Solution to Economic Dispatch by Equal Incremental Cost Criterion

RM Saloman Danaraj, EEE Department, Aurora Technological Research Institute, Hyderabad, India

The basic economic dispatch problem can be described mathematically as a minimization of problem

Minimize \( \sum_{i=1}^{n} F_i(P_i) \)  \hspace{1cm} (1.1)

\( F_i(P_i) \) is the fuel cost equation of the \( i \)'th plant. It is the variation of fuel cost (\$ or Rs) with generated power (MW). Normally it is expressed as a quadratic equation.

\[ F_i(P_i) = a_i P_i^2 + b_i P_i + c_i \]  \hspace{1cm} (1.2)

If \( a_i \geq 0 \) then the quadratic fuel cost function is monotonic. The total fuel cost is to be minimized subject to the following constraints.

\[ \sum_{i=1}^{n} P_i = D + P_i \]  \hspace{1cm} (1.3)

\[ P_i = \sum_{j=1}^{n} B_{ij} P_j \]  \hspace{1cm} (1.4)

By Lagrangian multipliers method and Kuhn Tucker conditions and the following conditions for optimality can be obtained

\[ 2 a_i P_i + b_i = \lambda (1 - 2 \sum_{j=1}^{n} B_{ij}) \], \( i = 1, 2, \ldots, n \)  \hspace{1cm} (1.5)

\[ \sum_{i=1}^{n} P_i = D + P_i \]  \hspace{1cm} (1.6)

\[ P_i = \sum_{j=1}^{n} B_{ij} P_j \]  \hspace{1cm} (1.7)

\[ P_i^{\text{min}} \leq P_i \leq P_i^{\text{max}} \]  \hspace{1cm} (1.8)

The non-linear equations and inequalities are solved by the following procedure.

1. To initialize the procedure allocate lower limit of each plant as generation, evaluate the transmission loss and incremental loss coefficients and update the demand.

\[ P_i = P_i^{\text{min}}, x_i = 1 - \sum_{j=1}^{n} B_{ij} P_j, D^{\text{new}} = D + P_i^{\text{old}} \]  \hspace{1cm} (1.9)

2. Substitute the incremental cost coefficients and solve the set of linear equations to determine the incremental fuel cost.

\[ \lambda = \frac{\sum_{i=1}^{n} b_i}{2a_i} \quad \frac{D^{\text{new}} + \sum_{i=1}^{n} x_i}{2a_i} \]  \hspace{1cm} (1.10)

3. Determine the power allocation of each plant
If plant violates its limits it should be fixed to that limit and the remaining plants only should be considered for next iteration.

4. Check for convergence

\[
\left| \sum_{i=1}^{a} P_i - D^{\text{new}} - P_i \right| \leq \varepsilon
\]  

(1.12)

\( \varepsilon \) is the tolerance value.

5. Carry out the steps 2-4 till convergence.

**Quadratic programming methodology to implement Equal Incremental Cost Criterion**

Quadratic Programming is an effective optimization method to find the global solution if the objective functions is quadratic and the constraints are linear. It can be applied to optimization problems having non-quadratic objective and nonlinear constraints by approximating the objective to quadratic function and the constraints as linear. For all the four problems the objective is quadratic but the constraints are also quadratic so the constraints are to be made linear.

\[
\text{Minimize } X^T H X + f^T X, \text{ subject to } KX \leq R, X^\text{min} \leq X \leq X^\text{max}
\]  

(1.13)

\( X = [x_1, x_2, x_3, \ldots, x_n]^T \)  \( f = [f_1, f_2, f_3, \ldots, f_n]^T \)  \( R = [R_1, R_2, R_3, \ldots, R_m]^T \)

\( H \) is a Hessian matrix of size nxn and \( A \) is a mxn matrix representing inequalities.

To solve the economic dispatch with losses the QP can be effectively implemented by defining the matrices \( H, f, K \) and \( R \).

\[
H = \text{diag} \left( \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ x_1 & x_2 & x_3 \end{bmatrix} \right), f = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \]

(1.14)

\( K = [1, 1, 1, \ldots, 1] \) \( 1 \times n \) matrix, and \( R = D + P^\text{old} \)

(1.15)

Instead of steps 2&3 QP can be used to determine the allocation.

Case study 1.

A six plant system with quadratic monotonic fuel cost functions (System-A) is considered for simulation.

**Economic & Emission dispatch Problems**

There are so many ways for including emission into the formulation of economic dispatch. One approach is combined economic and emission dispatch (CEED), which is formulated as a multi objective optimization problem, which should minimize both, fuel cost and emission subject to meet the demand and losses. Another approach is emission controlled economic dispatch (ECED), which is minimizing the economy subject to that particular emission limit for particular demand.
Combined Emission and Economic Dispatch

The combined economic and emission dispatch problem can be formulated as:

\[
\text{Minimize } f(\text{FC, EC}), \sum_{i=1}^{N} P_i = D + P_i, P_i^{\text{min}} \leq P_i \leq P_i^{\text{max}}
\]  

(2.1)

\[
\text{FC} = \sum_{i=1}^{n} a_i P_i^2 + b_i P_i + c_i
\]  

(2.2)

\[
\text{EC} = \sum_{i=1}^{n} d_i P_i^2 + e_i P_i + f_i
\]  

(2.3)

\[
P_i = \sum_{j=1}^{n} \sum_{l=1}^{n} B_{ij} P_j P_l
\]

(2.4)

FC is the total fuel cost and EC is the total emission. The transmission losses \( P_l \) can be found either from load flow or using \( B_{mn} \) coefficients. The multi-objective optimization problem is converted as single objective optimization problem by The price penalty factor of each plant can be found for a particular demand can be found as follows:

1. The ratio between the average fuel cost and the average emission for maximum power capacity of that plant is found \( h_i = \frac{FC_i(P_i^{\text{max}})}{EC_i(P_i^{\text{max}})} \), \( i = 1, 2, n \).

(2.5)

2. Based on the value of price penalty factor found the plants are arranged in ascending order

3. The maximum capacity of each unit (\( U_i \)) is added one at a time, starting from the smallest \( h_i \) unit until \( \sum P_i \geq D \).

(2.6)

4. At this stage \( h_i \) associated with the last unit in the process is the price penalty factor ‘h’ (Rs/Kg) for the given load demand using price penalty factor as follows

\[
\text{Minimize } f(\text{FC, EC}) = \text{Minimize } (\text{FC} + h.\text{EC})
\]

(2.7)

Economic emission dispatch problem can be solved by the same procedure described for basic economic dispatch with transmission loss included case.

clear;
clc;

% This program solves the economic emission dispatch dispatch with Bmn coefficients by
% quadratic programming and equal incremental cost critetion
% the ellddata matrix should have 5 columns of fuel cost coefficients and plant limits.
% 1.a ($/MW^2) 2. b $/MW 3. c ($) 4.lower lomit(MW) 5.Upper limit(MW)
%no of rows denote the no of plants(n)
ellddata=[0.15247 38.53973 756.79886 10 125
% the emidata matrix should have 3 columns of fuel cost coefficients and plant limits.  
% 1.a (Kg/MW^2) 2. b Kg/MW 3. c (Kg)

emidata=[
0.00419 0.32767 13.85932
0.00419 0.32767 13.85932
0.00683 -0.54551 40.2669
0.00683 -0.54551 40.2669
0.00461 -0.51116 42.89553
0.00461 -0.51116 42.89553
];

% h1 and h2 are the weightage factor for economy and emission
h1=1; h2=44.788;

% Demand (MW)
Pd=700;

% Loss coefficients it should be squarematrix of size nXn where n is the no of plants
B=1e-4*[1.4 .17 .15 .19 .26 .22
 .17 .6 .13 .16 .15 .2
 .15 .13 .65 .17 .24 .19
 .19 .16 .17 .71 .3 .25
 .26 .15 .24 .3 .69 .32
 .22 .2 .19 .25 .32 .85];

[P Fcost Emi Pl]=emield(elddata,emidata,h1,h2,B,Pd)
P =

62.1045
61.6732
119.9718
119.4721
178.1939
175.6409

Fcost =

3.7501e+004

Emi =

439.6075

Pl =

17.0565