The algorithm is very similar to the basic Gale-Shapley algorithm from the text. At any point in time, a student is either “committed” to a hospital or “free.” A hospital either has available positions, or it is “full.” The algorithm is the following:

While some hospital $h_i$ has available positions
    $h_i$ offers a position to the next student $s_j$ on its preference list
    if $s_j$ is free then
        $s_j$ accepts the offer
    else ($s_j$ is already committed to a hospital $h_k$)
        if $s_j$ prefers $h_k$ to $h_i$ then
            $s_j$ remains committed to $h_k$
        else $s_j$ becomes committed to $h_i$
        the number of available positions at $h_k$ increases by one.
        the number of available positions at $h_i$ decreases by one.

The algorithm terminates in $O(mn)$ steps because each hospital offers a position to a student at most once, and in each iteration, some hospital offers a position to some student.

Suppose there are $p_i > 0$ positions available at hospital $h_i$. The algorithm terminates with an assignment in which all available positions are filled, because any hospital that did not fill all its positions must have offered one to every student; but then, all these students would be committed to some hospital, which contradicts our assumption that $\sum_{i=1}^{m} p_i < n$.

Finally, we want to argue that the assignment is stable. For the first kind of instability, suppose there are students $s$ and $s'$, and a hospital $h$ as above. If $h$ prefers $s'$ to $s$, then $h$ would have offered a position to $s'$ before it offered one to $s$; from then on, $s'$ would have a position at some hospital, and hence would not be free at the end — a contradiction.

For the second kind of instability, suppose that $(h_i, s_j)$ is a pair that causes instability. Then $h_i$ must have offered a position to $s_j$, for otherwise it has $p_i$ residents all of whom it prefers to $s_j$. Moreover, $s_j$ must have rejected $h_i$ in favor of some $h_k$ which he/she preferred; and $s_j$ must therefore be committed to some $h_k$ (possibly different from $h_k$) which he/she also prefers to $h_i$.

\footnote{ex304.339.892}