Photonic bandgap analysis in 1D porous silicon photonic crystals using transfer matrix method

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There has been much interest in the physics and applications of one-dimensional spatially periodic, quasi-periodic and random photonic bandgap (PBG) structures. In this paper, we had employed an accurate and efficient transfer matrix method (TMM) to analyze the photonic bandgap variation in one-dimensional porous silicon photonic crystals (1DPSPCs). The optical properties dependence on imperative parameters angle of incidence, lattice constant and refractive index contrast have been analyzed. The forbidden bandwidth and the reflectance results show great agreement to each other, which verifies the accuracy obtained by us, using TMM for the computations. The increment in the angle of incidence and lattice constant shifts the photonic bandgaps towards the higher frequency range. The shift in forbidden bandgap and reflectance with the decrease of the lattice constant is clearly depicted in Fig. 5 and Fig. 6. The investigation of 1D photonic crystal for these parameters shows that the bandwidth varies from 0.11 to 0.32 as revealed in Fig. 4.

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1. Introduction

Considerable research efforts have been devoted to the miniaturization and high speed performance of integrated circuits but high resistance, higher power dissipation and signal synchronization problems are associated with them. The solution to overcome these problems lies in the use of a new class of materials where it is expected to turn photons instead of electron as the information carriers [1-2]. Photonic crystals (PCs) have become a key subject of today’s engineering and technological research. The photonic crystals are the optical analogy to electronic crystals in which electromagnetic waves propagating in a periodic dielectric structure can be separated by a gap where the propagation of the electromagnetic waves is prohibited [3-4]. Depending on periodicity they are classified as one, two and three dimensional photonic crystals [3-4]. A gap in the photonic crystals are created with appropriate selection of three parameters lattice topology, spatial period and dielectric constants of the constituent materials in which propagation of electromagnetic waves is prohibited. If a forbidden gap exists in all polarizations and directions; then it is said to be complete photonic bandgap. The ability to trap or localize light of photonic crystals provides a versatile platform for the design and fabrication of photonic devices in which a variety of active and passive components can be incorporated. Such unique properties of photonic crystals have focused the development of a new class of active and passive components for advanced photonics. The integrated photonic devices fabricated using photonic crystals are being used in today’s VLSI/ULSI and communication technology to achieve high speed performance [2]. The widening and shrinking of photonic bandgap shows great dependence on lattice constant, dielectric contrast, thickness of individual layers, number of bilayers, angle of incidence etc. Thus, photonic crystals have been explore for their advance applications in scientific and technological areas such as photonic band edge lasers, omnidirectional reflectors, microcavities, optical filters, optical switches, waveguides, antennas, channel drop filters, multiplexers/demultiplexers etc [3-6]. A photonic band gap can be extended by introduction of disorder in one-dimensional photonic crystals [7-8] by choosing proper layer thickness and keeping the average optical length in each layer as quarter wavelength, such structures constructed represents a wider bandgap. V. A. Tolmachev et al has reported the other way to extend the photonic bandgap, by mixing the periodic structures with two or more consecutively placed photonic crystals [9]. Yablonovitch and John suggested that the structures with periodicity of dielectric constant could influence the nature of electromagnetic modes within materials [10-11]. The control of radiative properties of materials and photon localization effect by introducing a random modulation in refractive index were the aim of Yablonovitch and John. The one-dimensional photonic crystals (1DPCs) fabrication is possible through various techniques. Therefore, they attracted much attention of researchers towards 1DPCs materials for applications in photonic devices.

Silicon has been the choice for microelectronics technology because of various reasons such as its cost, compatible with mass production and availability. Silicon based photonic devices are very significant from commercial point of view and are much compatible with established technology for microelectronic (CMOS) processing [12]. The most attractive aspects of fabricating conventional and nano-photonic devices on silicon are the availability of well established microelectronic tools and processes in general and starting materials technology, nano lithography and dry etching processes etc. The silicon based photonics technology is fully capable to
integrate electrical and optical components on a single chip. Porous silicon (PS) offers major potential for integrated optoelectronics technology and can accept the new challenges to fabricate silicon based photonic devices and retain advantages of silicon technology [13]. It is observed that a partial dissolution of silicon wafer become apparent with increasing current beyond some threshold level which indicates the formation of porous silicon. Due to room temperature luminescence PS provides the possibility to fabricate light emitting diodes [14]. Porous silicon has been used for the fabrication of 1D and 2D photonic crystals to achieve hundred percent reflection of light in one or two directions [15-17]. The possibility of molding the flow of electromagnetic waves by reflecting and transmitting them through one-dimensional porous silicon (1DPS) photonic crystal has opened enormous application areas in photonics.

The theoretical analysis carried out by simulation works has been reported by various researchers particularly for single layer of porous silicon [18]. Some research groups have been intensively engaged for the fabrication of photonic crystals by using porous silicon layers [15, 17]. But less effort has been taken in the area of designing and simulation of 1D porous silicon photonic crystals. However, in the analysis of light localization modes at the interface of PS layers Kavokin et al. [19] have reported the simulated results of bandgap, reflectance and electric field by transfer matrix method, Lugo et al. [17] and Ferrand et al. [20] have reported the same results but by planewave method and transfer matrix method in order to compare the experimental work. The analysis bandgap variation due to the change in the optical parameters with polarization condition is not reported. In this paper, we are reporting the detailed analysis of bandgap in one-dimensional photonic crystals for TE polarization condition and its dependency on optical parameters such as lattice constant, refractive index contrast and incident angle of light.

Detail analysis of dispersive properties has been carried by applying transfer matrix method. The transfer matrix method has been involved to achieve efficient accurate computation of the photonic band structure as a function of normalized frequency, lattice constant and incident angle in one-dimensional photonic crystals. In section two, the Maxwell equations and Bloch theory for periodic media aspects has been appropriately explained for better realization of physical phenomena involved in 1D photonic crystals. In section third, the dispersion relation of 1D photonic crystal for transverse electric (TE) waves based on transfer matrix approach has been depicted. As the variation of photonic bandgap is one goal to be pursued in present work, obtained results and discussions are summarized in section fourth. Section fifth, concludes the paper.

2. Mathematical approach

The dispersion of allowed and forbidden modes results in corresponding field distribution in 1D photonic crystal leading to field quantization which has forced us to analyze 1D porous silicon photonic crystals. To realize spatially periodic system, Maxwell equations have been involved to obtain the Helmholtz equation as given below which governs the electromagnetic phenomena for electric field macroscopically.

\[ \nabla^2 E + \frac{\omega^2}{c^2} \varepsilon_r E = 0 \quad (1) \]

At this juncture E is electric field, \( c^2 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \) and \( \varepsilon_r = \varepsilon / \varepsilon_0 \).

Bloch-Floquet theorem for Maxwell equations implies the existence of photonic bands which consists of allowed and forbidden band regions for electromagnetic waves. The wave propagation in 1D photonic crystal can be described in term of Bloch waves which are generated when incident light falls in the forbidden band and reflects. Hence, the Bloch-Floquet theorem has been applied to Equation (1) along with appropriate boundary conditions the solution in terms of Bloch waves is obtained as given in equation (2).

\[ E_K(x,z) = E_K(x)e^{ikz}e^{-ikx} \quad (2) \]

where \( E_K(x) \) and K are the periodic function and Bloch wave number which indicates the dependency of function \( E_K(x) \) on K, \( \beta \) is the z component of wavevector, which remains constant in entire medium.

To realize dispersive properties of photonic crystal a Fourier expansion of normal electromagnetic modes equation has to be substituted into the Maxwell equations which lead to imperative changes in the Bloch waves as given in equation (2). Hence, the dispersion relation and Bloch wavevector of 1D photonic crystal show great dependence on the incident light frequency.

The light propagation through the entire multilayer can be described by transfer matrix method. The transfer matrix method usually relates the amplitudes of electric field corresponding to forward and backward propagating waves at the interfaces between adjacent dielectric layers. Here, we had assumed a spatially periodic structure of porous silicon layers with refractive indices \( n_1 \) and \( n_2 \) and thicknesses \( d_1 \) and \( d_2 \) respectively with lattice constant \( \Lambda \) where subscripts 1 and 2 are corresponding to low and high refractive index layers respectively. The simple geometry of 1D photonic crystal is shown in Fig. 1. Here we assumed that the propagation of wave is in XZ-plane. K is the amplitude of the wavevector in the x-direction of the periodicity and \( \beta \) is the amplitude of wavevector parallel to the layers in z-direction.
Fig. 1. Schematic diagram of 1D porous silicon photonic crystal of alternate layers of low and high refractive index $n_1$ and $n_2$, thicknesses $d_1$ and $d_2$ respectively and lattice constant $\Lambda = d_1 + d_2$. The wave vector diagram is parallel to the layers in $z$-direction.

In this case medium is periodic in one direction therefore the refractive index profile can be given as

$$n(x) = \begin{cases} n_1, & 0 < z < d_1, \\ n_2, & d_1 < z < \Lambda, \end{cases}$$

with $n(x) = n(x + \Lambda)$, where, $z$ axis is normal to the layers interface, $n_1$ and $n_2$ are the refractive indices, $d_1$ and $d_2$ are the thicknesses of the corresponding layers respectively, $c$ and $\omega$ are the speed and angular frequency of the light and $\Lambda = d_1 + d_2$ is lattice constant.

In each medium the electric field is superposition of incident ands reflected waves. The electric field in layer $i$ (i=1, 2) of the $n^{th}$ unit cell can be represented by column vector

$$\begin{pmatrix} a_n^i \\ b_n^i \end{pmatrix}, i = 1, 2$$

Equation (2) of electric field in the same layer can be rewritten as

$$E(x, z) = \begin{pmatrix} a_n^i \\ b_n^i \end{pmatrix} e^{-ikz} e^{-ikL} + \begin{pmatrix} a_n^i \\ b_n^i \end{pmatrix} e^{ikz} \frac{1}{\sqrt{\gamma}}$$

The $z$ component of the wave vector in $i^{th}$ layer can be represented as

$$k_{ix} = \left( \frac{n_i \omega}{c} \right)^2 - \beta^2 \frac{1}{2}$$

with $i = 1, 2$ and $\omega/c$ is the wave number electromagnetic wave in the vacuum where $\omega$ and $c$ are the angular frequency and speed of light in vacuum.

By applying appropriate boundary conditions at the interface, the solutions can be obtained for TE polarization condition [22]. All these discrete equations can be recast by transfer matrix method which connects the electric and magnetic fields. These equations can be combined into a $2 \times 2$ matrix form, which is known as translational matrix, and is represented by

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix} = \begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix}$$

This above matrix relates the incident and reflected waves in one layer of a unit cell to those of equivalent layer in the next unit cell. The matrix elements for TE polarization case for two successive cells are

$$A = e^{-ikd_1} \left[ \cos\left(k_2d_2 + \frac{k_1}{k_2} \right) \right]$$

$$B = e^{ikd_1} \left[ \frac{1}{2} \left( k_2 - k_1 \right) \sin\left(k_2d_2 \right) \right]$$

$$C = e^{-ikd_1} \left[ \frac{1}{2} \left( k_2 + k_1 \right) \sin\left(k_2d_2 \right) \right]$$

$$D = e^{ikd_1} \left[ \cos\left(k_2d_2 + \frac{k_1}{k_2} \right) + \frac{1}{2} \left( k_2 - k_1 \right) \sin\left(k_2d_2 \right) \right]$$

where, the electric field $E_k$ is periodic with lattice constant $\Lambda$ and $K$ is called the Bloch wavevector. In order to get the bandgap, $E_k$ and $K$ must be found as function of frequency $\omega$ and wavevector $\beta$. By applying the periodicity condition to equation (7) and solving for eigenvalue of matrix we can have eigenvalue equation of translation matrix and their corresponding eigenvectors as follows

$$e^{iK\Lambda} = \frac{1}{2} \left[ (A + D) \pm \sqrt{\left[ \frac{1}{2} \left( A + D \right) \right]^2 - 1} \right]$$

$$\begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = \begin{pmatrix} B \\ e^{iK\Lambda - A} \end{pmatrix}$$

The Bloch waves that results from equation (13) can be considered as eigenvectors of translational matrix and eigenvalues are given by equation (12). Using equation (12) we can obtain a dispersion relation between frequency and tangential component of the wavevector and Bloch wavevector $K$. This dispersion relation defines the band structure of periodic dielectric media i.e. 1D photonic bandgap and expressed as

$$KA = \cos^{-1} \left[ \frac{1}{2} \left( A + D \right) \right]$$

The solution of above equation for $K$ is real in the regimes $\left| \frac{1}{2} (A + D) \right| < 1$, which are corresponding to the propagating Bloch waves (pass band) and when $\left| \frac{1}{2} (A + D) \right| > 1$, the values of $K$ become complex which consists of an imaginary and a real part corresponding to the evanescent and propagating Bloch waves. However the regimes $\left| \frac{1}{2} (A + D) \right| = 1$, defines the band edges.

The final expression for Bloch wave which completes the solution in the $n_1$ layer of the $n^{th}$ unit cell is expressed as

$$E_k(x) e^{-ikz} = \begin{pmatrix} a_0 e^{-ikL} + b_0 e^{ikL} \end{pmatrix} e^{ik(x-nA)}$$

This above equation contains all the information about photonic crystal eigenmodes which describes the properties of Bloch waves where $a_0$ and $b_0$ are amplitudes of electric field defined in equation (13).

At the band edges the reflectivity can be expressed as
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\[
|\rho_n|^2 = \frac{|C|^2}{|C|^2 + \left( \frac{1}{N^2} \right) \sinh \text{Im}(K) \Lambda}
\]  

(16)

Here, \( N \) is the number of bilayers for TE polarization condition. The values of Bloch wavevector in photonic bandgap becomes complex. Therefore the expression for reflectivity is given as

\[
|\rho_n|^2 = \frac{|C|^2}{|C|^2 + \left( \frac{\sinh \text{Im}(K) \Lambda}{\sinh \text{Im}(K) \Lambda} \right)}
\]  

(17)

4. Results and discussion

The systems which exhibit photonic bandgap analogous to the semiconductor bandgap in the electronic states of periodic solids have been the subject of intense investigations. The allowed and forbidden photonic bandgaps in the frequency spectrum results due to a periodic modulation of the refractive index, frequency, lattice constant and incident angle of light. Here, we had carried the analysis of the optical properties of photonic crystals by using the transfer matrix method. The computation carried out through TMM is efficient and accurate. The complexity in obtaining the results has been minimized by using MATLAB software.

To design 1D photonic crystal the appropriate selections of optical parameters are important to realize the variation in the bandwidth and the position of the photonic bandgaps. To see these effects some parameters such as angle of incidence, lattice constant and refractive index contrast have been numerically analyzed and graphically presented. In Fig. 2a the Bloch wavevector has been revealed as a function of normalized frequency (in the units of \( \omega d/2 \pi c \)). The variation in Bloch wavevector with respect to frequency which provides allowed and forbidden bandgap has been analyzed for different lattice constants. The result shows that the lattice constant of higher value the forbidden gap is narrower. The reduction of 66.67% of lattice constant results in an increase of 66.12% of forbidden bandgap, this is due to increase in wavevector. The normalized frequency shift is 0.142 for change in lattice from 1.5 micron to 1 micron, here, the dispersion relation is observed for first Brillouin zone [0, \( \pi \)]. In Fig. 2b the refractive indices of consecutive layers are considered to be different of porous silicon layers which can be achieved by electrochemical etching of silicon wafer in HF based electrolyte [19-21]. In the first case, we had kept refractive indices difference to be 1.3, which results in a broader forbidden bandgap. The results divulges that with the decrease in difference of a refractive indices of the periodic layers decreases the forbidden bandgap. The maximum forbidden bandgap achieved in both cases is 0.161 while the minimum forbidden bandgap is found to be 0.027. The results shown in Fig. 2a and 2b have been obtained for incident angle \( \theta = 45^\circ \).

This article reports on the numerical analysis of optical properties of 1D porous silicon photonic crystal by considering reflectance as a function of angle of incidence. For the analysis and optimization of optical and physical parameters of 1D photonic crystals, the reflectance with respect to phase shift have been numerically obtained. The results showed in Fig. 3 reveals the variation in the forbidden bandwidth and reflectance as function of a frequency. In Fig. 3 it is clearly observable that at first forbidden bandwidth 100 % reflectance is achieved for the assumed incidence angle. Further, it is observed that the increase in a frequency reduces the reflectance for the incidence angle of 20\(^\circ\) and 40\(^\circ\), but for the angle of incidence 80\(^\circ\) the reflectance seems to be almost similar for the corresponding forbidden bandwidths. Hence, 1D PCs based on dielectric multilayer can be used as planar antennae, in which complete reflectance at desired frequencies is achieved and, due to the photonic bandgap, the emission efficiency is strongly enhanced compared with ordinary planar antennas whose metallic substrates absorb much energy.

![Fig. 2. The photonic band structure i.e. Bloch wavevector as function of normalized frequency of 1D Porous Silicon photonic crystal. (a) photonic band structure for varying lattice constant and (b) photonic band structure for different refractive index contrast, with fixed incident angle (\( \theta = 45^\circ \)).](image-url)
Photonic crystal application for vertical cavity surface-emitting laser (VCSEL), has been recognized as an important light source for optoelectronic systems. Therefore, it is necessary to understand the physical phenomena of a 1D photonic crystal. Hence, we had computed the photonic bandgap along with reflectance for the varying refractive indices, lattice constant and incidence angle the results are given in Fig. 4. The stop bandwidth observed to be increasing nonlinearly with the increase in incidence angle, while the stop bandwidth decreases linearly with the increase in a lattice constant and refractive index (n₁). The value of effective refractive index becomes closer to the refractive indices of the constituent layers in which photons are scattered between the dielectric layers. The significant variation of forbidden bandwidth from 0.194 to 0.112 revealed in Fig. 4b corresponds to refractive index contrast 1 and 1.5 respectively has been observed. Figure 4a, 4c and 4b shows dependence of forbidden bandgap on lattice constant and incidence angle. These results are very useful for designing the desired frequency Bragg reflector.

The dispersive properties in 1D porous silicon photonic crystal has been simulated numerically by considering reflectance as a function of angle of incidence and frequency. The proposed structure is proficient to reflect a large portion of electromagnetic wave for TE polarizations and for the incident angles of range 10° to 90° which is depicted in Fig. 5 and Fig. 6. The Fig. 5 reveals the variation in reflectance for varying incidence angle and frequency in 1D PC with lattice constant assumed to be 1.5 micron. From Fig. 5 it is observed for wide range of frequency the 100% reflectance has been achieved at high incidence angle. Outside the forbidden bandwidth the reflectance oscillates as the frequency varies. This oscillation phenomenon originates from the fact that the electromagnetic field changes periodically as a function of the frequency. These features make this an attractive structure in the design of high-reflectance coatings.

In Fig. 6 the lattice constant assumed to be 1 micron, the reduction in lattice constant dimension results in a major shift of photonic bandgap with respect to frequency. To observe one complete reflectance spectrum in this case the frequency range has been increases up to 0.7. The incidence angle analysis has been carried out in a same manner. Here, the lattice constant discloses major shift in the forbidden bandgap and in this bandgap a high reflectance is achieved due to a Brag reflection for a periodic λ/4 reflector. The measured curves follow the predictions, the maximum reflectance for varying incidence angle and frequency in a 1D photonic crystal. The analysis of optical properties such as reflectance is essential for the optimization of physical parameters such as lattice constant, incidence angle and frequency for the realization of scope of porous silicon based photonic crystals in advent applications in optical components.
5. Conclusions

There were presented a detailed theoretical study of one-dimensional photonic bandgap structures, and the corresponding reflectance spectra are obtained by using transfer matrix method simulations. A better analysis of dispersion relation of 1D photonic crystal for optical and physical parameters has been presented. The forbidden bandwidth and reflectance has been deduced as a function of lattice constant, incidence angle and refractive index contrast. The investigation of these significant parameters shows that the first frequency forbidden bandwidth varies from 0.11 to 0.32. The results reveal that 100% reflectance has been achieved for the first stop band, which is due to the exponential decrease in electric field amplitude within the photonic band structures.

References


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