On the MUSIC-derived Approaches of Angle Estimation for Bistatic MIMO Radar

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Abstract

We investigate the topic for the direction of departure (DOD) and direction of arrival (DOA) estimation in bistatic multiple-input-multiple-output (MIMO) radar systems with the exploitation of array invariance. Several MUSIC-derived algorithms for angle estimation in MIMO radar have been presented and compared for their complexity costs against that of ESPRIT. The proposed scheme of multi-invariance multiple signal classification (MI-MUSIC) has the best performance and also can be considered as a generalization of MUSIC. Simulations verify the collaborative usefulness of our algorithm.

Keywords-

Array signal processing, MUSIC, angle estimation, MIMO radar
• **Inspiration from the literal beauty of Greece Folk**

----the will, the strength and human’s creativity

• **The concept & geometry essence displayed in array signal processing**

----of symmetric beauty, rational and equilibrium

*(Show the models)*

• **Several relevant fields and the applications for array signals**

----MIMO, Sensor Array Processing & Networks, Radar Waveform Design, Underwater Sonar Detection
A brief summary for signal parameter estimation

• Mainly focus on the high-order of signal identification
• Independent Component Analysis (ICA) / Blind source separation
• Polarization-sensitive array from Far-fields
• Deterministic or statistical characteristics?
• Waveform design and beamforming
• Identifiability concern via capacity limits
• Comparable measurements

Signal parameter estimation: of paramount importance

Our Approach: MUSIC-derived algorithms for Direction-Of-Departure (DOD) and Direction-of-Arrival (DOA) estimation
Distinguished milestone and representative works in collaboration with array signal processing

(Mar. 00) Blind PARAFAC receivers in DS-CDMA systems, (Aug. 00) Parallel factor analysis in sensor array processing, (2001' Distinguished milestones, Best paper award in sensor array processing, SP society),
(Nov. 01) Exploiting Arrays with Multi-invariances Using MUSIC and MODE;

Summary in Ch4 PARAFAC Separation Techniques in Signal Processing, Vol II, Signal processing advances in wireless and mobile communications

Notable development of research interests in the recent decades

**Blind source separation, channel estimation, multiuser detection, multicarrier communications**

**MIMO techniques**
- (Capacity scaling, OFDM-based, MIMO radar signal processing)

**Underwater sonar, beamforming, sensor remotes, multi-scale Image,**

**Angle and time delay estimation, Joint DOD/DOA estimation, Target detection**

**Ultra-wideband communication**
- Cooperative communication
- Cognitive radio

**Wireless Sensor Network (WSN)**
- Physical-Layer Design
- Wimax, Virtual networks

**Interference channel, Network Coding**

**Distributed linear/nonlinear Optimization**

**PHY-APP Layer Interaction, Protocol Design**
I. Introduction

Outstandingly established by an innovative concept which utilizes multiple antennas for simultaneously transmitting diverse waveforms and receiving reflected signals in similar ways, multiple-input-multiple-output (MIMO) radar has been demonstrated for its potential advantages for radar systems via showing more degrees of freedom over conventional phased-array counterparts [1]–[4].

The algorithms of multiple signal classification (MUSIC) and its relevant approaches [9] for DOD and DOA estimation have been tried out in collaboration with a variety of subspace optimal methods, and also match some kind of irregularly-spaced array with high popularity [9].
II. Model and Constructions

Model: A bistatic MIMO radar system with both ULAs for its transmit/receive array, where \( M \) and \( N \) elements are orderly arranged with half-wavelength spacing between adjacent antennas

\[
X = [a_r(\phi_1) \otimes a_t(\theta_1), a_r(\phi_2) \otimes a_t(\theta_2), ..., a_r(\phi_K) \otimes a_t(\theta_K)]B^T
\]  
(1)

Critical Parameter: transmit/receive angles (elevation-azimuth), phases and amplitudes of the \( K \) sources, \( a_r(\phi_k) \otimes a_t(\theta_k) \) represents the Kronecker product of the transmit and the receive steering vectors for the \( k \)th target.

\[
X = [A_T \circ A_R]B^T = \begin{bmatrix}
X_1 \\
X_2 \\
M \\
X_M
\end{bmatrix} = \begin{bmatrix}
A_R D_1 (A_T) \\
A_R D_2 (A_T) \\
M \\
A_R D_M (A_T)
\end{bmatrix} B^T
\]  
(2)

\[
X_m = A_R D_m (A_T) B^T, \quad m = 1, \ldots, M
\]  
(3)
III. Music-derived Algorithms for angle estimation in Bistatic MIMO Radar

A. Multi-invariance MUSIC (MI-MUSIC) algorithm for angle estimation

The covariance matrix $R_x$ can be given by

$$R_x = XX^H = E_s D_s E_s^H + E_n D_n E_n^H$$  \hspace{1cm} (4)

The matrix $E_s : E_s = \Lambda T =$

$$[a_r(\phi_1) \otimes a_t(\theta_1), a_r(\phi_2) \otimes a_t(\theta_2), ..., a_r(\phi_K) \otimes a_t(\theta_K)]$$  \hspace{1cm} (5)

The subspace fitting :  $\hat{T}, \hat{\Lambda} = \text{arg min} \text{tr}(\Lambda^H \prod_{E_s}^\perp \Lambda)$  \hspace{1cm} (6)

The minimization of Eq. (6):

$$a_r(\phi), a_t(\theta) = \text{arg min} \sum_{k=1}^K [a_r(\phi_k) \otimes a_t(\theta_k)]^H \prod_{E_s}^\perp [a_r(\phi_k) \otimes a_t(\theta_k)]$$  \hspace{1cm} (7)
The minimization for Eq.(7) via searching the deepest K minimum:

\[ V(\phi, \theta) = [a_r(\phi) \otimes a_t(\theta)]^H \prod_{E_s} a_r(\phi) \otimes a_t(\theta) \]

\[ = a_t(\theta)^H [a_r(\phi) \otimes I_M]^H \prod_{E_s} a_r(\phi) \otimes I_M] a_t(\theta) = a_t(\theta)^H Q(\phi) a_t(\theta) \] (8)

The optimization problem comes with the linear constraint minimum variance solution

\[ \min_\phi a_t(\theta)^H Q(\phi) a_t(\theta), \quad \text{s.t.} \quad e^T a_t(\theta) = 1 \] (9)

Make solution to Eq.(9):

\[ \hat{\phi} = \arg \min_\phi \frac{1}{e^T Q(\phi)^{-1} e} = \arg \max_\phi e^T Q(\phi)^{-1} e \] (10)

Searching \( \phi \in [0, 360^\circ] \) we find the K largest peak of the (1, 1) element of \( Q(\phi)^{-1} \). Note that the K largest peak corresponds to the receive angle.
Another denotation can be given by

\[
V(\phi, \theta) = [a_r(\phi) \otimes a_t(\theta)]^H \prod_{E_s}^{\perp} [a_r(\phi) \otimes a_t(\theta)]
\]

\[
= a_r(\theta)^H [I_N \otimes a_t(\theta)]^H \prod_{E_s}^{\perp} [I_N \otimes a_t(\theta)] a_r(\phi) = a_r(\theta)^H P(\theta) a_r(\theta) \quad (11)
\]

where \( P(\theta) = [I_N \otimes a_t(\theta)]^H \prod_{E_s}^{\perp} [I_N \otimes a_t(\theta)] \).

Similarly, the solution for \( \theta \) is shown as

\[
\hat{\theta} = \text{arg max}_\theta e^T P(\theta)^{-1} e \quad (12)
\]

We also find the K largest peak of the \((1, 1)\) element of \( P(\theta)^{-1} \) via searching \( \theta \in [0, 360^\circ] \). Note that the K largest peak corresponds to the transmit angle.
The major steps of MI-MUSIC:

1) Perform eigen-decomposition operations for covariance matrix $R_x$ to $\hat{E}_s$, then calculate $\prod_{\hat{E}_s}^\perp$;

2) Search $\phi$ to find the K largest peak of the (1, 1) element of $Q(\phi)^{-1}$ (from Eq.(10)), then get the estimate of receive angle;

3) Search $\theta$, similarly find the K largest peak of the (1, 1) element of $P(\theta)^{-1}$ (from Eq.(12)), and obtain the estimate of transmit angle.
B. 2D-MUSIC and 1D-MUSIC for angle estimation

Construct the 2D-MUSIC spatial spectrum function in this form

\[
f_{2dmusic}(\phi, \theta) = \frac{1}{[a_r(\phi) \otimes a_t(\theta)]^H E_n E_n^H [a_r(\phi) \otimes a_t(\theta)]}
\]

(13)

2D-MUSIC requires an exhaustive 2D search, their approaches are normally inefficient due to high computational costs.

Comparing to 2D-MUSIC, 1D-MUSIC exploits the trilinear decomposition [11] method and thus reduces the range of spectrum searching:

The signal model is denoted as the trilinear model [11]

\[
x_{m,n,l} = \sum_{k=1}^{K} A_t(m,k) A_r(n,k) B(l,k), \ (m = 1, \ldots, M; \ n = 1, \ldots, N; \ l = 1, \ldots, L)
\]

(14)
\[ X_m = A_R D_m (A_T)^T B^T \]

\( m = 1, \ldots, M \), can be interpreted as slicing the 3-D data in a series of slices (2-D data) along the spatial direction. The symmetry of the trilinear model in (4) allows another matrix system rearrangement, \( Y'_n = BD_n (A_R)A_T^T \)

\( n = 1, \ldots, N \). Similarly we get

\[ Y_n = Y'_n = A_T D_n (A_R)B^T, \quad n = 1, \ldots, N \quad (15) \]

With respect to (3), we form the following matrix

\[ X' = \begin{bmatrix} X_1 & X_2 & \cdots & X_M \end{bmatrix} = A_R \begin{bmatrix} D_1 (A_T)^T B^T & D_2 (A_T)^T B^T & \cdots & D_M (A_T)^T B^T \end{bmatrix} \quad (16) \]

Construct the MUSIC spectrum function for DOA estimation as follows

\[ f_{doa-music} = \frac{1}{a_r(\phi)^H E_{n_1} E_{n_1}^H a_r(\phi)} \quad (17) \]
The $Y$ matrix can also be constructed by

$$
Y = \begin{bmatrix}
Y_1 & Y_2 & \cdots & Y_N
\end{bmatrix}
= A_T \begin{bmatrix}
D_1(A_R)B^T & D_2(A_R)B^T & \cdots & D_N(A_R)B^T
\end{bmatrix}
$$

(18)

Construct covariance matrix by $R_Y = YY^H$, obtain the noise subspace $E_{n_2}$ and estimate its DOD by constructing the MUSIC spectrum function

$$
f_{\text{dod-music}} = \frac{1}{a_t(\theta)^H E_{n_2}^H E_{n_2}^H a_t(\theta)}
$$

(19)

The detailed steps can be summarized as follows:

1) Perform eigen-decomposition for the covariance matrix $R_X$ to get the noise subspace $E_n$, where DOA is estimated by adopting of Eq. (17).

2) Operate the eigen-decomposition for cov. matrix $R_Y$ to get the noise subspace $E_{n_2}$, where DOD estimation is similarly obtained by Eq. (19).
### C. Complexity Analysis

**TABLE I. The algorithmic comparisons (n is the total searching times)**

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Complexity</th>
<th>The dimension of searching</th>
<th>Identifiable targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D-MUSIC</td>
<td>$O{LM^2N^2 + M^3N^3 + n^2[2MN (MN - K) + MN]}$</td>
<td>2</td>
<td>$MN - 1$</td>
</tr>
<tr>
<td>1D-MUSIC</td>
<td>$O{LMN^2 + LM^2N + M^3 + N^3 + n [2N (N - K) + 2M (M - K) + N + M]}$</td>
<td>1</td>
<td>$\min(M-1, N-1)$</td>
</tr>
<tr>
<td>MI-MUSIC</td>
<td>$O{LM^2N^2 + M^3N^3 + KM^2N^2 + n[M^3N^2 + M^2N^3 + M^3N + MN^3 + N^2 + M^2]}$</td>
<td>1</td>
<td>$MN - 1$</td>
</tr>
<tr>
<td>ESPRIT</td>
<td>$O(LM^2N^2 + M^3N^3 + 2K^2(M-1)N + 2K^2 (N-1) M + 6K^3)$</td>
<td>None</td>
<td>$MN - 1$</td>
</tr>
</tbody>
</table>
IV. Simulation Analysis

Monte Carlo trials are experimented to assess the angle estimation performance of the methods aforementioned. Set its trial numbers as 1000. \( L \) stands for the number of snapshots; \( M \) and \( N \) represent the number of transmit antennas and receive antennas, respectively.

Define root mean squared error (RMSE) as

\[
\text{RMSE} = \sqrt{\frac{1}{1000} \sum_{m=1}^{1000} (\hat{\phi}^m - \phi_0)^2},
\]

where \( \hat{\phi}^m \) is the estimated transmit / receive angle of the \( m \)-th Monte Carlo trial, \( \phi_0 \) is the perfect transmit / receive angle.

Define SNR:

\[
\text{SNR} = 10 \log_{10} \left( \sum_{m=1}^{M} \frac{\left\| A_R D_m (A_T) B^T \right\|_F^2}{\sum_{m=1}^{M} \left\| W_m \right\|_F^2} \right) dB
\]  

(20)
Fig. 1. DOD and DOA estimation performance comparison for target 1 with $L = 50$, $M = N = 8$

It is verified in Fig. 1 that among the three algorithms, MI–MUSIC has the best performance. 1D-MUSIC even does not perform better than ESPRIT, while the MI-MUSIC algorithm that we presented has much better performance than both ESPRIT and 1D-MUSIC.
The DOD and DOA estimation performance for target 3 with $L = 100$ is displayed in Fig. 2, it is also depicted that MI–MUSIC performs better than ESPRIT, and the latter even better than 1D-MUSIC.
Fig. 3. DOD and DOA estimation performance for target 2 with different $L$.

We confirm that the performance of angles estimation for MIMO radar becomes better in collaboration with $L$ increasing. The proposed MI-MUSIC algorithm even supports small sampling sizes.
Fig. 4. DOD and DOA estimation performance for target 2 with different $M$ and $N$ ($L = 50$)

Fig. 4 illustrates the DOD and DOA estimation performance by MI-MUSIC algorithm for target 2 with different transmit / receive antennas, respectively. It is obviously shown that the estimation performance of MI-MUSIC is gradually enhancing with the number of antennas increasing due to diversity gain.
Fig. 5. DOD and DOA estimation performance under different $K$ $(L = 50)$

Fig. 5 displays the algorithmic performance of MI-MUSIC under different $K$ when $M = 8$, $N = 8$, and $L = 50$. From Fig. 5, we conclude that DOD and DOA estimation performance levels down with the increment of target numbers.
V. Conclusions

2D-MUSIC and 1D-MUSIC rely on the necessity of peak searching and thus have higher costs. Due to the necessity of two-dimension searching, 2D-MUSIC has the highest complexity. Notably, it is suggested that MI-MUSIC represents the balanced approach from complexity evaluation.

• Derived the algorithms of MI-MUSIC, 2D-MUSIC, and 1D-MUSIC for angle estimation in bistatic MIMO radar systems.
• The usefulness of our methods has been illustrated as compatible for the MIMO radar system with three identified targets.
• The MUSIC-derived approaches as stated above works well and exhibits expansions for their adoptions in other array manifolds.
• The proposed MI-MUSIC can be viewed as a generalized approach of multiple signal classification.
VI. Future Work

• *The beampattern designs via convex optimization for bistatic MIMO radar*

• *The predicts of angle estimation schemes*

• *The corresponding identifiability conditions in MIMO radar systems.*
References


Thank you for your attention!

Dec. 28, 2009, Shanghai, China