(a) Let’s look at a given machine \( p \). In order for it to have no job, every job must be assigned to a different machine. As the jobs are assigned randomly and uniformly, the probability that a given job \( j \) is not assigned to \( p \) is \((1 - \frac{1}{k})\) and therefore the probability that \( p \) doesn’t get any job is \((1 - \frac{1}{k})^k\). Therefore the expected number of machines with no jobs is \( N(k) = k(1 - \frac{1}{k})^k \).

Finally \( N(k)/k = (1 - \frac{1}{k})^k \), which goes to \( 1/e \) as \( k \) goes to infinity. Also notice that in the limit the number of machines with no jobs is \( k/e \).

(b) There is a very simple solution to this problem. We notice that the number of rejected jobs (denote it by \( N_{\text{rej}} \)) is the number of total jobs \( k \) minus the number of accepted jobs \( N_{\text{acc}} \) (\( N_{\text{rej}} = k - N_{\text{acc}} \)). The number of jobs accepted is the \( k \) minus the number of machines with no jobs \( N_{\text{nojob}} \) (since the rest of the people do exactly 1 job). Therefore \( N_{\text{rej}} = k - N_{\text{acc}} = (k - N_{\text{nojob}}) - N_{\text{nojob}} \). Therefore the answer to part (b) is the same as the answer to part (a).

(c) This part will involve slight calculations. We know that the number of machines with no jobs is \( k/e \) (from the first part). We first calculate the number of machines with exactly one job. Again look at a machine \( p \). The probability that only 1 job is assigned to that machine is \( \frac{k}{k}(1 - \frac{1}{k})^{k-1} \). (The chance of a given job \( j \) being assigned to \( p \) is \( 1/k \) and the probability that the remaining jobs will not be assigned to \( p \) is \((1 - \frac{1}{k})^{k-1} \). Finally there are \( k \) choices of the “given” job \( j \) which puts the coefficient \( k \) in the beginning). Notice that this also in the limit \( 1/e \) therefore the number of machines with exactly 1 jobs is also \( k/e \).

Finally the remaining machines regardless of how many jobs they were assigned will perform exactly two jobs. There are \( k - \frac{2k}{e} \) of these.

The final tally is \( k/e \) machines with one job and \( k - \frac{2k}{e} \) people with two jobs. Subtracting this from \( k \) (the total number of jobs) we get that \( \frac{k(3-e)}{e} \) jobs are rejected, which is approximately 11%.

\(^1\text{ex16.34.694}\)