Performance Study of OFDMA vs. OFDM/SDMA

Zihhua Guo and Wenwu Zhu
Microsoft Research, Asia
3F, Beijing Guo, No. 49, Zhichun Road
Haidian District, Beijing 100080, P. R. China
{zhguo, wwzhu}@microsoft.com

Abstract: In this paper, we first consider the OFDM/SDMA system which admits several OFDM users simultaneously by reusing the same frequency band. With the linear Minimum Mean Squared Error (MMSE) receiver, it has been shown that such a system can improve the system capacity significantly compared to the single antenna OFDM system. However, the number of required linear filters is huge. Thus, in this paper, we propose two reduced-complexity MMSE receivers for OFDM/SDMA system; namely, interpolated MMSE and partial MMSE. We demonstrate that both methods perform much better than the previously proposed reduced-complexity receiver. Other than OFDM/SDMA, an alternative to support multiple users with high spectrum efficiency is to use Orthogonal Frequency Division Multiple Access (OFDMA) with high-level modulation. With both theoretical analysis and simulation, we find that the relative BER performance of these two systems depends on the receiver design of OFDM/SDMA. In particular, when the MMSE receiver is used, OFDM/SDMA is usually worse than or comparable to OFDMA. However, the former has the potential to outperform the latter because when the optimal (and the most complicated) ML detector is used, it is superior to the latter. On the other hand, OFDMA has the advantage of simple receiver structure.

I. INTRODUCTION

In recent years, there has been substantial research interest in applying Orthogonal Frequency Division Multiplexing (OFDM) to high-speed wireless communications due to its advantage in mitigating the severe effects of frequency-selective fading [1]. It increases the symbol duration by dividing the entire channel into many narrow orthogonal subchannels and transmitting data in parallel. At the same time, to increase the system capacity and cancel the cochannel interference, wireless communications with antenna diversity have been studied widely [2]. In particular, Space Division Multiple Access (SDMA) which employs a receiver antenna array can improve the spectrum efficiency significantly by allowing several users to reuse the same spectrum. It separates the users with their spatial signature. An array with P receiver antennas essentially can support P simultaneous users with approximate diversity order of 4. In this paper, we first consider the combined SDMA and OFDM system. On the receiver side of such a system, a linear MMSE filter operating on the signals in P antennas is usually used to suppress the multi-user interference (MUI) and estimate each user’s data for each subcarrier [2-4]. However, the MMSE filter needs direct matrix inversion (DMI) and the DMI is done for all the K subcarriers. This makes the OFDM/SDMA receiver very complicated.

In order to decrease the complexity of the MMSE receiver, in [3, 4], a reduced-complexity grouped MMSE was proposed. However, we will show that it saturates quickly and its performance degradation is too much compared to DMI-MMSE. In this paper, two other reduced-complexity receivers: interpolated MMSE and partial MMSE will be presented. It is demonstrated that our proposed interpolated filter significantly outperforms the previous grouped approach. However, it still saturates at high SNR. On the other hand, the partial MMSE is very effective in reducing complexity and maintaining good performance without saturation.

Other than OFDM/SDMA, an alternative to support multiple users with high spectrum efficiency is to use OFDMA [6] with high-level signal constellation. For instance, with 4 receive antennas, up to 4 users can reuse all the K subcarriers with BPSK in OFDM/SDMA. These users with the same throughput can also be supported in OFDMA with 16QAM by allocating K/4 subcarriers to each user. In this sense, there is no multiple access interference (MAI) between users and every user can make full use of the antenna diversity with an order of 4. Meanwhile, the demodulation of OFDMA signal in each antenna is very simple by taking only the N-point FFT, which is much simpler than the OFDM/SDMA receiver as we mentioned above. The signal in every antenna branch is then combined with Maximum Ratio Combining (MRC). A question now immediately arises: is the performance of OFDMA comparable to that of OFDM/SDMA? In this paper, we will address this question. By both analysis and simulation, it is shown that OFDM/SDMA has the potential to outperform OFDMA because when the optimal or the most complicated ML detector is used, it is superior to OFDMA. However, when the DMI-MMSE is used, SDMA is usually worse than or comparable to OFDMA.

The rest of the paper is organized as follows. In Section II, the OFDM/SDMA and OFDMA system models are given. In Section III, the receiver structure for OFDM/SDMA and OFDMA are presented and compared. Finally, the conclusions are drawn in Section IV.

II. SIGNAL MODEL

Assume that there are N mobile stations with single antenna and M receiving antennas in the base station. The channel between the i-th user and j-th receiver is assumed to be a L-path channel, which is given by

\[
\begin{bmatrix}
H_{ij} (f)
\end{bmatrix} = H_{ij} + N_{ij}(f)
\]
\[ h_{i,j}(t) = \sum_{l=1}^{L} \beta_{i,j}(l) \delta(t - \tau_{i,j}^{(l)}) \]  

where \( \beta_{i,j}(l) \) and \( \tau_{i,j}^{(l)} \) are the complex path gain and delay of the \( l \)-th path. The path gain \( \beta_{i,j}(l) \) between different antennas and paths is assumed to be independently and identically distributed (i.i.d) and \( \sum_{l=1}^{L} E[|\beta_{i,j}(l)|^2] = 1 \). Now, assume that the entire channel for an OFDM system is divided into \( K \) subcarriers. It follows that the gain of the \( k \)-th subcarriers between the \( i \)-th user and \( j \)-th receiver is given by:

\[
H_{i,j}(k) = \sum_{l=1}^{L} \beta_{i,j}(l) \exp(-j2\pi(k-1)f_s\tau_{i,j}^{(l)})
\]

\[ = w_{i,j}^{(k)} \ H_{i,j}
\]

where \( f_s \) is the bandwidth of the subcarrier.

\[
w_{i,j}^{(k)} = \begin{bmatrix} e^{-j2\pi(k-1)f_s\tau_{i,j}^{(1)}} & \ldots & e^{-j2\pi(k-1)f_s\tau_{i,j}^{(L)}} \end{bmatrix}^T
\]

\[
h_{i,j} = \begin{bmatrix} \beta_{i,j}(1) & \ldots & \beta_{i,j}(L) \end{bmatrix}^T.
\]

Throughout this paper, we assume that the receiver knows the channel state information.

The received signal \( y[k] = [y_1[k] \ldots y_M[k]]^T \) in the \( M \) antennas for the \( k \)-th subcarrier is given by:

\[
y[k] = H[k]x[k] + n[k]
\]

where \( x[k] = [x_1[k] \ldots x_N[k]]^T \) is the \( N \) users’ signals in the \( k \)-th subcarrier. For simplicity, we have normalized the transmitted signal so that \( |x_i[k]|^2 = 1 \). Likewise, \( n[k] = [n_1[k] \ldots n_M[k]]^T \) is the complex noise vector with variance \( \sigma_n^2 = N_0/2 \) per element.

Finally, \( H[k] = [H_{i,j}(k)]_{i=1 \ldots N, j=1 \ldots M} \) is the channel between the \( N \) users and the \( M \) receiving antennas for the \( k \)-th subcarrier.

We now investigate the statistic characteristics of the subchannel gain. From (2), it is easy to see that \( H_{i,j}(k) \) is the summation of \( L \) i.i.d rotated complex Gaussian random variables. Thus, \( H_{i,j}(k) \) is still Gaussian and \( |H_{i,j}(k)|^2 \) is still exponentially distributed (hence, \( |H_{i,j}(k)|^2 \) is still exponentially distributed) with:

\[
E[H_{i,j}(k)^2] = 1, \ \text{var}(H_{i,j}(k)^2) = 1.
\]

For OFDMA system, each user is assigned a set of non-overlapped subcarriers. Therefore, the users’ signals are separated and no MAI is presented for each subcarrier. This results in the conventional simple OFDM receiver in the base station. The signals from \( M \) antennas will be MRC combined.

In order to achieve frequency diversity and interference diversity, the assigned subcarrier set to a specific user may be varied in both the time domain and the frequency domain [5]. At the same time, in order to get the same throughput as SDMA, a higher-level modulation scheme must be used.

### III. PERFORMANCE COMPARISON

#### A. MMSE Receiver for OFDM/SDMA

1) DMI-MMSE

In order to recover the signal in the receiver (base station), the linear MMSE filter is usually applied [2,3]. Because the subcarrier of OFDM is fully parallel, thus, the MMSE filter is applied in a per-carrier fashion. Assume the filter for the \( k \)-th subcarrier is given by \( A[k] \). That is, the MMSE estimation of the transmitted signal \( x[k] \) is given by:

\[
l[k] = A^H[k]y[k]
\]

where \( A[k] = R_{yy}^{-1}R_{yx} \) and

\[
R_{yy} = E[yy^H] = H[k]H^H[k] + 2\sigma^2 I_{MxM}
\]

with \( I_{MxM} \) denoting the \( M \times M \) identity matrix. Likewise,

\[
R_{yx} = E[xy^H] = H[k].
\]

From (7), we can see that the MMSE in each subcarrier requires a direct matrix inversion (DMI) with dimension \( MxM \). Note that the complexity of the matrix inversion is \( O(m^3) \), where \( m \) is the dimension of the matrix. Therefore, we can see that the complexity of the DMI-MMSE receiver is very high.

2) Grouped MMSE

In order to decrease the receiver complexity, there has been some work reported. In [3, 4], a reduced-complexity grouped MMSE receiver was proposed. It divides the \( K \) subcarriers into \( G \) groups. In each group, a filter is obtained by minimizing the summed MSE of all the subcarriers within this group and applies this filter to all the \( N/G \) subcarriers within this group. That is, the filter is to minimize:

\[
E\left[ \sum_{i=1}^{K} |y[k_0+i] - A^H[k]y[k_0+i]|^2 \right]
\]

where \( k_0+1 \) is the index of the first subcarrier within this group. It is easy to see that \( A \) is now given by:

\[
A = \left[ \sum_{i=1}^{K} H[k_0+i]H^H[k_0+i] + 2\frac{N\sigma^2 I_{MxM}}{G} \right]^{-1} \left[ \sum_{i=1}^{K} H[k_0+i] \right]
\]

For this grouped MMSE receiver, its performance depends heavily on the similarity of the gains of the subchannels within this group. Thus, the performance will degrade severely and saturate quickly when the frequency diversity is rich, as we will show in the following.
3) Interpolated MMSE
In order to overcome the drawbacks of the grouped MMSE, we propose using the linear interpolated MMSE filter. That is, we calculate one DMI-MMSE filter for each group. The filters between two successive DMI-MMSE filters are obtained by interpolation. The interpolation can be linear or nonlinear. Therefore, we can see that this approach also utilizes the frequency correlation between OFDM subcarriers. Both the grouped MMSE and the interpolated MMSE roughly reduce the complexity by a factor of $G$. In contrast to the grouped MMSE, we will show that the interpolated MMSE is more robust than the grouped MMSE. However, as shown in Fig. 1, the interpolated MMSE still saturates in the high SNR region and performs much worse than the DMI-MMSE.

4) Partial MMSE
From above discussion, we can see that both the grouped and interpolated MMSE is not robust to the frequency diversity. In order to reduce complexity and maintain the good performance, we here propose the partial MMSE receiver. Note that most of the complexity of DMI-MMSE comes from the inverse of the matrix with dimension $M$. Therefore, in our approach, instead of process the signals in the $M$ received antennas simultaneously, we divide the antennas into some small overlapping or non-overlapping sets. Then, the MMSE filter is calculated for each set. Without loss of generality, let $M=5$ and they are divided into two sets. They consist of antennas 1,2,3 and antennas 3,4,5, respectively. Now, we can obtain two MMSE estimates for $x[k]$, namely, $\hat{x}[k,1]$ and $\hat{x}[k,2]$. They are given by

$$\hat{x}[k,1] = A_H^I y[k,1]$$
$$\hat{x}[k,2] = A_H^I y[k,2]$$

where $y[k,1]=[y_1[k], y_2[k], y_3[k]]^T$ and $y[k,2]=[y_3[k], y_4[k], y_5[k]]^T$. $A_H^I$ and $A_H^I$ can be obtained as in (7)-(8) similarly. The final MMSE estimate of $x[k]$ is given by

$$\hat{x}[k] = \hat{x}[k,1] + \hat{x}[k,2].$$

Recall that the complexity of matrix inverse is $O(m^3)$. Thus, for this example, the complexity of partial MMSE is approximately 1/3 of the DMI-MMSE filter. The complexity reduction will be even larger when more receiving antennas are employed. For example, with $M=6$ and two small sets with 3 antennas in each set, the complexity will be reduced to 1/4. Note that the MMSE filter corresponds to the MRC combining [2]. Therefore, with the division into small sets, the MRC combination within each small group is maintained, but it may be destroyed between sets. However, we can imagine that the MRC coefficients between sets may not be altered too much compared to DMI-MMSE.

In Fig. 1, we show the performance of the above four MMSE filters. Here, the TU channel model is used [6]. There are totally 512 subchannels and each is with 10kHz bandwidth. The number of the receiving antenna is $M=4$ and two users with QPSK are simulated. The group size for interpolated and grouped MMSE is 8. The number of antennas in the two sets for partial MMSE is 3 and 2, respectively. We can see that our proposed interpolated MMSE filter significantly outperforms the grouped MMSE filter. But it still saturates quickly. The partial MMSE filter performs a little worse than the interpolated MMSE in the low SNR region. But the former performs much better than the latter in the high SNR region and it does not saturate. At the same time, partial MMSE reduces the complexity by a factor 2 and is 2 dB away from the DMI-MMSE filter.

![Fig. 1 Performance of different MMSE receivers for OFDM/SDMA](image)

B. The Optimal ML Receiver for OFDM/SDMA
As a benchmark, we now derive the BER performance of the Maximum Likelihood receiver for OFDM/SDMA. By referring to (4), we can see that the ML receiver is given by:

$$\hat{x}[k] = \min_{\hat{x}[k]} \left| y[k] - H[k] \hat{x}[k] \right|^2$$

(13)

In the following derivation, for simplicity, we assume BPSK is used. On occurring of an error event, the ML decoder decides erroneously in favor of a symbol set for $N$ users $e = e_1[k] e_2[k] ... e_N[k]$ assuming that $x = x_1[k] x_2[k] ... x_N[k]$ was transmitted. Then, the probability of the error decision is given by (note that the energy of the transmitted signal has been normalized to 1):

$$P(x \rightarrow e) = Q\left( \sqrt{d^2 (x,e) / (2N_0)} \right)$$

(14)

where

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp(-\frac{y^2}{2}) dy$$

$$d^2 (x,e) = \sum_{i=1}^{N} (x_i[k] - e_i[k])^2$$
When there is only one user in error, the product distance is given by:

\[ d^2(x, e) = 4 \sum_{j=1}^{M} |H_{i,j}(k)|^2 \]  

(16)

From (16), we can see that the ML detector is actually the MRC of the signals from \( M \) antennas when one user is in error. Recall in (5), we have shown that \( |H_{i,j}(k)|^2 \) is still exponentially distributed with unit mean. That is, the probability density function (p.d.f) of \( |H_{i,j}(k)|^2 \) is given by

\[ f(S) = \frac{S^{M-1} e^{-S}}{(M-1)!} \]  

(17)

With characteristic function, it is easy to show that the p.d.f. of the summation of \( M \) exponentially distributed random variable is given by

\[ f(S) = \frac{S^{M-1} e^{-S}}{(M-1)!} Q(\sqrt{2S / N_0}) dS \]  

(18)

Finally, the probability of one error is given by

\[ p_1 = \int_0^{\infty} \frac{S^{M-1} e^{-S}}{(M-1)!} Q(\sqrt{2S / N_0}) dS = \frac{Nq_1}{2} \]  

(19)

When there are two users, say, user 1 and user 2, in errors, the distance in (15) is now given by

\[ d^2(x, e) = 4 \sum_{j=1}^{M} |H_{i,j}(k) + H_{j,i}(k)|^2 \]  

(20)

Since \( H_{i,j}(k) \) and \( H_{j,i}(k) \) are uncorrelated complex Gaussian, it follows that their summation is still complex Gaussian with doubled mean power. Therefore, the product distance in (20) has a similar p.d.f as in (18) scaled by a constant. Finally, the probability of two errors is given by:

\[ p_2 = \int_0^{\infty} \frac{S^{M-1} e^{-S}}{(M-1)!} Q(\sqrt{4S / N_0}) dS \]  

(21)

Likewise, we can derive that the probability of \( u \) errors is given by

\[ p_u = \int_0^{\infty} \frac{S^{M-1} e^{-S}}{(M-1)!} Q(\sqrt{2uS / N_0}) dS \]  

(22)

In summary, the BER of the ML detector for an \( N \)-user OFDM/SDMA system can be given by:

\[ BER = \frac{1}{N} \sum_{i=1}^{N} p_i \]  

(23)

Because \( p_i \ll p_{i-1} \), thus, when the number of antennas is not large, (23) can be approximated by

\[ BER = p_1. \]  

(24)

In other words, the BER performance of the optimal ML detector is insensitive to the increase of number of users. This will be verified in Fig. 2. Following a similar approach, we can derive the BER when QPSK and Gray code are used (only up to two errors are considered).

\[ BER = \frac{1}{2N} \left( 2Nq_1 + \left( \frac{2N}{2} \right) q_2 \right) \]  

(25)

where

\[ q_1 = \int_0^{\infty} f(S) Q(\sqrt{S / N_0}) dS \]

\[ q_2 = \int_0^{\infty} f(S) Q(\sqrt{2S / N_0}) dS \]  

(26)

In Fig. 2, we show the performance of the optimal ML detector and the DMI-MMSE receiver for OFDM/SDMA. The parameters are similar to those in Fig. 1 except that BPSK is used. We can see that the MMSE receiver is much worse than the ML receiver. At the same time, the performance of the MMSE receiver degrades severely with the increase of the number of the users; while the performance of the ML receiver is rather insensitive to the number of users. In addition, the analytical results in (23) and (26) agree quite well with the simulation results for the ML detector, which are not shown here due to the limited space.

C. Performance of OFDMA

In order to have the same throughput as OFDM/SDMA, in OFDMA system, higher level modulation scheme must be used. Assume that the constellation size for OFDMA is \( 2^Q \). Then, the constellation size for OFDM is \( 2^{NQ} \). Meanwhile, the symbol energy of OFDMA should be \( N \) times larger than that of OFDM/SDMA. The signals in the \( M \) antennas are combined with MRC for each user.

\[ s_i[k] = \sum_{j=1}^{M} |H_{i,j}(k)|^2 x_i[k] + \sum_{j=1}^{M} H_{i,j}^*(k)n_j[k] \]  

(27)

Eqn. (27) is equivalent to

\[ s_i[k] = x_i[k] + \sum_{j=1}^{M} |H_{i,j}(k)|^2. \]  

(28)
Let
\[ \eta[k] = \frac{\sum_{j=1}^{M} H_{i,j}^*(k) n_j[k]}{\sum_{j=1}^{M} |H_{i,j}(k)|^2} \]  
(29)

It is easy to show that
\[ E[\eta[k]] = 0, \quad \text{var}[\eta[k]] = \frac{2\sigma^2}{\sum_{j=1}^{M} |H_{i,j}(k)|^2}. \]  
(30)

Therefore, we can see that OFDMA corresponds to the MRC of the distorted symbol with constellation size \(2^{NQ}\) and OFDM/SDMA (from (16)) corresponds to the MRC of the distorted symbol with constellation size \(2^Q\). Although the symbol energy of OFDMA increases linearly with the increase of \(N\) compared to OFDM/SDMA, the symbol distance in the constellation decreases almost exponentially. Thus, we can imagine that the performance of OFDMA should be worse than OFDM/SDMA using the ML detector.

Recall that the subchannel power is exponentially distributed. It follows that the BER for OFDMA with Gray code is given by (assume BPSK is used for OFDM/SDMA):

QPSK: \[ \text{BER} = \int_{0}^{\infty} f(S) Q\sqrt{2S/N_0} dS \]

8PSK: \[ \text{BER} = \int_{0}^{\infty} \frac{2}{3} f(S) Q(\sin \frac{\pi}{8} \sqrt{6S/N_0}) dS \]  
(31)

In Fig. 3, we show the performance comparison between OFDMA and OFDM/SDMA with two users. First, we apply BPSK to OFDM/SDMA and both the ML and the DMI-MMSE receivers are presented. We can see that the OFDMA (QPSK) performs much better than OFDM/SDMA employing the MMSE receiver, and is comparable to OFDM/SDMA employing the optimal ML receiver. Next, in Fig. 3, the performance of QPSK modulated OFDM/SDMA is also shown (note that the x-axis is still indexed by \(E_b/N_0\)). To maintain the same throughput as OFDM/SDMA, OFDMA should use 16QAM instead. We can see that OFDM/SDMA with DMI-MMSE receiver performs a litter better than OFDMA in low SNR region; while they are similar in high SNR region. At the same time, OFDM/SDMA with the ML detector is much better than OFDMA. From above, we can conclude that:

1. OFDM/SDMA has the potential to outperform OFDMA when the optimal ML detector is applied. However, the ML detector is too much complicated.
2. When the less complicated MMSE detector is used for OFDM/SDMA, it is usually worse than or comparable to OFDMA.
3. Either the ML or the MMSE detector for OFDM/SDMA is much more complex than the receiver for OFDMA which only requires MRC.
4. OFDM/SDMA is still an attractive choice for multiuser communications if less complicated multiuser detector with performance approaching that of the ML detector can be devised.

IV. CONCLUSIONS

In this paper, we analyzed and compared the performance of OFDMA and OFDM/SDMA system under the condition that they have the same throughput. We have shown that with the ML detector, the OFDM/SDMA system is much better than the OFDMA system. However, if DMI-MMSE is employed in OFDM/SDMA system, its performance is usually worse than or comparable to the performance of the OFDMA system. Compared with OFDM/SDMA, the main advantage of OFDMA is its simple receiver structure and reasonable performance. Therefore, this gives us tradeoff between complexity and performance for OFDMA and SDMA. At the same time, to reduce the complexity of the MMSE receiver for OFDM/SDMA, two reduced-complexity receivers, namely, interpolated MMSE and partial MMSE, were proposed in this paper.

REFERENCES: