A capon beamforming method for clutter suppression in colocated compressive sensing based MIMO radars

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ABSTRACT

Compressive sensing (CS) based multi-input multi-output (MIMO) radar systems that explore the sparsity of targets in the target space enable either the same localization performance as traditional methods but with significantly fewer measurements, or significantly improved performance with the same number of measurements. However, the enabling assumption, i.e., the target sparsity, diminishes in the presence of clutter, since clutters is highly correlated with the desire target echoes. This paper proposes an approach to suppress clutter in the context of CS MIMO radars. Assuming that the clutter covariance is known, Capon beamforming is applied at the fusion center on compressively obtained data, which are forwarded by the receive antennas. Subsequently, the target is estimated using CS theory, by exploiting the sparsity of the beamformed signals.

Keywords: Compressive sensing, MIMO Radar, DOA estimation, Chutter suppression

1. INTRODUCTION

Due to its potential to improve target detection and discrimination, multiple-input and multiple-output (MIMO) radars have received considerable attention in recent years. Unlike traditional phased-array radars, a MIMO radar system transmits multiple independent waveforms from its antennas. Depending on the transmit (TX) and receive (RX) antenna configuration, MIMO radar systems are classified as widely separated and colocated MIMO radars. The former view the target from multiple uncorrelated directions; the spatial diversity results in improved target detection performance. The latter exploit waveform diversity to form a long virtual array, much longer than traditional radar systems with the same number of TX and RX antennas, and as a result enjoy superior resolution.

Compressive sensing (CS) is a relatively recent development for finding sparse solutions to underdetermined linear systems. CS theory states that a K-sparse signal \( x \) of length \( N \) can be recovered exactly, with high probability, from \( O(K \log N) \) linearly compressed measurements. The recovery requires that the product of the measurement matrix and the sparsifying basis matrix, referred to as the sensing matrix, satisfies the uniform uncertainty principle (UUP); in other words, the sensing matrix exhibits low correlation between its columns.

CS in the context of MIMO radars has been studied in Refs. 14–20. Refs. 14–17 considered the application of CS to colocated MIMO radars with point targets, while Refs. 18–20 considered the application of CS to widely separated MIMO radars with extended targets. Both cases of CS-based MIMO radars exploit the sparsity of targets in the target space and enable target estimation based on a small number of samples obtained at the RX antennas.

The assumed sparsity, however, diminishes in the presence of clutter, and as result the performance of CS-based MIMO radars in the presence of clutter degrades. This is because clutter is highly correlated with the desired target echoes and thus, radar systems cannot distinguish the true targets from the clutter. In this paper we proposed a scheme to suppress clutter in the context of colocated MIMO radars. When clutter and signal of interest have different Doppler shifts, clutter suppression can be relatively easily done, especially when the source of clutter is static while the target is moving. In this paper, we will consider the more
challenging case, in which Doppler differences cannot be exploited to separate clutter from signal of interest, e.g., when both the target and the source of clutter are static. We will assume that the clutter covariance is known. Typically, clutter has high space correlation to its neighboring range cells\(^3\)\(^4\). Therefore, the received signals at neighboring range cells can be used to estimate the clutter covariance matrix of the current range cell. The proposed schemes consists of first applying Capon beamforming on the compressively obtained data, which are forwarded to the fusion center by the receive antennas, and second, recovering the target using CS theory, by exploiting the sparsity of the beamformed signals. The proposed scheme has been shown to significantly improve detection performance in the presence of strong clutter.

Notation: Lower case and capital letters in bold denote respectively vectors and matrices. Superscripts \((\cdot)^H\) and \((\cdot)^T\) denote respectively the Hermitian transpose and transpose. \(0_{L \times M}\) and \(I_{L \times M}\) respectively denote an \(L \times M\) matrix with “0” and “1” entries. \(I_M\) represents an identity matrix of size \(M\). \(\otimes\) denotes the Kronecker tensor product.

2. CS-BASED COLOCATED MIMO RADAR

Let us consider a MIMO radar system consisting of \(M_t\) TX antennas and \(N_r\) RX antennas that are closely spaced in an arbitrary configuration. Let \((r_j^t, \alpha_j^t) / (r_j^r, \alpha_j^r)\) denote the location of the \(i\)-th TX/RX antenna in polar coordinates. Let us also consider the presence of \(K\) point targets; the \(k\)-th target is at azimuth angle \(\theta_k\).

Let \(L\) denote the number of \(T_s\)-spaced samples of the transmitted waveforms. The effect of the compressive receiver in Figure 1 of Ref. 17 is equivalent to pre-multiplying by matrix \(\Phi\) a \(T_s\)-sampled version of the received signal. The size of \(\Phi\) is \(M \times L\).

Under the far-field and narrowband targets assumptions, the received baseband signal at the \(l\)-th receive antenna can be approximated by Ref. 21

$$r_l \approx \sum_{k=1}^{K} \beta_k e^{j2\pi f_c t_l / c} \Phi \eta^r_k(\theta_k) \Phi \eta^r_k(\theta_k) + \Phi \eta^t_k$$

(1)

where

1. \(X\) is an \(L \times M_t\) matrix that contains the transmit waveforms of \(M_t\) antennas as its columns and \(\text{diag}\{X^H X\} = [1, \ldots, 1]^T\); \(\beta_k\) is the target reflection coefficient with respect to the \(k\)-th target;
2. \(v_t(\theta_k) = [e^{j2\pi f_c t_l (\theta_k)}, \ldots, e^{j2\pi f_c t_l (\theta_k)}]^T\) is the transmit steering vector associated with target direction \(\theta_k\) and \(\eta^r_k(\theta_k) = r_l^{t/r} \cos(\theta_k - \alpha_l^{t/r})\);
3. \(n_l\) is the interference at the \(l\)-th receiver, arising due to the jammer signals and thermal noise.

Let us discretize the angle space into \(N\) discrete angles \([a_1, \ldots, a_N]\). The discretization step is small enough so that each target falls on some angle grid point. Then (1) can be rewritten as

$$r_l = \Phi_l \Psi_l s + \Phi_l n_l$$

(2)

where \(\Psi_l = [e^{j2\pi f_c (a_1)} X_v(a_1), \ldots, e^{j2\pi f_c (a_N)} X_v(a_N)]; s = [s_1, \ldots, s_N]^T\). Here, \(s_n\) equals \(\beta_k\) if the \(k\)-th target locates at \(a_n\); otherwise it equals zero.

By stacking the receive data vector from each antenna into a long vector, we have

$$y = [r_1^T, \ldots, r_{N_r}^T]^T = \left(\Phi \Psi_{1}^T, \ldots, \Phi \Psi_{N_r}^T\right) s + \left(\Phi n_1^T, \ldots, \Phi n_{N_r}^T\right)^T$$

(3)
According to the CS formulation, $\Theta$ is the sensing matrix and $\Psi_l$ is the basis matrix for the $l$-th antenna. If the number of targets is small as compared to $N$, then the positions of the targets are sparse in the angle space, and $s$ is a sparse vector. The locations of the non-zero elements of $s$ provide information on the target angles. A variety of CS methods can be applied to the recovery of $s$, e.g., basis pursuit\textsuperscript{11}, matching pursuit\textsuperscript{12} and Lasso methods\textsuperscript{13}.

3. CLUTTER REJECTION

Let us form the $(M \times N_r)$ matrix $Y = [r_1, ..., r_{N_r}]$, where $r_l$ is a vector containing the compressed samples forwarded to the fusion center by the $l$-th receive antenna.

It holds that

$$Y^T = \sum_{k=1}^{K} \beta_k v_r(\theta_k) v_t^T(\theta_k) X^T \Phi^T + Z$$

(4)

where

- $Y$: the received signal at $N_r$ RX antennas, of which the $i$-th column contains the received signal from the $i$-th RX antenna
- $v_t(\theta_k)$: the transmit steering vector at the direction of $\theta_k$, defined in (1)
- $v_r(\theta_k)$: the receive steering vector at the direction of $\theta_k$, defined similarly to $v_t(\theta_k)$
- $Z$: the clutter matrix, whose covariance $R_z$ is assumed to be known

The Capon beamformer, $w$, allows the signal from a particular direction $\theta$ to pass undistorted, while it minimizes the power coming from all other directions. The estimation of $w$ is formulated as

$$\min_w w^H R_z w \text{ s.t. } w^H v_r(\theta) = 1.$$  

(5)

The solution of (5) is

$$w(\theta) = \frac{R^{-1} v_r(\theta)}{v_t^H(\theta) R^{-1} v_r(\theta)}.$$  

(6)

Let $w_n$ denote the Capon beamformer that focuses on the discrete angle $a_n$ of the $N$-point angle grid. Let us apply $w_n$ on the received data matrix $Y$ to produce $\tilde{y}_n$, i.e.,

$$\tilde{y}_n = (w_n^H Y^T)^T = Y w_n^*$$

$$= \Theta_n s + Z^T w_n^* n = 1, ..., N$$

(7)

where

$$\Theta_n = \Phi X [v_t(a_1) v_r^T(a_1) w_n^*, ..., v_t(a_N) v_r^T(a_N) w_n^*]$$

and $s$ is a sparse vector, the non-zero elements of which indicate the target locations.

The vector $\tilde{y}_n$ $(M \times 1)$ is the beamformer output corresponding to angle $a_n$, computed based on $M$ snapshots of the data received at the fusion center. If there is a target at $a_n$ then all elements of $\tilde{y}_n$ will have a large magnitude, otherwise, they will have low magnitudes.
Stacking $\tilde{y}_n$, $n = 1, \ldots, N$ into a vector, we get a vector $\tilde{y}$, which has a block-sparse like appearance and can be expressed as:

$$\tilde{y} = [\tilde{y}_1^T, \ldots, \tilde{y}_N^T]^T = [\Theta_1^T, \ldots, \Theta_N^T]^T s + [w_1^H Z, \ldots, w_N^H Z]^T = \hat{\Theta} s + \hat{Z}. \tag{8}$$

Here, $\tilde{y}$ provides a high resolution target picture. However, by exploiting the structure of (8), we can further improve resolution. This can be achieved by compressing $\tilde{y}$ through a matrix $\hat{\Phi}$ and applying CS theory to recover the vector $s$. We should note that the compression through $\hat{\Phi}$ is not necessary but it reduces the dimensionality of the problem and thus the complexity of the recovery.

4. SIMULATIONS

We consider a MIMO radar system with TX and RX antennas uniformly located on a disk of radius 10m. The carrier frequency is $f = 5$GHz. Each TX antenna uses Hadamard orthogonal waveform sequence of length $L = 128$ and unit power. The received signal is corrupted by zero-mean Gaussian noise of unit power.

Three targets are present on the angle grid $[-30^\circ, -29^\circ, \ldots, 30^\circ]$. The signal-to-noise ratio (SNR) is set as 0dB. The clutter signal is constructed as the sum of returns reflected by 3000 reflectors, located in the DOA angle space $[-30^\circ, -30^\circ + 60/1500^\circ, \ldots, 30^\circ]$. The reflection coefficient of each reflector is 0.15. Random matrix is used as the measurement matrix, i.e., $\Phi \in \mathbb{R}^{M \times L}$, with $M = 50$.

Assume there are $K = 3$ targets in directions $[-20^\circ, 0^\circ, 15^\circ]$ and the target reflect coefficients are set as $[1, 1, 1]$. Figure 2 shows the contour of the beamformed receive data matrix $Y = [\tilde{y}_1, \ldots, \tilde{y}_N]$ for $N_t = 20$ and $N_r = 200$, respectively. In both cases, the number of TX antennas is set as $M_t = 20$. It can be found from Fig. 2 that, as $N_r$ is large enough, there are $K = 3$ columns with large amplitude in matrix $Y$ whose column indices correspond to the indices of each target’s DOA in the discretized angle space. This indicates that after applying the Capon-beamformer the stacked receive data vector $\tilde{y}$ defined in (8) is relatively block-sparse.

Figure 3 shows the averaged signal-to-clutter-plus-noise ratio (SCNR) versus the number of RX antennas. The SCNR without beamforming is obtained according to (4). The SCNR of CS-Capon is defined according to (8). Here, we also plot the maximum and minimum of SCNR for all beamformed receive vector $\tilde{y}_n$, $n = 1, \ldots, N$. In simulations, $K = 3$ targets are generated randomly on the DOA grid $[-30^\circ, -29^\circ, \ldots, 30^\circ]$ and the target reflect coefficients are set as $[1, 1, 1]$. The number of TX antennas is set as $N_t = 20$. For each $N_r$, 100 runs have been carried out and average SCNR is calculated. It can be found from Fig. 3 that, without beamforming, the SCNR of the received signal is around $-10$dB and does not change much with the increase of $N_r$. With beamforming, the SCNR of the CS-Capon estimator defined in (8) is around 20dB when $N_r = 50$, and increases with the increase of $N_r$. This indicates that the proposed CS-Capon method could achieve significant gain in SCNR performance, i.e., strong clutter suppression. We can also find from Fig. 3 that, the SCNR of the CS-Capon estimator falls in the middle between the maximum and minimum SCNR of all beamformed vector $\tilde{y}_n$, $n = 1, \ldots, N$. This is because the SCNR of the CS-Capon estimator is averaged over the entire scanned angle space.

Figure 4 plots the ROC curves of the angle estimates that are obtained based on 50 random and independent runs. In each run, $K = 3$ targets are randomly generated on the angle grid of interest $[-8^\circ, -8^\circ + 0.2^\circ, \ldots, 8^\circ]$ with random reflection coefficients $[1, 0.45, 0.8]$, respectively. In the simulations, $N_t = 20$, $N_r = 30$, $L = 256$ and each RX antenna collects 20 compressed measurement. A random Gaussian matrix is used as the measurement matrix. One can see that the proposed CS-Capon method outperforms the CS method and the Capon method. It is worth noting that if the reflection coefficients of the targets are identical, Capon performs similar to the proposed CS-Capon method. When the reflection coefficients of three targets are different, the ripples generated by Capon may mask the targets with smaller reflectivity.
5. CONCLUSIONS

We have proposed a Capon beamforming based approach to suppress clutter in the context of CS MIMO radars. With a prior knowledge of the clutter covariance, Capon beamforming is applied to the compressively obtained data. Subsequently, sparse signal recovery is used to extract target information from the beamformed data by exploiting the sparsity of the beamformed signals. It has been shown that the proposed scheme enables to significantly improve DOA estimation in presence of strong clutter.

REFERENCES

Figure 1: The diagram for the proposed CS-Capon method

Figure 2: Contour of the beamformed receive data matrix for $N_r = 20$ and $N_r = 200$, respectively.


Figure 3: SCNR comparisons with/without beamforming with $N_t = 20$.

Figure 4: ROC performances under for CS-Capon and Capon method with $N_t = 20$ and $N_r = 30$. 