Let $X$ be a random variable equal to the number of times that $b^*$ is updated. We write $X = X_1 + X_2 + \cdots + X_n$, where $X_i = 1$ if the $i^{th}$ bid in order causes $b^*$ to be updated, and $X_i = 0$ otherwise.

So $X_i = 1$ if and only if, focusing just on the sequence of the first $i$ bids, the largest one comes at the end. But the largest value among the first $i$ bids is equally likely to be anywhere, and hence $EX_i = 1/i$.

Alternately, the number of permutations in which the number at position $i$ is larger than any of the numbers before it can be computed as follows. We can choose the first $i$ numbers in \( \binom{n}{i} \) ways, put the largest in position $i$, order the remainder in $(i-1)!$ ways, and order the subsequent $(n-i)$ numbers in $(n-i)!$ ways. Multiplying this together, we have \( \binom{n}{i} (i-1)! (n-i)! = n!/i \). Dividing by $n!$, we get $EX_i = 1/i$.

Now, by linearity of expectation, we have $EX = \sum_{i=1}^{n} EX_i = \sum_{i=1}^{n} 1/i = H_n = \Theta(\log n)$.\footnote{ex547.67.324}