One algorithm is the following.

For $i = 1, 2, \ldots, n$
Receiver $j$ computes $\beta_{ij} = f(\beta_1^*, \ldots, \beta_{i-1}^*, \alpha_i^{(j)})$.
$\beta_i^*$ is set to the majority value of $\beta_{ij}$, for $j = 1, \ldots, k$.
End for
Output $\beta^*$

We’ll make sure to choose an odd value of $k$ to prevent ties.

Let $X_{ij} = 1$ if $\alpha_i^{(j)}$ was corrupted, and 0 otherwise. If a majority of the bits in \{ $\alpha_i^{(j)} : j = 1, 2, \ldots, k$ \} are corrupted, then $X_i = \sum_j X_{ij} > k/2$. Now, since each bit is corrupted with probability $\frac{1}{4}$, $\mu = \sum_j EX_{ij} = k/4$. Thus, by the Chernoff bound, we have

\[
\Pr[X_i > k/2] = \Pr[X_i > 2\mu] < \left(\frac{e}{4}\right)^{k/4} < (0.91)^k.
\]

Now, if

\[ k \geq 11 \ln n > \frac{\ln n - \ln 0.1}{\ln(1/0.91)}, \]

then

\[ \Pr[X_i > k/2] < 0.1/n. \]

(So it is enough to choose $k$ to be the smallest odd integer greater than $11 \ln n$.) Thus, by the union bound, the probability that any of the sets \{ $\alpha_i^{(j)} : j = 1, 2, \ldots, k$ \} have a majority of corruptions is at most 0.1.

Assuming that a majority of the bits in each of these sets are not corrupted, which happens with probability at least 0.9, one can prove by induction on $i$ that all the bits in the reconstructed message $\beta^*$ will be correct.

\[ ^1 \text{ex482.918.336} \]