Space-time clutter model for airborne bistatic radar with non-Gaussian statistics*

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(Received September 21, 2007)

Abstract: To validate the potential space-time adaptive processing (STAP) algorithms for airborne bistatic radar clutter suppression under nonstationary and non-Gaussian clutter environments, a statistically non-Gaussian, space-time clutter model in varying bistatic geometrical scenarios is presented. The inclusive effects of the model contain the range dependency of bistatic clutter spectrum and clutter power variation in range-angle cells. To capture them, a new approach to coordinate system conversion is initiated into formulating bistatic geometrical model, and the bistatic non-Gaussian amplitude clutter representation method based on a compound model is introduced. The veracity of the geometrical model is validated by using the bistatic configuration parameters of multi-channel airborne radar measurement (MCARM) experiment. And simulation results manifest that the proposed model can accurately shape the space-time clutter spectrum tied up with specific airborne bistatic radar scenario and can characterize the heterogeneity of clutter amplitude distribution in practical clutter environments.

Keywords: airborne bistatic radar, clutter model, geometry, non-Gaussian.

1. Introduction

Due to having many advantages over their monostatic counterparts[1], bistatic airborne radars are desirable surveillance systems in military applications. However, to detect stressing targets in interference backgrounds, airborne bistatic radar systems must be able to cope with the complicated, heterogeneous bistatic ground clutter returns compared with monostatic cases. Meanwhile, space-time adaptive processing (STAP)[2] is regarded as a leading technology candidate for improving the detection performance of bistatic airborne radars, because it can simultaneously distinguish targets from clutter in angle and Doppler domains.

However, most STAP algorithms based on clutter covariance matrix estimation generally assume that the secondary data and the primary data are statistically independent and identically distributed (IID). To the contrary, the measured bistatic clutter data are found heterogeneous (i.e. not IID) in terms of both clutter spectrum and amplitude. On one hand, for the separate motion effects of the transmitter and the receiver in airborne bistatic radar configurations, the bistatic clutter spectrum displays range dependence; on the other hand, the amplitude statistical distribution of bistatic clutter is not homogeneous because of spatial variation in practical scenarios about clutter reflectivity, terrain, shadowing and obstruction, and so forth. In such cases, the primary data and the secondary data do not share the same covariance matrix, and the bistatic STAP performance suffers a remarkable degradation.

Currently, most algorithms of bistatic STAP are mainly concerned with the problem of mitigating the clutter spectral range nonstationarity [3–4], while the amplitude heterogeneous or non-Gaussian clutter suppression problem is seldom considered in bistatic applications. In this paper, our objective is to elaborate both spectral and amplitude heterogeneous bistatic clutter model for potential STAP algorithms development and performance assessment. The example of its applications is used to study a class of recently proposed knowledge-aided STAP (KA-STAP) algorithms[5] for airborne bistatic radar clutter cancellation.

* This project was supported by the National Defense Advanced Research Foundation of China (51407020304DZ0223).
2. Geometrical model for varying airborne bistatic radar scenarios

To get the descriptions of bistatic geometry, Klemm\[^7\] classified bistatic scenarios into four types based on the relationship between the moving directions of bistatic platforms (i.e. aligned, parallel, orthogonal, and hybrid); Wang\[^8\] gave the bistatic geometrical model in the case of aligned flight. However, these models can only be used for a few special kinds of bistatic scenarios, the flexible and generalized bistatic geometry formulation, which can be applied in any bistatic scenario configurations, has not been reported till now. In the section, a coordinate system conversion method is proposed to set up a generalized bistatic geometrical model, and we can see its convenience for varying bistatic scenario modelling.

A typical airborne bistatic configuration is presented in Fig.1. The receiver is at point \(R_X\) at height \(H_R\) above the \(X - Y\) ground plane, and the transmitter is at the point \(T_X\) at height \(H_T\). The receiver moves in \(\delta_R\) direction at speed \(v_R\) and the transmitter moves in \(\delta_T\) direction at speed \(v_T\). A transmitter pulse hits the ground at a stationary point \(P\) after passing the transmit slant range \(R_{TS}\) and the reflected pulse is collected by the receiver after passing the receiver slant range \(R_{RS}\). The sum of two slant ranges represents the bistatic range sum \(R_{Sum}\), and the set of points having same range sums forms a group of range ellipsoids. The angles \(\varphi_R\) and \(\varphi_T\) are the azimuth angles of the scattering point \(P\) measured with respect to the bistatic baseline, and the angles \(\theta_R\) and \(\theta_T\) are elevation angles denoted by the angles between the slant range and the ground surface. The angles \(\psi_R\) and \(\psi_T\) are cone angles measured with the clutter scattering point to the moving direction of radar platforms.

Two coordinate systems are exploited in Figure 1: one is the ground-coordinate system \(O - XYZ\), and its origin is set at the point \(O\) which is the projecting point of \(R_X\) on the ground surface; the other is the local-coordinate system \(O' - X'Y'Z'\), which assumes its \(O'X'\) axis is on the bistatic baseline and its origin at the middle of points \(R_X\) and \(T_X\). The angles \(\alpha_A\) and \(\alpha_E\) are the rotation angles of azimuth and elevation direction from the \(OX\) axis to the \(O'X'\) axis, and we define the angle \(\alpha_R\) as the surplus angle measured by the rotated \(OX\) axis with respect to \(O'X'\) axis. Thus, through a series of transformation operations, the bistatic scenarios represented by the ground-coordinate system and the scenarios represented by the local-coordinate system can be mutually conversed.

![Fig.1 The geometry of an airborne bistatic radar](image)

2.1 Bistatic coordinate system conversion method

Image a virtual bistatic scenario where the centre of range ellipsoid is at the point \(O\) and the bistatic baseline coincides with the \(OX\) axis in the ground-coordinate system. In such a case, the local-coordinate system and the ground-coordinate system are absolutely superposed. Thus, the range sum ellipsoid of the virtual bistatic scenario represented by the local-coordinate system is expressed as

\[
\frac{x'^2}{a^2} + \frac{y'^2}{b^2} + \frac{z'^2}{c^2} = 1
\]  

(1)

where \(a, b\) and \(c\) denote the axes of ellipsoid and set \(c = b\), the distance between its focuses is equal to the length of baseline \((BL)\), and the range sum is equal to \(R_{Sum}\). The relationships of axes, \(R_{Sum}\) and \(BL\) can be expressed as

\[
a = \frac{R_{Sum}}{2}, \quad b = \sqrt{a^2 - (BL/2)^2}
\]  

(2)
Our object is to transform the supposed bistatic scenario described by the local-coordinate system to practical bistatic scenario in the ground-coordinate system shown in Fig. 1. The equivalent problem can be looked as making the major axis of assumed ellipsoid coincided with the bistatic baseline of range ellipsoid in practical scenario, which is illustrated in Fig. 2.

\[
\begin{bmatrix}
  x_1 & y_1 & z_1 \\
  x'_{1'} & y'_{1'} & z'_{1'}
\end{bmatrix} = \begin{bmatrix}
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  x_0 & y_0 & z_0 & 1
\end{bmatrix} \cdot T_{Sht} \cdot \begin{bmatrix}
  \alpha_A \\
  -\sin \alpha_A & \cos \alpha_A & 0 \\
  0 & 1 & 0 \\
  0 & 0 & 1
\end{bmatrix} \quad (3)
\]

where \((x_1,y_1,z_1)\) is the new coordinate in \(O_1 - X_1Y_1Z_1\) after shifting. This operation can be noted as \(T_{Sht}(x_0,y_0,z_0)\). Then, three rotation steps are required to coalign the baseline of the supposed scenario with that of real scenario: (1) rotating \(\alpha_A\) angle around the fixed axis \(O_1Z_1\); (2) rotating \(\alpha_E\) angle around the fixed axis \(O_2Y_2\); (3) rotating \(\alpha_R\) angle around the fixed axis \(O_3X_3\). Here, the numerical subscripts denote the indexes of new coordinate systems resulting from the previous transformation and the axes \(O_1Z_1, O_2Y_2, O_3X_3\) denote the axes \(Z, Y, X\) in corresponding coordinate system. These rotations are represented by the following expressions respectively

\[
T_{RotZ}(\alpha_A) = \begin{bmatrix}
  \cos \alpha_A & -\sin \alpha_A & 0 & 0 \\
  \sin \alpha_A & \cos \alpha_A & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix}
\]

where \(z_2 = z_1\), and which represents the operation of rotating \(\alpha_A\) angle around axis \(O_1Z_1\) and can be noted by \(T_{RotZ}(\alpha_A)\).

\[
T_{RotY}(\alpha_E) = \begin{bmatrix}
  \cos \alpha_E & 0 & -\sin \alpha_E & 0 \\
  0 & 1 & 0 & 0 \\
  \sin \alpha_E & 0 & \cos \alpha_E & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix}
\]

where \(y_3 = y_2\), and which represents the operation of rotating \(\alpha_E\) angle around axis \(O_2Y_2\) and can be noted by \(T_{RotY}(\alpha_E)\).

\[
\begin{bmatrix}
  x_{1'} & y_{1'} & z_{1'}
\end{bmatrix} = \begin{bmatrix}
  x_3 & y_3 & z_3
\end{bmatrix} \cdot T_{RotX}(\alpha_R)
\]

where \(x' = x_3\), and which represents the operation of rotating \(\alpha_R\) angle around axis \(O_3X_3\) and is noted by \(T_{RotX}(\alpha_R)\). Based on Eqs. (3) and (4), the total transformation can be expressed by

\[
[x \ y \ z] = \begin{bmatrix}
  x_{1'} & y_{1'} & z_{1'}
\end{bmatrix} \cdot T_{RotX}(\alpha_R) \cdot T_{RotY}(\alpha_E) \cdot T_{RotZ}(\alpha_A) \cdot T_{Sht}(x_0,y_0,z_0)
\]

and the inverse transformation is

\[
[x \ y \ z] = (T_{RotZ}(\alpha_A) \cdot T^{-1}_{RotX}(\alpha_R) \cdot T^{-1}_{RotY}(\alpha_E) \cdot T^{-1}_{Sht}(x_0,y_0,z_0) \cdot [x \ y \ z])
\]

where \(T^{-1}(\cdot)\) denotes the inverse of the transformation matrix.
2.2 Bistatic ellipse range bin

Substituting Eq. (6) into Eq. (1), the bistatic range ellipsoid equation for practical bistatic scenario is represented in the ground-coordinate system. Then, dictating $z = 0$, the expressions for elliptic range bins on the ground can be represented by
\[
\frac{(x - x_{eo})^2}{A^2} + \frac{(y - y_{eo})^2}{B^2} = 1 \tag{7}
\]
where $(x_{eo}, y_{eo})$ is the central point of the ellipse range bin, $A$ and $B$ are the semi-axes of the ellipse range bin on the ground after coordinate system conversion. Figure 3 shows the bistatic range bin of the virtual scenario and the practical scenario in the two coordinate systems, in which the bistatic range bin is at 150 km and the baseline is 100 km.

![Figure 3](image)

(a) The virtual scenario  (b) The practical scenario

3. Non-Gaussian clutter model for airborne bistatic radar

Consider a bistatic airborne radar system with $M$ spatial channels receiving $N$ pulses corresponding to the $N$ coherent pulses returning from a given range cell as shown in Fig.1. The true clutter covariance matrix $R_k = E[q_lq_l^H]$ for the $k$-th bistatic range bin is unknown and generally replaced by its maximum likelihood estimate (MLE) $\hat{R}_k$ which is defined as
\[
\hat{R}_k = \frac{1}{L} \sum_{l=1}^{L} q_lq_l^H, \quad q_l = c_l + n_l \in C^{MN \times 1} \tag{8}
\]
where $q_l$ is the space-time sample data vector which consists of the correlated clutter vector $c_l$ and the uncorrelated receiver noise vector $n_l$ with noise power $\sigma_n^2$, and $L$ is the number of the IID secondary data samples and is not less than $2MN$ for the desirable performance. Under such a condition, $\hat{R}_k$ is asymptotically equal to $R_k$ as follows
\[
E[\hat{R}_k] = \frac{1}{L} \sum_{l=1}^{L} E[q_lq_l^H] = \frac{1}{L} \sum_{l \neq k}^{L} R_l = R_k \tag{9}
\]
where $R_l = R_k \forall l$, and $l \neq k$.

However, for heterogeneous or non-Gaussian clutter, the above equation is not the case because of the clutter power variation in range and angle. According to the compound model in Ref. [6], the elements of single channel compound-Gaussian clutter vector $c$ can be described as follows
\[
c(n) = z(n)g(n), \quad n = 1, ..., N \tag{10}
\]
where $g(n)$ is a complex correlated, range and angle independent Gaussian process corresponding to time $n$ with zero mean and unit variance, and $z(n)$ is a statistically independent, slowly temporal varying, modulating process which represents the clutter amplitude variation with range and/or angle. For $M$ spatial channels, the clutter in each channel is still dictated by Eq. (10). Moreover, for the long temporal coherence of $z(n)$, it is a random variable, $z(k; \varphi, \theta)$ with respect to range and/or angle but constant over time. By discretizing the range bin with $N_p$ angle cells, Eq. (10) can be reformulated as
\[
c_i(n) = z_i(k; \varphi, \theta)g(n), \quad n = 1, ..., N, \quad i = 1, ..., N_p \tag{11}
\]
Thus, the space-time vector $z_{i,MN\times1}$ introduced by the modulating process of the $i$-th clutter cell $(\varphi_i, \theta_i)$ is
\[
z_{i,MN\times1} = z_k(j_i, q_i)[1 \ 1 \ ... \ 1]^T
\]
\[i = 1, ..., N_p \tag{12}
\]
where $[1 \ 1 \ ... \ 1]^T$ is the $MN \times 1$ dimension column vector, thus $z_k(\varphi_i, \theta_i)$ is a positive constant in term of each clutter cell, but is a random sequence meeting specific probability density distribution over range and angle. Therefore, the clutter statistics for each cell is typically Gaussian, but the variance changes randomly from one range-angle cell to another according to a non-Gaussian probability density distribution of $z(k; \varphi, \theta)$. Then, according to Eq. (8)–Eq. (12), the asymptotic MLE of $R_k$ of non-Gaussian clutter is
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\[ E[R_b] = \bar{p}_c R_g + s^2_{R MN}, \quad \bar{p}_c = E \left[ \frac{1}{L} \sum_{l=1}^{L} \sum_{i=1}^{N_p} p_{c,li} \right] \] (13)

where \( p_{c,li} \) is the power of clutter patch, \( R_g \) is the covariance matrix of the Gaussian process \( g(n) \) with unit variance, and \( \bar{p}_c \) is the mean clutter power over range and angle.

In the case of the airborne bistatic radar scenarios, based on R. Klemm’s bistatic clutter model [2], the bistatic clutter response of the \( k \)-th range bin at the \( m \)-th sensor at the \( n \)-th pulse is

\[
q_{nm,k} = \int_0^{2\pi} \sigma^2_{c,k}(\varphi_{R,k}, \theta_{R,k}) \Phi_m(f_{S,k}(\varphi_{R,k}, \theta_{R,k})) \times \\
\Phi_n(f_{D,k}(\varphi_{R,k}, \theta_{R,k})) |df_{R,k}\]

\[ m = 1, 2, ..., M, \quad n = 1, 2, ..., N \] (14)

where \((\varphi_{R,k}, \theta_{R,k})\) is the receive azimuth-elevation angle pair of clutter patch, \( \sigma^2_{c,k}(\varphi_{R,k}, \theta_{R,k}) \) is the received clutter power of clutter patch, \( \Phi_m(f_{S,k}) \) and \( \Phi_n(f_{D,k}) \) are the spatial and temporal phase terms as follows

\[
\Phi_m(f_{S,k}) = \exp[j2\pi(m-1)f_{S,k}(\varphi_{R,k}, \theta_{R,k})] \\
\Phi_n(f_{D,k}) = \exp[j2\pi(n-1)f_{D,k}(\varphi_{R,k}, \theta_{R,k})] \] (15)

where \( f_{S,k}(\varphi_{R,k}, \theta_{R,k}) \) and \( f_{D,k}(\varphi_{R,k}, \theta_{R,k}) \) are the spatial frequency and the Doppler frequency of each clutter patch seen from the receiver as follows

\[
f_{S,k}(\varphi_{R,k}, \theta_{R,k}) = (d/\lambda) \cos \varphi_{R,k} \cos \theta_{R,k} \\
f_{D,k}(\varphi_{R,k}, \theta_{R,k}) = (v_T T/\lambda) \cos \varphi_{T,k} \cos \theta_{T,k} + \\
(\gamma_{T} T/\lambda) \cos \varphi_{R,k} \cos \theta_{R,k} \] (16)

where \( d \) is the sensor spacing, \( \lambda \) is the wavelength, and \( T \) is the pulse repetition interval (PRI). Based on Skolnik’s radar equation, the clutter power term contained in Eq. (14) follows as

\[
\sigma^2_{c,k}(\varphi_{R,k}, \theta_{R,k}) = \kappa_C G_T(\varphi_{T,k}, \theta_{T,k}) g_{R}(\varphi_{R,k}, \theta_{R,k}) \gamma_{C,k}(\varphi_{R,k}, \theta_{R,k}) \\
R_{sum,k}^2 \] (17)

where \( \kappa_C \) is a constant related with radar system parameters, \( G_T(\cdot) \) is the transmit antenna gain, \( g_{R}(\cdot) \) is the receive antenna gain, \( R_{sum,k} \) is the bistatic range sum, and \( \gamma_{C,k}(\varphi_{R,k}, \theta_{R,k}) \) represents the reflectivity of the clutter patch at range bin \( k \). Hence, the elements of the bistatic clutter covariance matrix are calculated as follows

\[
r_{lp} = E[q_{nm} q_{iq}^*] \\
l = (m-1)N + n, \quad m = 1, ..., M, \quad n = 1, ..., N \] (18)

\[ p = (q-1)N + i, \quad q = 1, ..., M, \quad i = 1, ..., N \]

Note that the bistatic clutter reflectivity in Eq. (17), \( \gamma_C(\varphi_{R,k}, \theta_{R,k}) \) determines the clutter amplitude statistical variation over range and angle. Because the exact statistical distribution family of the bistatic clutter reflectivity has been in study [9], without loss of generality of the proposed modelling method, suppose the range-angle variation of clutter reflectivity follows the \( K \) distribution PDF which is

\[
f(\gamma) = \frac{2}{\alpha \Gamma(v+1)} \left( \frac{\gamma}{2\alpha} \right)^{v+1} K_{v-1}\left( \frac{2\alpha}{\gamma} \right), \quad \gamma \geq 0 \]

\[
E[\gamma^k] = (2\alpha)^k \left[ \Gamma\left( \frac{k}{2} + v + 1 \right) / \Gamma(v+1) \right] \] (19)

where \( \gamma \) is the bistatic clutter reflectivity for each clutter patch, \( v \) is the shape parameter, \( \alpha \) is the scale parameter, \( \Gamma(\cdot) \) is the gamma function, \( K_{v-1}(\cdot) \) is the second type modified Bessel function, and the moments are given by \( E[\gamma^k] \). Such selection of the \( K \) distribution represents a likely choice for overall surveillance [10].

4. Simulation results

4.1 Geometrical model validation

In order to demonstrate the validation of the proposed bistatic geometrical model in Section 2, two bistatic scenarios respectively offered by R. Klemm [7] (Scenario 1) and used in MCARM measurement [3] (Scenario 2) are employed in the simulation. In the two bistatic configurations, the simulated clutter trajectories on the angle-Doppler plane are identical to the corresponding results in references.

In Scenario 1, the receiver’s velocity is 90 m/s while the transmitter’s velocity is 90 m/s, with an offset angle of 90°. The radar wavelength is 0.03 m. The heights of both the receiver and the transmitter are 1 km, and the baseline separation is 2 km. The pulse repetition frequency (PRF) is assumed to be 12 kHz. And the test range bins are 3 km, 5 km and 10 km respectively.

In Scenario 2, both the receiver and the transmitter are moving at the velocity of 200 m/s, with an
offset angle of 135°. The radar operation frequency is 1.24 GHz. The receiver height is 6 km, the transmitter height is 9 km, and the baseline length is 100 km. And the test range bins are 150 km, 230 km and 310 km respectively.

Figure 4 plots the angle-Doppler trajectories of the two cases by exploiting the proposed bistatic geometrical model, which shows the same results as the conclusions in references. Due to the dispersion effects of the angle-Doppler trajectories corresponding to distinct range bins, the range dependence or heterogeneous characteristic of bistatic spectrum appears on the angle-Doppler plane.

4.2 Non-Gaussian amplitude clutter simulations

In the simulations, the variation in clutter reflectivity of clutter patches is modelled by a compound-Gaussian process with $K$ amplitude distribution and Gaussian spectral shape. Figure 5 illustrates the amplitude PDFs of clutter, in which the shape parameters $v$ are set by 0.5, 0.8, 1.5, and 3 respectively, the scale parameter $a$ in each case can be drawn by the moment expression of Eq. (19), and the variance of spectral bandwidth $\sigma_b^2$ is 30 Hz.

![Fig.4](image1.png)

**Fig.4** The bistatic range bins in the two coordinate systems

![Fig.5](image2.png)

**Fig.5** The amplitude PDF of bistatic clutter reflectivity with $K$ probability density
4.3 Space-time bistatic clutter spectrum

Figure 6(a) and Figure 6(b) present the space-time minimum variance (MV) bistatic clutter spectrum of Scenario 1 and Scenario 2 respectively. In these cases, the transmit and receive antenna array are both uniform linear array with 16 elements, each receive spatial channel collects 32 coherent pulse, the PRFs are 12 kHz and 8 kHz for the two scenarios respectively, and the range ambiguity is not considered in the simulation.

As we can see in the Fig. 6, the tracks of clutter power spectra in these scenarios are consistent with that of angle-Doppler trajectories shown in Fig.4. The bistatic spectrum exhibits heterogeneous on angle-Doppler plane, which proves that bistatic clutter not only is range-dependent but also is angle-dependent.

5. Conclusion

In this paper, a space-time bistatic clutter model with non-Gaussian amplitude statistics for airborne bistatic radar is proposed. Such bistatic clutter model captures the heterogeneous characteristics both in terms of spectrum and amplitude for pragmatic bistatic clutter environments. The suggested coordinate system conversion method for bistatic geometrical scenarios modelling not only provides more accurate descriptions of range-dependent spectra but also is convenient to model the diverse airborne bistatic radar scenarios. The non-Gaussian amplitude distribution of clutter is modelled by the bistatic clutter reflectivity variation in range-angle clutter patches based on a compound-Gaussian model. The proposed model can be used to evaluate the performance of bistatic STAP algorithms in non-Gaussian or compound-Gaussian clutter environments.

References

[10] Gini F. Sub-optimum coherent radar detection in a mixture of K-distributed and Gaussian clutter. IEE Proc. F,
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