Checking whether $G$ is 1- or 2-colorable is easy. For $k = 3, 4, \ldots, w + 1$, we test whether $G$ is $k$-colorable by dynamic programming. We use notation similar to what we used in the Maximum-Weight Independent Set problem for graphs of bounded tree-width. Let $(T, \{V_t : t \in T\})$ be a tree decomposition of $G$. For the subtree rooted at $t$, and every coloring $\chi$ of $V_t$ using the color set $\{1, 2, \ldots, k\}$, we have a predicate $q_t(\chi)$ that says whether there is a $k$-coloring of $G_t$ that is equal to $\chi$ when restricted to $V_t$. This requires us to maintain $k^{(w+1)} \leq (w + 1)^{(w+1)}$ values for each piece of the tree decomposition.

We compute the values $q_t(\chi)$ when $t$ is a leaf by simply trying all possible colorings of $G_t$. In general, suppose $t$ has children $t_1, \ldots, t_d$, and we know the values of $q_{t_i}(\chi)$ for each choice of $t_i$ and $\chi$. Then there is a coloring of $G_t$ consistent with $\chi$ on $V_t$ if and only if there are colorings of the subgraphs $G_{t_1}, \ldots, G_{t_d}$ that are consistent with $\chi$ on the parts of $V_t$ that intersect with $V_t$. Thus we set $q_t(\chi)$ equal to true if and only if there are colorings $\chi_i$ of $V_{t_i}$ such that $q_{t_i}(\chi_i) = true$ and $\chi_i$ is the same as $\chi$ when restricted to $V_t \cap V_{t_i}$.

\footnote{ex897.854.812}