Beamspaces ESPRIT algorithm for bistatic MIMO radar

Y.D. Guo, Y.S. Zhang and N.N. Tong

A novel beamspaces ESPRIT (B-ESPRIT) algorithm is proposed to estimate the direction of departures (DODs) and direction of arrivals (DOAs) for bistatic MIMO radar. It restores the rotational invariance structure lost in the beamspace transformation for both the transmit array and the receive array, and then the DODs and DOAs can be estimated through ESPRIT. The proposed algorithm can achieve a significant computational saving over the element-space ESPRIT (E-ESPRIT) algorithm for bistatic MIMO radar. Simulation results validate these conclusions.

Introduction: Multiple-input multiple-output (MIMO) radar [1] can transmit orthogonal waveforms to enhance parameters, identifiability and resolution by virtual array which would greatly increase the degrees of freedom (DOF) of MIMO radar. The DOF of MIMO radar is often proportional to the product of the number of transmitters and the number of receivers, so it can be very large. In [2, 3], the ESPRIT algorithm is used for direction of departures (DODs) and direction of arrivals (DOAs) estimation in bistatic MIMO radar, which necessitates eigendecomposition of the sample covariance matrix. But huge computation will be involved when the DOF is very large. Beamspaces transformation is one way of reducing computation and sometimes improving the estimated robustness. As a consequence of beamspace transformation being performed, arrays such as uniform linear arrays (ULAs) would lose their rotational invariance structure. As a result, computational complexity may actually increase since the computationally efficient ESPRIT algorithm cannot be applied directly. In this Letter, we show how the beamspaces ESPRIT (B-ESPRIT) algorithm exploits the beamspaces invariance property of both the transmit array and receive array for DODs and DOAs estimation in the bistatic MIMO radar system. Notation: (·)' denotes transpose, conjugate-transpose, conjugate operators; ⊗ denotes the Kronecker product; |·| denotes the modulus operation.

Problem formulation: Consider a bistatic MIMO radar system with $M_t$ transmitters and $M_r$ receivers, both of which are half-wavelength-spaced ULAs. The transmit antennas transmit orthogonal waveforms with identical bandwidth and central frequency. Assume that there are $P$ noncoherent targets located at the same range. The DOD and DOA of the $p$th target with respect to the transmit array normal and the receive array normal are denoted by $\varphi_p$ and $\theta_p$, respectively. Therefore, the outputs of all the matched filters in all receivers can be expressed as

$$x(t) = A\cdot \tilde{x}(t) + n(t)$$

(1)

where $A = A_0 \otimes A = [a_0, a_1, \ldots, a_N]$ is an $M_M \times P$ matrix composed of $P$ steering vectors, $a_p = a_j(b_a) \otimes a(t_j)$, $s_j(t)$ = $[s_1(t), \ldots, s_J(t)]^T$ is a column vector, of which $s_j(t)$ = $\beta_a e^{j2\pi f_0 t}$ represents the envelope of the reflected signal with $f_0$ being the Doppler frequency and $\beta_a$ being the amplitude including the reflection coefficients and radar losses and so on. $n(t)$, the noise, is modelled as a zero-mean, spatially white Gaussian process of covariance matrix $\sigma^2_{\text{noise}} I_{M_M} \otimes I_{M_M}$, $I_{M_M}$ denotes an $M_M \times M_M$ unity matrix.

B-ESPRIT for angle estimation: Define $M_M \times L_r$ beamspace transformed matrix as $B = B_0 \otimes B$, where $B_0$ is the $M \times L_r$ ($L_r < M$) transmit beamspace transformed matrix and $B$ is the $M \times L_r$ ($L_r < M$) receive beamspace transformed matrix. Then the transformation from $M \times M$ dimensional element-space to an $L_r \times L_r$ dimensional beamspace can be described as

$$y(t) = B^H x(t) = C \tilde{x}(t) + B^H n(t)$$

(2)

where $C = [B^H_0 A \otimes B^H_0 A]$. The beamspace covariance matrix can be estimated by $R = 1/L \sum_{l=1}^L y(l)y^H(l)$. Let $U_r$ be the signal subspace composed of the $P$ largest eigenvectors corresponding to the $P$ largest eigenvalues of $R$. The relationship between $U_r$ and $C$ can be determined by a unique nonsingular matrix $T$ as

$$U_r = CT$$

(3)

In the above beamspace processing, the rotational invariance structures in the transmit and receive array are altered by the row transformation $B^H_0$ and $B^H_0$. To restore the lost rotational invariance structures, $B_0$ and $B$ must have the same rotational invariance structures, so they should satisfy [4]

$$J_{1, i} B_i = J_{2, i} B_i F_i$$

$$J_{1, i} B_i = J_{2, i} B_i F_i$$

(4)

where $J_{1, i}$ and $J_{2, i}$ are the first $M_t - 1$ rows and the last $M_t - 1$ rows of an $M_t \times M_t$ unity matrix, $J_{1, i}$ and $J_{2, i}$ are the first $M_r - 1$ rows and the last $M_r - 1$ rows of an $M_r \times M_r$ unity matrix, $F_i$ and $F_i$ are nonsingular.

Therefore, we can have the beamspace rotational invariance property for both the transmit array and receive array as follows

$$Q B_i^H A_t = Q B_i^H J_{i} A_t = Q F_i J_{i} A_t \Phi_r$$

(5)

$$Q B_i^H A_r = Q B_i^H J_{i} A_r = Q F_i J_{i} A_r \Phi_r$$

(6)

Here we use the facts that $J_{i} A_t = J_{i} A_r \Phi_r$, $J_{i} A_r = J_{i} A_r \Phi_r$, where $\Phi_r$ and $\Phi_r$ are matrices with the main diagonals $\gamma_{rr} = e^{j2\pi f_0 t_i}$ and $\gamma_{rr} = e^{j2\pi f_0 t_i}$. $p = 1, \ldots, P$, respectively, and zeros elsewhere. Define the respective matrices $W_1 Q \otimes Q W_2 = Q F_2^H \otimes Q$. Then, $W_1 = Q \otimes Q F_2^H$. According to (3) and (6), we can obtain the following matrices

$$G_1 = W_1 U_t = (Q B_i^H A_t \otimes Q B_i^H A_t) T$$

$$G_2 = W_2 U_r = (Q B_i^H A_r \otimes Q B_i^H A_r) T = G_1 \Psi_r$$

$$G_3 = W_1 U_r = (Q B_i^H A_r \otimes Q B_i^H A_r) T = G_1 \Psi_r$$

(7)

where $\Psi_r = T^{-1} F_2 \otimes T$. $\Psi_r = T^{-1} F_2 \otimes T$. Therefore, we can obtain the DODs and DOAs through $\Psi_r$ and $\Psi_r$ using the TLS ESPRIT method. Since $\Psi_r$ and $\Psi_r$ have the same eigenvectors, the estimated DODs and DOAs can be paired by their corporate eigenvectors.

A simple and effective choice of $F_1$, $F_2$, $Q$ and $Q$ is as discussed before, we know that the choice of $F_1$, $F_2$, $Q$ and $Q$ is the basis of the proposed B-ESPRIT algorithm. Here we show how to choose $F_1$, $F_2$, $Q$, and $Q$, to satisfy the requirement. If $B_0$ and $B_0$ are the collection of standard Fourier basis vectors, the $i$th and $i$th column of $B_0$ and $B_0$ are denoted as $B_0 [i, i] = 1/\sqrt{M_1} [e^{j2\pi f_0 t_i}, \ldots, e^{j2\pi (M_1-1) t_i}]$ and $B_0 [i, i] = 1/\sqrt{M_1} [e^{j2\pi M_1 t_i}, \ldots, e^{j2\pi (M_1-1) M_1}]$.

Simulation results: To demonstrate the performance of the B-ESPRIT algorithm, two sets of simulations compared with the E-ESPRIT algorithm are provided. For the first simulation, we assume that there exist $P = 2$ uncorrelated stationary targets, which are located at angles $(\varphi_1, \theta_1) = (10, 20)$ and $(\varphi_2, \theta_2) = (30, -30)$, and the number of snapshots is $L = 100$ for an M = 8 and $M_r = 6$ bistatic MIMO radar. Define the RMSE of the $p$th target as $\text{RMSE}(\varphi_p, \theta_p) = \sqrt{\text{E}[\varphi_p - \varphi_p]^2 + (\theta_p - \theta_p)^2}$, where $\hat{\varphi_p}$ and $\hat{\theta_p}$ are the estimated DOD and DOA for the same target, respectively. The variation of angle estimation RMSE of the B-ESPRIT and the E-ESPRIT with SNR is presented in Fig. 1, where 200 Monte-Carlo simulations are used. From Fig. 1, we know that the performance of the E-ESPRIT algorithm is slightly better than the B-ESPRIT algorithm. However, we should also observe that the computational complexity of B-ESPRIT is much smaller than the E-ESPRIT algorithm, which will be seen from the second simulation.
Fig. 1 RMSE of angle estimation from 200 Monte-Carlo simulations for two targets

For the second simulation, we compare the computational complexity of the B-ESPRIT with the E-ESPRIT algorithm. Fig. 2 presents an evaluation of the computational complexity using TIC and TOC instruction in MATLAB for the proposed algorithm and the E-ESPRIT algorithm, for TIC and TOC instruction can be used to calculate the runtime of an algorithm. The runtime is plotted against the number of antennas in Fig. 2. To facilitate our presentation, here we consider $M_t = M_r$, and the other simulated conditions are not changed. We can observe from Fig. 2 that the runtime of the B-ESPRIT is much smaller than the E-ESPRIT algorithm.

Fig. 2 Runtime of both B-ESPRIT and E-ESPRIT algorithm against $M_t = M_r$ with two targets

Conclusion: We have presented a novel B-ESPRIT algorithm which is used for angle estimation in bistatic MIMO radar. The restored rotational invariance structures of the transmitting and receiving array are exploited to estimate the DODs and DOAs. Compared with the E-ESPRIT algorithm, the B-ESPRIT algorithm can reduce the computational cost from $o(M_t^3 M_r^3)$ to $o(L_t^3 L_r^3)$, which would achieve a significant computational saving. Simulation results verify the performance of the proposed method.

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One or more of the Figures in this Letter are available in colour online.

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References