Yes, $\mathcal{H}$ will always be connected. To show this, we prove the following fact.

(1) Let $T = (V, F)$ and $T' = (V, F')$ be two spanning trees of $G$ so that $|F - F'| = |F' - F| = k$. Then there is a path in $\mathcal{H}$ from $T$ to $T'$ of length $k$.

Proof. We prove this by induction on $k$, the case $k = 1$ constituting the definition of edges in $\mathcal{H}$. Now, if $|F - F'| = k > 1$, we choose an edge $f' \in F' - F$. The tree $T \cup \{f'\}$ contains a cycle $C$, and this cycle must contain an edge $f \not\in F'$. The tree $T' \cup \{f'\} - \{f\} - T'' - (V, F'')$ has the property that $|F'' - F| = |F' - F''| = k - 1$. Thus, by induction, there is a path of length $k - 1$ from $T''$ to $T'$; since $T$ and $T''$ are neighbors, it follows that there is a path of length $k$ from $T$ to $T'$. \blacksquare