(a) For every node \( v_k \) that comes later than \( v_j \), i.e. \( k > j \), it has probability \( \frac{1}{k-1} \) to link to \( v_j \), since \( v_k \) chooses from the \( k - 1 \) existing nodes with equal probabilities. For all the nodes coming before \( v_j \), such probability is obviously zero.

So the expected number of incoming links to node \( v_j \) is

\[
\sum_{k=j+1}^{n} \frac{1}{k-1} = \sum_{k=1}^{n-1} \frac{1}{k} - \sum_{k=1}^{j} \frac{1}{k} \\
= H(n-1) - H(k-1) \\
- \Theta(\ln n) - \Theta(\ln k) \\
= \Theta(\ln \frac{n}{k})
\]

(b) Consider a node \( v_j \), every node \( v_k \) with \( k > j \) has probability \( \frac{1}{k-1} \) not to link to \( v_j \). So if we have random variable \( X_j \) s.t.

\[
X_j = \begin{cases} 
1 & \text{node } v_j \text{ has no in-coming links} \\
0 & \text{otherwise}
\end{cases}
\]

then

\[
\text{Exp}[X_j] = Pr[\text{no nodes links to } v_j] \\
= \prod_{k=j+1}^{n} \left( 1 - \frac{1}{k-1} \right) \\
= \frac{j-1}{j} \cdot \frac{j}{j+1} \cdot \frac{j+1}{j+2} \cdots \frac{n-2}{n-1} \\
= \frac{j-1}{n-1}
\]

Therefore, by linearity of expectations, we get the expected number of nodes without in-coming links

\[
\sum_{j=1}^{n} \text{Exp}[X_j] = \sum_{j=1}^{n} \frac{j-1}{n-1} = \frac{1}{n-1} \sum_{j=1}^{n} (j-1) = \frac{1}{n-1} \cdot \frac{n(n-1)}{2} = \frac{n}{2}
\]