Fast-Varying Doppler Compensation for Underwater Acoustic OFDM Systems

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Abstract—This paper presents a practical low-density parity-check (LDPC) coded OFDM system designed for the underwater acoustic channel with its attendant multipath channel and large Doppler shift. The Doppler shift and channel state information (CSI) are assumed unavailable to both to the transmitter and the receiver. Due to the slow speed of sound in water, even small platform motions can affect the performance of the wideband system significantly, thus tracking the time-varying Doppler is essential for mobile underwater communication. An iterative receiver is proposed, which performs a Doppler estimation, channel estimation, and decoding in an iterative manner. The Doppler and channel tracking is done based on symbol by symbol, thus desirable for high speed mobile platform.

I. INTRODUCTION

THE fundamental obstacles to robust underwater acoustic communications (UAC) are the long multipath delay and large Doppler effects. The main advantage of Orthogonal Frequency Division Multiplexing (OFDM) is that since each subcarrier only experiences flat fading, complex time-domain equalizers are not necessary. Thus, OFDM is an attractive choice for such a channel as the cyclic prefix (CP) eliminates intersymbol interference (ISI) and high data rates using coherent transmission can be achieved. In terrestrial communications, OFDM has been adopted for next-generation wireless standards including IEEE 802.11a/g, 802.16, 802.20, Digital Audio Broadcasting (DAB) and Digital video broadcasting (DVB). Recent applications of OFDM to underwater communications appear in [1]–[6].

In a narrowband system, all subcarriers suffer from approximately same Doppler shift, and most analysis for the terrestrial OFDM systems is based on the narrowband assumption. However, the underwater acoustic channel is inherently wideband since the signal bandwidth is not negligible with respect to the carrier frequency. In this case, the Doppler effect causes each subcarrier to suffer a different frequency shift. Due to the slow speed of sound in water (c = 1500 m/s), even small platform motions can affect the performance of the wideband system significantly [1]. Therefore OFDM for the UAC requires agile and accurate tracking of the Doppler effect with wideband model. We focus on robust receiver solutions where the Doppler and channel are estimated on a symbol-by-symbol basis, which is well-suited to mobile high-speed platforms such as AUVs.

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The main contribution of the paper is the development of a computationally efficient Doppler estimator (DE) and channel estimator for UAC OFDM systems. The receiver is designed such that the rapidly varying Doppler shift and multipath channel are estimated in an iterative manner in combination with LDPC decoding. Doppler estimation (DE) is accomplished by oversampling, interpolation, and rate conversion (resampling). Channel estimation (CE) is implemented in the time-domain least squares (LS) in order to facilitate iterative processing. Computation savings are achieved by feeding back the hard-decisions from the LDPC decoder in the iterative receiver. Reference [3] also addresses the Doppler effect for wideband OFDM systems, where only non-iterative receivers are considered. In [3], the resampling rate is estimated by a special-purpose preamble and postamble. Since the preamble and postamble are added between data packets (multiple OFDM symbols), the algorithm in [3] may not be robust for rapidly varying Doppler.

The rest of this paper is organized as follows: Section II gives the mathematical formulation of the signal and the receiver architectures. In Sections III and IV, we design the Doppler and channel estimator. Simulation results are presented in Section V, and Section VI gives conclusions.

II. SIGNAL AND SYSTEM MODEL

The system specification for the underwater OFDM system considered in this paper is designed as in Table I based on the target environment below. The target channel is assumed to

| FFT size (NFFT) | 512 |
| Number of Data subcarriers | 336 |
| Number of pilots (NLp) | 128, index:[-254:4:-2 2:4:254] |
| Number of nulls | 48 |
| Cyclic prefix Ratio (LP/CP) | 1/4 |
| Carrier Frequency (f0) | 24 KHz |
| Bandwidth | 12 MHz (18 - 30 KHz) |
| Subcarrier BW (Δf) | 23.44 Hz |
| Cyclic prefix duration | 10.7 ms |
| Symbol duration with CP | 53.3 ms |
| Data rate (encoded/uncoded) | 12.6 Kbps, 6.3Kbps |
| ADC/DAC Frequency | 96 KHz |
| Modulation Order | QPSK |
| search resolution | 128 points |
| LDPC decoder | 872/336 half-rate IEEE 802.16e |

TABLE I: OFDM System specifications
have 10 ms maximum delay spread, which is reasonable for short-to-medium (1 - 5 km) range in shallow water [2], [7]. Paths with delays greater than 10 ms are treated as noise. The pilot subcarriers have 10 ms maximum delay spread, which is reasonable for shallow water purposes. This simplified model is applicable to shallow water transmission with range much greater than depth, where the multipath arrival angles are similar, and the Doppler shift effect is dominated by transmitter/receiver relative motion.

The bandpass received signal after removing the CP is modeled as,

$$y(t) = Re \left\{ e^{j2\pi f_c(1+\beta)t} \sum_k r_k e^{j2\pi k\Delta f(1+\beta)t} g((1+\beta)t) \right\}.$$  (2)

where $r_k = h_k \cdot s_k$, where $h_k$ is the channel gain in the frequency domain. This model is valid since the CP converts the linear convolution into a circular convolution. The baseband signal after down conversion is given by,

$$y(t) = e^{j2\pi f_c\beta t} \sum_k r_k e^{j2\pi k\Delta f(1+\beta)t} g((1+\beta)t).$$  (3)

We observe that a) the received waveform in (3) is compressed or dilated by $\beta$, b) each subcarrier experiences a Doppler shift which depends on the frequency of the subcarrier, and c) the carrier frequency offset (CFO) term is introduced.

The effect of waveform dilation can be reduced using the resampling technique in [11] as follows. Suppose $\beta$ is known at the receiver and the waveform is sampled with the period $\Delta f N_\beta(1+\beta)$, yielding

$$y_n = e^{j2\pi \epsilon n/N_s} \sum_k r_k e^{j2\pi k\epsilon n/N_s}.$$  (4)

where $\epsilon = \frac{\beta}{f_s N_s}$ is the normalized CFO. The OFDM symbol can be demodulated via FFT after CFO correction and channel equalization.

From (4), we observed that the Doppler effect can be compensated for by sampling with appropriate rate and CFO correction. Once the Doppler effect is removed, it is convenient to express the compensated samples in vector form. The channel matrix $\mathbf{F} \in \mathbb{C}^{N_s \times N_s}$ is circulant with first column given by $[f_0, f_1, \cdots, f_{N_s-1}]^T$. Assuming the Doppler rate $\beta$ is found from the Doppler estimator, and assuming $\hat{\beta} \simeq \beta$, the corresponding resampled sequence is approximated as $\hat{z}(\beta) \simeq y$, where $y = [y_0, y_1, \cdots, y_{N_s-1}]^T$. Then the residual CFO is further compensated by

$$z(\hat{\beta}) = \mathbf{E}^H(\hat{\epsilon}) \hat{z}(\hat{\beta}),$$  (5)
where $E(\hat{\epsilon}) = \text{diag} \{1, e^{i2\pi\hat{\epsilon}/N_s}, \cdots, e^{i2\pi(N_s-1)\hat{\epsilon}/N_s}\}$ and $\hat{\epsilon} = \hat{\beta}/(1+\hat{\beta}f_{c}/\Delta f)$. The corrected vector in the presence of noise is,

$$z(\hat{\beta}) = FW^Hs + n,$$

where $W \in \mathbb{C}^{N_s \times N_s}$ is the FFT matrix, with $W_{nm} = \exp(-i2\pi(n-m)/N_s), 0 \leq n, m \leq N_s - 1$. The additive Gaussian noise with covariance matrix $\sigma_n^2I_{N_s}$ is denoted by $n$.

**B. Iterative Receiver**

For the iterative receiver in Fig. 1, multiple inner LDPC decoder iterations $N_{in}$ are run for every $N_{out}$ outer estimator/detector $\Rightarrow$ LDPC decoder iteration. At the first iteration, the Doppler shift is estimated based on null subcarriers and the channel is estimated on pilots as discussed in Section III-A and IV-A. After LDPC decoding, the tentative decisions are made using the a posteriori decoder L-values. The Doppler and channel estimates are further refined as discussed in Section III-B and IV-B followed by another decoding process.

1) Symbol Demapper (Detector): Assume the Doppler effect is compensated and the estimate of the channel in the frequency domain $h = \sqrt{N_s}W[f^T0]^T$ is available from CE. Then $z(\hat{\beta})$ is converted into the frequency-domain via FFT, $r = Wz(\hat{\beta})$. The symbol demapper produces extrinsic L-values of bits and the $i$th coded bit of the $j$th data symbol $d_j$ ($c_{j,i}$) is given by

$$L_{E,DET}(c_{j,1}) = \frac{1}{\sigma_n^2}\text{Re}(r_jh_2^*)$$

$$L_{E,DET}(c_{j,2}) = \frac{1}{\sigma_n^2}\text{Im}(r_jh_2^*)$$

for Gray mapping QPSK modulation.

2) Symbol Mapper: The tentative decisions, $\hat{d}$ are made from the a posteriori decoder L-values ($L_{APP,EST}$) via hard decisions and fed back to the “symbol mapper”, which generates the OFDM symbols, $\hat{s} = p + \hat{d}$ for use by the Doppler and channel estimators of next iteration. While soft decisions also can be utilized, the performance improvement using soft instead of hard decisions has been shown to be minimal in [12]. We will show that significant computations are saved if hard-decisions are used.

**III. DOPPLER ESTIMATION**

For $\beta > 0$ (the receiver is moving toward the transmitter and the waveform is compressed), we need to sample the waveform at least $1/(1+\beta)$ times the Nyquist rate in order to reconstruct the transmitted signal. From (3) and (4), we observe the sampling rate conversion (resampling) is required in order to detect the data successfully. The rate conversion can be performed via upsampling by a factor of $P$ and downsampling by a factor of $Q$, where $P$ and $Q$ are relatively prime integers and can be implemented using polyphase filters [13]. However, the implementation of the different polyphase filters for each search point of $\beta$ is a demanding operation.

Efficient rate conversion is achieved by oversampling the received signal (3) through the sampling period $1/(\Delta f / N_{r_o})$; the waveform is sampled at $r_{o}$ times faster than the Nyquist rate. Finally the samples at times $n\Delta f / (N_{r_o}1+\beta), 0 \leq n \leq N_{cp}+N_s - 1$, are obtained using linear (straight-line) interpolation for each candidate $\beta$. Then the first $N_{cp}$ samples (the CP portion) are discarded. The oversampling factor $r_{o}$ is chosen sufficiently large so that the error from linear interpolation is minimized. The interpolator’s performance is measured by the signal-to-distortion ratio (SDR) at the output [11], [13], which for a given signal frequency ($f$) is approximated as,

$$\text{SDR (dB)} \approx 40 \log_{10}\left(\frac{2f_s}{f}\right),$$

We choose $r_{o} = 8 = 2f_{c}$, thus the SDR equals 36 dB for our system specification.

In [3], the Doppler parameter $\beta$ is estimated using a dedicated preamble and postamble, and the CFO is compensated independently, thus may not be robust for rapid variations in $\beta$. Our proposed algorithm is based on symbol by symbol estimation, thus agile tracking of the speed is possible. The proposed algorithm is especially attractive for underwater communications. First, the interpolation method can reduce computations with minimal degradation. Second, high-rate oversampling rates $r_{o}$ are feasible at the bandwidths used in underwater acoustic communications.

A. Initial Doppler estimation

In initial Doppler estimation, we use null subcarriers to estimate $\beta$. Define $w_m = [1, e^{i2\pi m/N_s}, \cdots, e^{i2\pi(N_s-1)m/N_s}]^T$. The sum of energy on the null subcarriers is used as the cost function,

$$J(\beta) = \sum_{m \in S_N} |w_m^Hz(\beta)|^2,$$

where $S_N$ is the set of null subcarrier indices and $z(\beta)$ is the sequence that (3) is sampled at the rate of the candidate $\beta$ using interpolation followed by CFO compensation as in (5). At the true value of $\beta$, $J(\beta)$ is ideally zero in the absence of additive noise. Therefore, $\beta$ is estimated as

$$\hat{\beta} = \arg \min_{\beta} J(\beta),$$

which can be found via one-dimensional search. A similar cost function (9) is used in [3], [9] for CFO-only search. However, the MUSIC-like algorithm in (9),(10) searches the actual Doppler velocity ratio $\beta$, rather than just a pure CFO.

B. Iterative Doppler Estimation

After decoding, hard decisions computed using a posteriori L-values ($L_{APP,EST}$) are fed back to the “symbol mapper” in Fig. 1. Then the estimated symbols $\{\hat{d}_j\}$ and pilots are multiplexed such that $\hat{s} = p + \hat{d}$ and converted to the time domain via the IFFT $\hat{t} = W^H\hat{s}$. In order to estimate the Doppler rate, the channel distorted signal without Doppler shift is approximated as

$$\hat{z} \approx FW^H\hat{t} + n = \hat{T}f + n,$$

where $\hat{T} \in \mathbb{C}^{N_s \times N_s}$ is a truncated circulant matrix with first column given by $\hat{t}$.  

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To obtain a robust CFO estimation algorithm, we use the unconstrained LS channel estimate \( \hat{f}_{LS} = \left( \hat{H}^H \hat{T} \right)^{-1} \hat{T}^H \hat{z} \). Substituting \( \hat{f}_{LS} \) into the cost function \( ||z(\beta) - \hat{f}_{LS}||^2 \) yields the CFO estimate,

\[
\hat{\beta} = \arg \max_{\beta} z(\beta)^H \hat{T} \left( \hat{H}^H \hat{T} \right)^{-1} \hat{T}^H z(\beta).
\]  

(12)

Thus \( \hat{\beta} \) can be found through one-dimensional search.

In practical implementations, we show that the matrix inverse operation can be eliminated, thus significant computational saving is possible if constant modulus modulation is used (e.g. m-PSK) and hard-decisions are fed back from the decoder. Consider the real symmetric Toeplitz matrix \( \hat{T}^H \hat{T} \) in (12). The first column of \( \hat{T}^H \hat{T} \) is the circular auto-correlation of \( \hat{T} \), and the matrix can be completely defined by its first column. Recall the “circular correlation property of FFT”, if \( X(k) \) and \( Y(k) \) are the DFT of the \( x(n) \) and \( y(n) \), then the DFT of the circular correlation of \( x(n) \) and \( y(n) \) is given by \( X(k) \cdot \overline{Y(k)} \). Define \( m = \{ |\hat{s}_{-2/N_s}|^2, |\hat{s}_{-1/N_s}|^2, \ldots, |\hat{s}_{N_s-1}|^2 \}^T \) since we know \( m \) (the power of the OFDM symbol \( \{ \hat{s}_k \}^2 \) for all subcarriers \( k \)), the first column of \( \hat{T}^H \hat{T} \) is known and given by \( \hat{W}^H m \). Therefore \( \hat{T}^H \hat{T} \) and its inverse can be predefined regardless of the tentative decisions. Note that \( A = \hat{T}^H \hat{T} - 1 \) can be predefined only for hard-decision feedback, thus the matrix inverse at every iteration is required for the soft-decisions. As reviewed previously, hard-decisions offer approximately the same performance as soft for iterative estimation. Therefore very computationally efficient algorithm can be implemented by using hard-decisions without performance degradations.

IV. CHANNEL ESTIMATION

A. Initial Estimation

Since there is no information from the decoder at the first stage, the initial CE is performed based on only pilots. A total of \( N_p \) pilots are inserted in the OFDM symbol at known locations \( \{ n : 0 \leq n \leq N_p - 1 \} \). Define the Doppler compensated OFDM symbol in the frequency domain \( \hat{r} = \hat{W} z(\beta) \) and \( \hat{h} = \{ h_{n_0}, h_{n_1}, \ldots, h_{N_p-1} \} \) where \( h_n = r_n / p_n \). Then the channel impulse response can be obtained by [14]

\[
\hat{f}_{LS} = (B^H B)^{-1} B^H \hat{h}
\]  

(13)

where \( B_{n,m} = e^{-j2\pi n m / N_s} \). The matrix \( (B^H B)^{-1} B^H \) in (13) also can be precomputed.

B. Iterative Channel Estimation

After decoding, hard decisions computed using \( L_{APP,EST} \) are fed back to the “symbol mapper” in Fig. 1. Then the estimated symbols \( \{ \hat{d}_j \} \) and pilots are multiplexed such that \( \hat{s} = p + d \) and converted to the time-domain via the IFFT \( \hat{t} = \hat{W}^H \hat{s} \).

Using (11), the least square (LS) channel estimate, \( \hat{f}_{LS} \) is given by,

\[
\hat{f}_{LS} = \left( \hat{T}^H \hat{T} \right)^{-1} \hat{T}^H \hat{z}.
\]  

Again, the matrix \( A = \left( \hat{T}^H \hat{T} \right)^{-1} \) can be precomputed.

V. SIMULATION RESULTS

The system specification for simulations is in Table I. The system is designed so that one OFDM symbol contains one codeword. In simulations, the Doppler rate \( \beta \) is uniformly generated in the range of \([-1/256, 1/256] \) corresponding \pm 11.39 knots. The 128-tap channel (corresponding to the 10.6ms multipath delay spread) is generated with each tap undergoing Rayleigh fading independently. The Doppler rate and the channel is generated independently between symbols. We set \( N_{in} = 25 \) and \( N_{out} = 20 \). After LDPC encoding, we apply a bit-wise block interleaving (\( \Pi_i \)) adopted from the IEEE 802.16e standard [10].

Define \( E_p = |p_k|^2 \) is the power of each pilot subcarrier and \( E_d = |d_k|^2 \) is the power of each data subcarrier. The BERs with \( E_p/E_d = 0.707, 1, 2, \) and 3 are in Fig. 2. We refer to [6] for the detailed power allocation strategy using EXIT charts. We observe that the iterative process yields progressively lower error rates as \( N_{out} \) increases. However, the gain from successive outer iterations becomes negligible as \( E_p/E_d \) increases. Note that more than \( N_{out} = 20 \) iterations hardly improve the performances for all \( E_p/E_d \) schemes. Therefore, even though the gain from successive outer iterations is significant for \( E_p/E_d = 0.707 \), the overall performance after sufficient outer iterations is limited due to the poor initial estimates. \( E_p/E_d = 2 \) achieves best performances and allocating more power to the pilots \( (E_p/E_d = 3) \) again deviates from the optimal performances since too less data power is allocated.

VI. CONCLUSION

In this paper, iterative LDPC-coded OFDM receiver with Doppler tracking and channel estimation is designed. A computationally efficient Doppler estimator and channel estimator are developed by using hard-decision feedback from the LDPC decoder. The pilot power allocation strategy for the wideband underwater system is compared through the simulations. The proposed receiver is robust for high-speed mobile platform in underwater, and can be utilized where Doppler effect is significant such as in AUV underwater communications.

REFERENCES


Fig. 2: BERs with different pilot power allocations


