat{\gamma}_\beta \cos \hat{\theta} \right) - \frac{\hat{T}_l}{J} \\
\frac{\hat{P}}{L} \hat{\alpha} + \hat{\gamma}_\beta \sin \hat{\theta} + \frac{\omega}{L} \\
\frac{-\hat{P}}{L} \hat{\beta} - \hat{\gamma}_\alpha \cos \hat{\theta} + \frac{\omega}{L} \\
0
\end{bmatrix}
+ \Gamma \begin{bmatrix}
\hat{\alpha} \\
\hat{\beta}
\end{bmatrix}
\]  
(23)

with

\[
\Gamma = \begin{bmatrix}
\frac{L}{\phi} \cos \hat{\theta} & \frac{L}{\phi} \sin \hat{\theta} & 0 & 0 \\
\frac{L}{\phi} \sin \hat{\theta} & \frac{-L}{\phi} \cos \hat{\theta} & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
G = \begin{bmatrix}
g_{11} & g_{12} \\
g_{21} & g_{22} \\
g_{31} & g_{32} \\
g_{41} & g_{42}
\end{bmatrix}
\]

where \( \hat{\theta}, \hat{\omega}, \hat{\alpha}, \hat{\beta} \) are the estimated states and \( \hat{T}_l \) is the estimated load torque. In the observer driving gain \( \Gamma \), \( G \) is a constant matrix and \( k_1 \) and \( k_2 \) are constant values. The output equation is given by

\[
y = [\hat{\alpha} \hspace{1cm} \hat{\beta}]^T
\]  
(24)

V. SENSORLESS SPEED CONTROL

Two nested loops are used to implement the speed control. The inner loop provides feedback linearization and the outer loop generates a linear control law to track a speed reference \( \omega_{ref} \). Indeed, the command signal \( u_q \) is calculated so that the following equation is satisfied

\[
\frac{d^2 \varepsilon}{dt^2} + \gamma_1 \frac{d\varepsilon}{dt} + \gamma_2 \varepsilon = 0
\]  
(25)

where \( \varepsilon = \omega_{ref} - \omega \). The constants \( \gamma_2 \) and \( \gamma_2 \) are chosen so that the roots of the polynomial \( P(s) = s^2 + \gamma_1 s + \gamma_2 \) have negative real part. In such a case, the error \( \varepsilon \) tends exponentially to zero with a speed convergence that depends on \( \gamma_1 \) and \( \gamma_2 \). Differentiating (19) (with the term \( B \omega = 0 \), since it is included in the load torque) and using (20) the following equation is obtained

\[
\frac{d^2 \omega}{dt^2} = \frac{3}{2} \frac{\phi}{J} u_q - \frac{d\phi}{dt}
\]  
(26)

The auxiliary control law \( u_q \) is obtained replacing (26) and (19) into (25). Obviously, this law could be constructed if the variables \( \theta, \omega, \hat{\alpha}, \hat{\beta}, \hat{T}_d \) and \( \hat{T}_l \) were known. The proposed speed controller assumes that there are no mechanical sensors for \( \theta \) and \( \omega \) so the values estimated by the observer (23) replace the true ones in the control law. Then, the auxiliary control \( u_q \) becomes:

\[
u_q = \frac{2J}{3\phi} \left[ \gamma_1 \left( -\frac{3}{2} \hat{\gamma}_\alpha \hat{\theta} + \hat{\gamma}_\beta \hat{\beta} \right) + \frac{\phi}{J} \right] - \gamma_2 \hat{\omega} + \frac{\partial \omega_{ref}}{dt} + \gamma_1 \frac{\partial \omega_{ref}}{dt} + \gamma_2 \omega_{ref} \]

(27)

where

\[
\hat{\lambda}_q = -\hat{\gamma}_\alpha \sin \hat{\theta} + \hat{\gamma}_\beta \cos \hat{\theta}
\]

The auxiliary control \( u_d \) is given by

\[
u_d = -K_i \hat{\lambda}_d + \hat{\lambda}_{ref} \]

(28)

with

\[
\hat{\lambda}_d = \hat{\gamma}_\alpha \cos \hat{\theta} + \hat{\gamma}_\beta \sin \hat{\theta}
\]

The innermost loop achieves feedback linearization, from \( u_q \) to \( (\theta, \omega) \), when the stator voltages \([v_{\alpha}, v_{\beta}]^T\) are calculated from (10), (11), (16) and (17). Replacing the state variables by estimated values, the stator voltages \( (v_{\alpha}, v_{\beta}) \) become:

\[
v_{\alpha} = L(u_d \cos \hat{\theta} - u_q \sin \hat{\theta}) + R\hat{\alpha} - L\hat{\omega}_\beta - \phi \hat{\omega} \sin \hat{\theta}
\]

(29)

\[
v_{\beta} = L(u_d \sin \hat{\theta} + u_q \cos \hat{\theta}) + R\hat{\beta} + L\hat{\omega}_\alpha + \phi \hat{\omega} \cos \hat{\theta}
\]

(30)

The implementation of the proposed sensorless speed control requires the real time integration of five nonlinear differential equations for the observer (23). The non linearities consist of sinusoidal functions and products of variables, but from the computation complexity, the product of two variables is the same as the product of a constant and a variable. The control itself requires four algebraic equations; speed loop (27), flux loop (28), and feedback linearization (29) (30). No mechanical sensors are needed, and so the additional hardware and software associated to them are avoided. On the other hand a vector control of a PMSM requires at least the real time integration of three differential equations for the speed and current control loops. The current synchronization is obtained from optical encoders, and a dedicated hardware or additional software is needed to obtain the speed feedback from the encoder. So we conclude that considering a digital control system, the proposed control eliminates every external hardware at the expense of some more computational complexity which is easy to obtain with nowadays microcontrollers of DSP systems.

VI. PERFORMANCE EVALUATION

The proposed observer-based controller with disturbance torque estimation, was designed for a 1 kW drive, as shown in Fig. 1. The performance was verified by means of simulations. Parameter values of the PMSM, the observer and the controller are given in the appendix. The speed reference \( \omega_{ref} \) was chosen to be piecewise polynomial, glued together to satisfy
up to second derivative continuity. The maximum $\dot{\omega}_{\text{ref}}$ slope is designed to be near the maximum acceleration of the drive, to make full use of the actuator and tending to avoid its saturation. The final values of reference speed, were chosen for three different ranges, nominal speed (150 rad/sec), medium speed (50 rad/sec) and low speed (1.5 rad/sec). Figs. 2 and 3 illustrate the transient response when the motor is started for different values of final reference speed. The initial estimated speed was set at 0.1 rad/sec, while the estimated position was set to 0 rad and the actual rotor position was varied between $\pm 0.8$ rad. In Fig. 2 the actual and reference speeds are almost undistinguishable, showing the very good performance of the drive in the whole speed range regardless of the observer’s initial conditions. It is seen that the observer estimates the viscous load torque, $B\dot{\omega}$, as a disturbance torque, following the acceleration process. In Fig. 3 the curves describing the amplitude of the stator current space vector, when the system runs under the above conditions are presented. It is clearly seen that its value remains well below the its nominal value (30 A). In a second test, we apply a variable load torque as shown in Fig. 4, when the motor is running at constant speed. Fig. 5 shows the results of this test through the speed error $(\dot{\omega}_{\text{ref}} - \dot{\omega})$. In Fig. 5 it can be seen that the speed error is very small and almost independent of the running speed. Its maximum value coincides with the maximum slope of the load torque, which is in full agreement with the assumption that the observer tracks constant torques. It can be shown that the frequency of the ripple on the speed error corresponds to twice the rotor speed. Figs. 6 and 7 show the result of simultaneous variation of load torque (with profile shown in Fig. 4) and reference speed, when the PMSM is operating at 100% and 1% of nominal speed, respectively. A very good performance of the drive is also observed in this case. Fig. 8 reports the effectiveness of the extended observer against uncertainties in the inertia coefficient of the motor. We show the transient response starting the motor with the final
reference speed set to its nominal value. The rotor inertia ($J$) is supposed varying between $-50\%$ to $+100\%$ of its nominal value. There is a speed error during the acceleration of the motor, then the transient response then converges to zero, as expected. The results of these tests show that, a slowly varying disturbance torque, as well as uncertainties in the mechanical parameters, are well estimated and compensated with the proposed control law. On the other hand, the electrical parameters are not estimated by the proposed observer, but they are better known since they are obtained from the motor data, and/or the normal tests. Fig. 9 reports the test of sensitivity of the drive against uncertainties in the electrical parameters. Start up of the motor to nominal speed is reproduced assuming differences in the stator resistance ($\Delta R$) in Fig. 9a), the stator inductance ($\Delta L$) in Fig. 9b), and the permanent magnet flux linkage ($\Delta \phi$) in Fig. 9c). From this figure, it can be pointed out that: 1) variations of $R$ and $\phi$ give rise to a steady state speed error, 2) variations of $L$ only modifies the transient behavior, 3) big uncertainties of $R$ or $L$ may compromise the stability of the whole system specially for negative variations of them. There exists a trade-off in the choice of the observer gains since a decrease of these gains will improve the sensitivity of the system to variations of the electrical parameters, but will decrease the speed of convergence slowing down the whole system. If the uncertainties in the electrical parameters and the required speed of the system cannot satisfy this trade off, further improvements in the parameter knowledge can be obtained implementing a self commissioning stage. If this is not sufficient, the parameters can be estimated “on-line” in a similar way as was done with the disturbance torque. This will require more calculus power from the controller.

VII. CONCLUSIONS

A sensorless nonlinear control strategy was introduced for controlling the speed of a PMSM motor driving an unknown torque. Exact linearization was proposed as the control law in the innermost loop; while a linear trajectory tracking strategy was proposed for the outer loop. The observer introduced in [7] was used to estimate the variables to be fed back to both the linear and nonlinear controllers. The proposed control scheme was tested by simulations, presenting a very satisfactory performance in the whole speed range, with slowly varying load torque and uncertainties in the mechanical parameters. The dynamics of the scheme can become unstable because of electrical parameters variations, but a judicious selection of the nominal values reduces the importance of this problem. Our control scheme also delivers an estimate of motor speed that becomes inaccurate in
the presence of electrical parameters variations. These inaccuracies and the overall performance could still be improved using for instance, a higher order extended nonlinear observer.

APPENDIX

The data and parameters of the devices considered in the paper are:

PMSM: \( T_N = 7 \, \text{Nm}; \, \Omega_N = 1500 \, \text{rpm}; \) Number of pole pairs: 1; \( L = 20.5 \, \text{mH}; \, R = 1.5 \, \Omega; \, \phi = 0.22 \, \text{Nm} / \text{A}; \, J = 2.2 \times 10^{-5} \, \text{Kgm}^2; \, B = 2.2 \times 10^{-2} \, \text{Nm} / \text{s} / \text{rad}. \)

Observer: \( g_{11} = g_{22} = 6 \times 10^6; \, g_{12} = g_{21} = 0; \, g_{41} = g_{42} = g_{44} = 0; \, k_1 = k_2 = -3 \times 10^5; \)

Controller: \( \gamma_1 = 400; \, \gamma_2 = 4 \times 10^4; \, K_{x_4} = 600. \)

REFERENCES