AUTOMOBILE
SUSPENSION DETAILED ANALYSIS
TERM PROJECT REPORT
(2012-2013)

DYNAMICS OF MACHINES, ME22004
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GROUP MEMBERS:
1) MOHIT GOENKA : 10ME30021
2) JITENDRA CHORDIA: 10ME30013
3) MOHIT GHELAWAT: 10ME10028
INTRODUCTION:

The vehicle suspension system is an important system of any vehicle and is responsible for driving safety and comfort. It carries the vehicle body and transmits all the forces between the road and the body. The system consists of the wishbones, the spring and the shock absorber. The spring carries the body mass and isolates the body from road disturbances. The task of the damper is the damping of the body and wheel oscillations.

Key Terms:

1. **Camber** – The small angle made by the wheels of the vehicle with respect to the vertical.
2. **Kingpin Angle** - The angle between the steering axis and the vertical line.
3. **Scrub Radius** – It is the distance between the steering axis and the wheel’s contact patch.
**Working Principle**

Tyres are the most important parts of a race car. They have to transmit all drive, brake and steering forces to the road through a very small contact patch. This makes it very important for a car to keep the tyres in optimal contact with the road at all times. That is the task of the suspension system. In case of a race car, the suspension system can be designed specifically for that goal, at the cost of driver comfort. The wheel and brake disc are connected to the upright by bearings. Carbon fibre rods with ball joints on each end connect the upright to the chassis. One of these six rods (the inclined one) is not mounted to the chassis, but to a rocker. If the wheel moves up with respect to the vehicle body, the upright pulls on the rod, which in turn causes the rocker to rotate about its pivot point. A spring-damper is connected to the bell crank on one end, and to the chassis on the other. So by rotating the rocker, the spring-damper is compressed.

The spring-damper system consists of two subsystems: the coil-over damper itself (see below figure, and the anti-roll system. The coil spring absorbs the energy from a bump by compressing, and releases it again at an uncontrolled rate. The spring will continue to bounce until all of the energy originally put into it is dissipated. Dampers are used to control this energy dissipation. They slow down and reduce the amplitude of the wheel motion by converting kinetic energy into heat.

The anti-roll system consists of a torsion bar with a lever on each end. If the movements on each end of the bar are not exactly the same, it will be twisted. This results in a reaction force.

The main rocker connects the pull rod to the two subsystems. The geometry of this rocker determines the movement of the spring-damper and the anti-roll bar as a result of the movement of the pull rod. Two operating conditions will be explained to show how the subsystems work to control the movements of the suspension.

1) **Bump situation:**

If the car drives over a threshold, the left and right wheels will move up an equal amount. This results in the same angular rotation of the left and right main rocker. The spring-damper are therefore actuated equally. Since left and right levers of the anti-roll bar are rotated to the same angle in the same direction, it will not create a reaction moment.

2) **Cornering situation:**

If the car drives through a corner, its body will roll to the outside of the bend. As a result, the outer wheel moves up with respect to the chassis, and the inner wheel will move down. This means the main rockers are rotated in opposite directions. The load on the outside spring will become higher, while the inside spring will be (partially) unloaded. The anti-roll bar will be twisted, which results in an opposing moment that tries to keep the vehicle body level.
TYPES OF DIFFERENT SUSPENSION DESIGN USED:

LEAF- SPRING:

A leaf spring is a simple form of spring commonly used for the suspension in wheeled vehicles. It takes the form of a slender arc-shaped length of spring steel of rectangular cross-section. The centre of the arc provides location for the axle, while tie holes are provided at either end for attaching to the vehicle body. For very heavy vehicles, a leaf spring can be made from several leaves stacked on top of each other in several layers, often with progressively shorter leaves. Leaf springs can serve locating and to some extent damping as well as springing functions. While the interleaf friction provides a damping action, it is not well controlled and results in stiction in the motion of the suspension. For this reason manufacturers have experimented with mono-leaf springs.

Fig 2 (showing the schematic of a leaf-spring suspension)
**MacPherson Strut:**

MacPherson struts consist of a wishbone or a substantial compression link stabilized by a secondary link which provides a bottom mounting point for the hub or axle of the wheel. This lower arm system provides both lateral and longitudinal location of the wheel. The upper part of the hub is rigidly fixed to the inner part of the strut proper, the outer part of which extends upwards directly to a mounting in the body shell of the vehicle.

To be really successful, the MacPherson strut required the introduction of unibody construction, because it needs a substantial vertical space and a strong top mount, which unibodies can provide, while benefiting them by distributing stresses. The strut will usually carry both the coil spring on which the body is suspended and the shock absorber, which is usually in the form of a cartridge mounted within the strut. The strut also usually has a steering arm built into the lower inner portion. The whole assembly is very simple and can be preassembled into a unit; also by eliminating the upper control arm, **it allows for more width in the engine compartment**, which is useful for smaller cars, particularly with transverse-mounted engines such as most front wheel drive vehicles have.

**Overall:**

Simple design with wide placed anchor points providing good transverse rotational stiffness (good for isolating chassis against acceleration and braking torques).

- Low packaging room -very popular for front wheel drive cars with transverse mounted engines.
- High overall height means that designs usually end up higher hood and fender line.
DOUBLE WISHBONE SUSPENSION:
(Along with push/pull rod mechanism)

In automobiles, a double wishbone (or upper and lower A-arm) suspension is an independent suspension design using two (occasionally parallel) wishbone-shaped arms to locate the wheel. Each wishbone or arm has two mounting points to the chassis and one joint at the knuckle. The shock absorber and coil spring mount to the wishbones to control vertical movement. Double wishbone designs allow the engineer to carefully control the motion of the wheel throughout suspension travel, controlling such parameters as angle, caster, toe pattern, roll centre height, scrub radius, scuff and more.

PUSH/PULL ROD SUSPENSION (Along with double wishbone suspension):

Generally a pushrod suspension at the front and a pullrod suspension at the rear is used in almost all the racing cars.

1) A Pullrod is mounted on or near the upper wishbone and runs down to a bell crank near the bottom of the chassis. The orientation of this rocker can be adjusted to mount the damper vertical as in the figure, or horizontally in x-direction as used for the front suspension system. Other orientations are of course possible, but these are the two most common ones. The main advantage of a horizontally mounted damper is height of the centre of gravity. Secondly, a pullrod is in tension, so it can be lighter than an equivalent pushrod that might fail due to buckling.

2) A Pushrod is mounted near lower A-arm and runs up to a rocker near the top of the rear frame. The damper can be mounted vertically as in the figure, or in a different orientation.
Situation analysis as per Bumpy roads/Cornering:

a) During cornering, the outside upper A-arm is in tension, while the lower one is in compression. This compressive force is >1 times higher than the lateral tyre force, because of the leverage between the road surface and the upper and lower wishbone. To make matters worse, the lower connection rods of the multilink system are longer than the upper ones to achieve the desired camber gain, which makes them even more prone to fail. Therefore, the toe link is placed on the same level as the lower connection rods to reduce the load per rod. This load per rod can be reduced even further by choosing a pushrod configuration.

![Fig a (Showing Pullrod Suspension) & fig b (Showing Pushrod Suspension)](image)

b) When a car drives over a bump, the upper connection rods will be loaded with a compressive force, and the lower ones with a tensile force. In case of a pullrod system, the upper connection rods gain an extra compressive load due to the pullrod. In case of a pushrod configuration, the lower connection rods gain an extra tensile load. These two cases can of course also be combined: a car hits a bump while cornering. Then, the upper connection rods, which are under tension from the lateral tyre forces, are (partially) unloaded, or even change to a push rod. The compressive cornering force on the lower ones will also be reduced or switched to tensile.
DETAILED ANALYSIS OF SUSPENSION DESIGN:
(Both linear & non-linear model)

ELECTRONIC CONTROLLED SUSPENSION:

Everyone knows about the Passive case (linear) suspension where the damping coefficient is a constant “b” but due to some technological advancement today there exits better damper with Variable damping coefficients “b(t)”.

Vehicle suspension serves as the basic function of isolating passengers and the chassis from the roughness of the road to provide a more comfortable ride. In other words, a very important role of the suspension system is the ride control. Due to developments in the control technology, **electronically controlled suspensions** have gained more interest. These suspensions have active components controlled by a microprocessor. By using this arrangement, significant achievements in vehicle response can be carried out. Selection of the control method is also important during the design process. The design of vehicle suspension systems is an active research field in which one of the objectives is to improve the passenger’s comfort through the vibration reduction of the internal engine and external road disturbances.

The present work aims at developing an active suspension for the quarter car model of a passenger car to improve its performance by using a proportional integral derivative (PID) controller. The controller design deals with the selection of **proportional, derivative gain and integral gain parameters** (kp, ki, and kd). The results show that the active suspension system has reduced the peak overshoot of sprung mass displacement, sprung mass acceleration, suspension travel and tire deflection compared to passive suspension system.

**General Tips for Designing a PID Controller**

When you are designing a PID controller for a given system, follow the steps shown below to obtain a desired response.

a) Obtain an open-loop response and determine what needs to be improved
b) Add a proportional control to improve the rise time
c) Add a derivative control to improve the overshoot
d) Add an integral control to eliminate the steady-state error

**TERMS:**

a) **Sprung Mass**: It is the portion of the vehicle's total mass that is supported above the suspension, including in most applications approximately half of the weight of the suspension itself.

b) **Unsprung Mass**: It is the mass of the suspension, wheels or tracks (as applicable), and other components directly connected to them, rather than supported by the suspension. (The mass of the body and other components supported by the suspension is the sprung mass.) Unsprung weight includes the mass of components such as the wheel axles, wheel bearings, wheel hubs, tires, and a portion of the weight of driveshaft, springs, shock absorbers, and suspension links. Even if the vehicle's brakes are mounted outboard (i.e., within the wheel), their weight is still considered part of the unsprung weight.
PID CONTROLLERS:

![PID Controller Diagram]

**Effects of increasing a parameter independently**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Rise time</th>
<th>Overshoot</th>
<th>Settling time</th>
<th>Steady-state error</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td>Decrease</td>
<td>Increase</td>
<td>Small change</td>
<td>Decrease</td>
<td>Degrade</td>
</tr>
<tr>
<td>$K_i$</td>
<td>Decrease</td>
<td>Increase</td>
<td>Increase</td>
<td>Eliminate</td>
<td>Degrade</td>
</tr>
<tr>
<td>$K_d$</td>
<td>Minor change</td>
<td>Decrease</td>
<td>Decrease</td>
<td>No effect in theory</td>
<td>Improve if $K_d$ is small</td>
</tr>
</tbody>
</table>

Basic pre-requisites assumptions before the analysis (shown in the below figures a & b):

- Considering a basic spring-damper system with spring and damper constant as “$k_1 & b_1$” (for Passive Case) whereas “$k_1 & b_1(t)$” (for semi-active case).
- Quarter of the **sprung mass** acting on 1 spring-damper system is “$M_1$” and the **unsprung mass** being “$M_2$”.
- In our analysis we have considered the tyre as a combination of spring and damper with having coefficients as “$k_2 & b_2$”.
- Taken all the Motion in the Vertical direction (x-axis), let M1 move $x_1$, M2 by $x_2$ and the Base/road excitation is “$w$”.
- The **input excitation** is in the form of a **Step Input** which characterizes a vehicle coming out of a pot hole (for say). The pot hole has been represented such that the “step height” is 0.1m and “step time” is 5 seconds.
- The entire calculation is done for one spring-damper system such that only quarter of the mass is to be considered while analysis.
**REQUIRED DATA FOR ANALYSIS** (Suppose for an “X” Automobile):

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1 (Quarter car sprung mass), Kg</td>
<td>241.5 Kg</td>
</tr>
<tr>
<td>M2 (unsprung mass), Kg</td>
<td>41.5 Kg</td>
</tr>
<tr>
<td>k1 Spring constant (spring-damper), N/m</td>
<td>6000 N/m</td>
</tr>
<tr>
<td>k2 Equivalent tyre spring constant, N/m</td>
<td>140000 N/m</td>
</tr>
<tr>
<td>b1 damper coefficient, Ns/m</td>
<td>300 Ns/m</td>
</tr>
<tr>
<td>b2 Equivalent tyre damping coefficient, Ns/m</td>
<td>1500 Ns/m</td>
</tr>
</tbody>
</table>

**PID Controller Coefficients (using Ziegler–Nichols method)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional gain, kp</td>
<td>0.552</td>
</tr>
<tr>
<td>Integral gain, ki</td>
<td>5.52</td>
</tr>
<tr>
<td>Derivative gain, kd</td>
<td>22.08</td>
</tr>
</tbody>
</table>
GOVERNING EQUATION OF MOTION (for fig a & b):

\[ M_1 \dddot{x}_1 + b_1 (\dot{x}_1 - \dot{x}_2) + k_1 (x_1 - x_2) = 0 \]

\[ M_2 \dddot{x}_2 + b_1 (\dot{x}_2 - \dot{x}_1) + k_1 (x_2 - x_1) + b_2 \dot{x}_2 + k_2 x_2 = b_2 \ddot{w} + k_2 w \]

The above equation is for linear spring-damper system. For the case of PID-Controlled Suspension, the damping coefficient is a variable.

So, \( b_1 \rightarrow b_1 (t) \).

Here:
- \( x_1 \): Sprung mass vertical displacement, m
- \( x_2 \): Unsprung mass vertical displacement, m
- \( \dddot{x}_1 \): Sprung mass acceleration, m/s^2
- \( x_1 - x_2 \): Suspension travel, m
- \( x_2 - w \): Tire deflection, m

In order to find the various output responses for the above system, we used MATLAB 7.12.0 (R2011a), SIMULINK plug-in (we did by both block diagrams and Laplace method).

Applying Laplace transform:

There are several methods to solve the above equation but one of the most efficient being using the Laplace transform. Therefore switching from time domain to operational domain gives:

\[ M_1 s^2 x_1 + b_1 s (x_1 - x_2) + k_1 (x_1 - x_2) = 0 \]  
\[ ........equation 1 \]

\[ M_2 s^2 x_2 + b_1 s (x_2 - x_1) + k_1 (x_2 - x_1) + b_2 s x_2 + k_2 x_2 = s b_2 w + k_2 w \]  
\[ ........equation 2 \]

From above equation 1 we get:

\[ x_2 (s) = x_1 (s) \frac{M_1 s^2 + b_1 s + k_1}{b_1 s + k_1} \]

Putting this in above equation 2, we have:

\[ M_1 s^2 x_1 (s) + x_1 (s) \frac{(M_1 s^2 + b_1 s + k_1)(M_2 s^2 + b_2 s + k_2)}{b_1 s + k_1} = w (s) (b_2 s + k_2) \]

Now obtaining the transfer functions from above, that is:

\[ H_1 (s) = \frac{x_1 (s)}{w (s)} \]

\[ H_2 (s) = \frac{x_2 (s)}{w (s)} \]

\[ H_3 (s) = \frac{x_1 (s) - x_2 (s)}{w (s)} \]

\[ H_4 (s) = \frac{b_1 b_2 s^2 + s (k_1 b_2 + k_2 b_1) + b_2 k_1}{M_1 M_2 s^4 + s^2 (M_1 b_1 + M_1 b_2 + M_2 b_1) + s^2 (M_1 k_1 + M_1 k_2 + M_2 k_1 + b_1 b_2) + s (b_1 k_2 + k_1 b_2) + k_1 k_2} \]
Above is the Simulink Schematic block diagram along with the Laplace transform function to find the response for Step Input for "Passive case (linear)". As it is clearly seen from the figure that the final output response \(x_1\) is fed into the Scope block which gives the response graphs.

Similarly below is the figure for the Simulink Schematic for semi-active case (non-linear) where we have found out several responses and tried to compare the responses of "linear" and "non-linear" cases for the same Excitation input.
Simulink Schematic for semi-active case (non-linear):
RESPONSES TO BE FOUND:

1) $x_1$: Vertical displacement of sprung mass, M1.
2) $x_2$: Vertical displacement of unsprung mass, M2.
3) $x_1 - x_2$: Suspension Travel. This depicts that for a particular Input what is the maximum expansion/compression in the suspension. Our aim is to minimize it so as to reduce the fatigue loading.
4) $x_2 - w$: Tyre deflection. It’s is a very important analysis to check for the tyre-ground contact.
5) $\dot{x}_1$: Sprung mass velocity v/s time.

Note:
For any of the Excitation Input (Step, Sine or arbitrary), we check the above responses for both Passive case (linear suspension) and Semi-active case (non-linear suspension) and henceforth try to compare them on the basis of **riders comfort, tyre-ground contact, settling time** *(The time required for the step response to settle within a certain percentage of its final value. A frequently used figure is 2%) and spring failure due to fatigue loading.*

A. **PLOTS FOR STEP INPUT** (step time: 5 sec & step height: 0.1 m):

   I. $X_1$ (sprung mass displacement) v/s time Plot for both linear and Non-linear case:

   ![PLOT](Image)

   1) **Pink plot**: Passive Case; 2) **Yellow plot**: Semi-active case & 3) **Green**: Road Input

<table>
<thead>
<tr>
<th></th>
<th>Maximum Overshoot of displacement (m)</th>
<th>Settling Time (sec)</th>
<th>Rise time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive Case</td>
<td>0.0712</td>
<td>7.08</td>
<td>0.3099</td>
</tr>
<tr>
<td>Semi-Active Case(PID)</td>
<td>0.0076</td>
<td>2.32</td>
<td>0.2014</td>
</tr>
<tr>
<td>% reduction due to PID Controller</td>
<td><strong>89.32 %</strong></td>
<td><strong>67.23 %</strong></td>
<td><strong>35.01 %</strong></td>
</tr>
</tbody>
</table>
II. **X1 – X2 (suspension travel) v/s time plot for both linear and Non-linear case:**

![Graph showing suspension travel vs time for both linear and non-linear cases.](image)

1) **Pink plot:** Semi-active case [b(t)]; 2) **Yellow plot:** Passive case (constant damping coefficient)

Here settling time remains the same.

<table>
<thead>
<tr>
<th></th>
<th>Maximum Spring compression (m)</th>
<th>Maximum Spring Elongation (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive Case</td>
<td>0.1177</td>
<td>0.0683</td>
</tr>
<tr>
<td>Semi-Active Case(PID)</td>
<td>0.0693</td>
<td>0.007172</td>
</tr>
<tr>
<td>% reduction due to PID Controller</td>
<td><strong>44.12 %</strong></td>
<td><strong>89.49 %</strong></td>
</tr>
</tbody>
</table>

Here it is clear that the implementation of Semi-active suspension system has brought down both maximum expansion and compression of suspension spring by a huge amount minimizing the possibility of **Spring Failure due to Fatigue Loading**.

III. **X2 – w (Tyre deflection) v/s time plot for both linear and Non-linear case:**

<table>
<thead>
<tr>
<th></th>
<th>Maximum Tyre lift from Ground (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive Case</td>
<td>0.0231</td>
</tr>
<tr>
<td>Semi-Active Case(PID)</td>
<td>0.0131</td>
</tr>
<tr>
<td>% reduction due to PID Controller</td>
<td><strong>43.29 %</strong></td>
</tr>
</tbody>
</table>
So we can conclude that in the Semi-active case, the tyre lift from the ground as well as the settling time is much less in comparison with Passive case.

Here also, although velocity overshoot is higher for PID Controller but still it lasts for an extremely short period of time which once again makes it a better option.
V. $X_1$ v/s $t$ plot keeping $k_i$, $k_p$ constant and varying $k_d$ gain in PID Controller:

- It clearly seems that keeping other parameters constant; decreasing the value of “$k_d$” decreases the sprung mass maximum overshoot and the output response gets smoother (comfort zone).

B. Sprung Mass displacement response for Inclined Ramp Ground Input (slope $\tan 30^\circ$):

1) Pink plot: Passive Case; 2) Yellow plot: Semi-active case & 3) Green: Road Input
- It seems that in semi-active mode response is in sync with input giving maximum comfort.
C. Sprung Mass Response for **Arbitrary Excitation** as Input (mean as 0.009167 and variance as 0.003827):

1) **Pink plot**: Passive Case; 2) **Yellow plot**: Semi-active case

- Once again in the above graph, output maximum overshoot response for Semi-active is much lower than that for passive case which concludes that it is much comfortable than Passive case.
CONCLUSION:
In this report Suspension is designed for a quarter car model of a passenger car to improve the ride comfort and road holding ability. For PID Controller, Zeigler and Nichols tuning rules are used for series of tunings to improve the ride comfort and road holding ability. For step input the of 0.1 m, the sprung mass displacement has been reduced by 89.32 % which shows the improvement in ride comfort. The Maximum suspension compression and elongation has been reduced by 44.12 % and 89.49 % respectively. The tire deflection has reduced by 43.29%. We also see that the sprung mass settling time and rise time decreases by 67.23 % and 35.01.

Even for Inclined ramp and arbitrary excitation the output response is much desirable for PID Controllers as compared to the passive one.

Therefore, it is concluded that the Semi-active suspension system has better performance capabilities over passive suspension system.