An SNR Estimation Algorithm Using Fourth-Order Moments

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Abstract — An algorithm is presented that allows an estimation of the SNR just by the observation of the noisy signal. For the estimation shape factors of the signal's and the noise's probability density functions are used. The algorithm is based on the second and fourth order moments of the observed noisy signal.

I INTRODUCTION

We assume that the noisy signal \( y(t) \) can be described by the superposition of a stationary (wanted) signal \( x(t) \) and stationary noise \( n(t) \). Signal and noise are assumed to be real, zero mean and mutually independent, the latter at least with respect to fourth order statistics.

II REAL RANDOM PROCESSES

Let \( f_z(t) \) denote the probability density function (pdf) of a random process \( z(t) \). We expand the characteristic function in a series of its moments

\[
\phi_z(\omega) = \int_{-\infty}^{\infty} f_z(t)e^{-j\omega t} dt = \sum_{k=0}^{\infty} (-j)^k m_{z,k} \frac{\omega^k}{k!},
\]

where \( m_{z,k} \) is the k-th order moment of \( z(t) \).

The kurtosis \( K_z \) of \( f_z(z) \) is defined by:

\[
K_z = m_{z,4}/m_{z,2}^2.
\]

For the following, it is important to note that the process \( z(t) \) and a scaled version \( \alpha z(t), \alpha > 0 \) have the same kurtosis:

\[
K_{\alpha z} = m_{\alpha z,4}/m_{\alpha z,2}^2 = (\alpha^4 m_{z,4}/(\alpha^2 m_{z,2})^2 = K_z.
\]

Hence the kurtosis characterizes the shape of a pdf, but is completely independent of the actual power.

More useful than the kurtosis is in our case the Gaussian-unlikeness [1]

\[
G_z = K_z - 3
\]

which is equal to zero for (zero mean) Gaussian processes.

III THE SUM OF SIGNAL AND NOISE

The summation \( y(t) = x(t) + n(t) \) of independent random processes corresponds to the convolution of the respective pdf's or to the product of the characteristic functions:

\[
\phi_y(\omega) = \phi_x(\omega)\phi_n(\omega)
\]

It requires some simple algebra to show using (1) and (5) that the k-th order moment of the sum is given by:

\[
m_{y,k} = \sum_{\ell=0}^{k} \binom{k}{\ell} m_{x,\ell} m_{n,k-\ell}.
\]

IV ESTIMATION OF THE SNR

If we evaluate (6) for \( k = 2 \) and \( k = 4 \) we find (remember that both, signal and noise, are zero mean)

\[
\begin{align*}
\{ m_{y,2} &= m_x,2 + m_n,2 \\
\{ m_{y,4} &= m_x,4 + m_n,4 + 6m_x,2 m_n,2
\end{align*}
\]

Using (4) we obtain from (7)

\[
G_y = G_x k^2 + G_n (1 - k)^2,
\]

where

\[
k = m_{x,2}/(m_x,2 + m_n,2)
\]

is the wanted signal to total signal power ratio. Hence the SNR of the received signal is given by \( SNR = k/(1 - k) \).

It can be shown that at least one solution for \( k \) always exists, regardless of \( G_x \) and \( G_n \).

V IMPLEMENTATION CONSIDERATIONS

In practical applications such as SNR measurement in communication systems the observed noisy signal \( y(t) \) is e.g. the sampled output of a matched filter. \( G_y \) is given by the statistics of the modulation scheme used, \( G_n \) is given by the expected noise statistics; for Gaussian noise \( G_n = 0 \). \( G_y \) is usually approximated by second and fourth order time averages \( \bar{y}^2(t) \) and \( \bar{y}^4(t) \), resp. (see [2] for details). This algorithm was successfully tested in a CPM transmission system [3, 4].

REFERENCES


