Consider an ordered triple \((S, i, j)\), \(1 \leq i, j \leq n\) and \(S\) is a subset of the vertices that includes \(v_i\) and \(v_j\). Let \(B[S, i, j]\) denote the answer to the question, “Is there a Hamiltonian path on \(G[S]\) that starts at \(v_i\) and ends at \(v_j\)?” Clearly, we are looking for the answer to \(B[V, 1, n]\).

We now show how to construct the answers to all \(B[S, i, j]\), starting from the smallest sets and working up to larger ones, spending \(O(n)\) time on each. Thus the total running time will be \(O(2^n \cdot n^3)\).

\(B[S, i, j]\) is true if and only if there is some vertex \(v_k \in S - \{v_i\}\) so that \((v_i, v_k)\) is an edge, and there is a Hamiltonian path from \(v_k\) to \(v_j\) in \(G[S - \{v_i\}]\). Thus, we set \(B[S, i, j]\) to be true if and only if there is some \(v_k \in S - \{v_i\}\) for which \((v_i, v_k) \in E\) and \(B[S - \{v_i\}, k, j]\) is true. This takes \(O(n)\) time to determine.