Multi-invariance ESPRIT-based Blind DOA Estimation for MC-CDMA with Antenna Array

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Abstract: In this paper, we address the problem of Direction Of Arrival (DOA) estimation for MC-CDMA system with antenna array. We reconstruct the received signal to form data model with multi-invariance property, and then multi-invariance ESPRIT algorithm for DOA estimation is proposed. This algorithm has improved DOA estimation performance and identified more DOAs compared to ESPRIT algorithm. Moreover, our algorithm enables DOA estimation of a large number of impinging waves. Simulation results illustrate performance of this algorithm.

Index Terms: MC-CDMA, antenna array, DOA estimation, multi-invariance, ESPRIT

I. INTRODUCTION

Recently, Multi-Carrier Code Division Multiple Access (MC-CDMA) [1], [2] has received extensive attentions in the context of 4G communication systems [3]. MC-CDMA has its capabilities of achieving high capacity over a frequency-selective fading channel [4]-[6]. Antenna array in MC-CDMA base station exploits the spatial domain to provide an extra way of co-channel interference cancellation and thus tends to improve system capacity [7]-[9]. Blind Direction Of Arrival (DOA) estimation methods for array antenna MC-CDMA system contain ESPRIT [10], [11] and MUSIC [12]. In contrast to training-based methods, the blind DOA estimators improve bandwidth efficiency. Notably, ESPRIT method [10], [11] exploits the inherent shift-invariant structure of received signal, and has the high-accuracy estimation performance. In this paper, we reconstruct the received signal to form data model with multi-invariance property, and then derive a multi-invariance ESPRIT algorithm for DOA estimation. Comparing to ESPRIT algorithm, our proposed algorithm has improved DOA estimation performance, and identifies more DOAs than ESPRIT. It also enables DOA estimation of a large number of impinging waves.

The following sections of this paper are structured as follows. Section II develops data model, while Section III derives Multi-Invariance ESPRIT algorithm. Error analysis is presented in Section IV. Section V and VI offer our simulation results and conclusions, respectively.

Notation: We denote by $(\cdot)^*$ the complex conjugation, by $(\cdot)^T$ the matrix transpose, and by $(\cdot)^H$ the matrix conjugate transpose. The notation $(\cdot)^+$ refers to the Moore–Penrose inverse (pseudo inverse). $\text{real}\{\cdot\}$ is to get real part of complex number.

II. DATA MODEL

Let us assume there are $K$ users in the MC-CDMA system, and the receiver is equipped with a uniform linear array containing $I$ antennas. The transmitter structure of the $k$th user is shown in Fig. 1. The symbol sequence of the $k$th user is $\mathbf{b}_k = [b_k(1), b_k(2), \ldots b_k(L)]^T$. The data symbol $\mathbf{b}_k$ is binary phase shift-keying (BPSK) modulated signal and transmitted in parallel over $N$ subcarriers, each multiplied by a different element of the spread sequence $\mathbf{c}_k$, where $\mathbf{c}_k = [c_k(1), c_k(2), \ldots c_k(N)]^T$ is the spread code of the $k$th user. The output signal of spread spectrum is shown as $\mathbf{U}_k = \mathbf{c}_k \mathbf{b}_k^T$, and the signal $\mathbf{U}_k$ is processed under multicarrier modulation, which can also be denoted as an inverse fast Fourier transform (IFFT). Hence, the output signal of multicarrier modulation is hereby expressed as

\[ \mathbf{D}_k = \mathbf{F}^H \mathbf{U}_k = \mathbf{F}^H \mathbf{c}_k \mathbf{b}_k^T \]

(1)

where $\mathbf{F}$ is the fast Fourier transform (FFT) matrix with $N \times N$. $\mathbf{F}^H$ stands for the IFFT.

![Fig. 1. The transmitter structure of the $k$th user](image)

We also assume that each of the $K$ users has only a single path, through which they synchronously arrive at antenna array. The array spacing is $d = \lambda_c/2 = c/2f_c$, where $f_c$, $\lambda_c$, and $c$ are the carrier frequency, wavelength and light speed, respectively. Meanwhile, we define the $n$th subcarrier
frequency \( f_c = f_e + (n - 1) \Delta f \), where \( \Delta f \) is subcarrier spacing. Considering MC-CDMA system with a carrier frequency of 5 GHz, 32 subcarriers and subcarrier spacing of 25 kHz, we suppose that the signal of the \( k \)th user impinges the array antenna with angle \( \theta_k \), and the channel response between the \( n \)th subcarrier of \( k \)th user and the \( n \)th antenna should be

\[
h_{k,n}(t) = e^{-j2\pi f_c(n-1)\Delta t} e^{-j2\pi f_c(n-1)\sin \theta_k} h_{k,n}, n = 1, 2, \ldots, N
\]

(2)

where \( j = \sqrt{-1} \), \( h_{k,n}(t) \) is the channel response between the \( n \)th carrier of the \( k \)th user and the first antenna. Compared with the carrier frequency \( f_c \), subcarrier spacing \( \Delta f \) is extremely small. And carrier frequency \( f_c \) is much larger than transmission band width (BW). In general, \( \Delta f / f_c < 0.0001 \) and \( BW / f_c < 0.01 \), for example, \( f_c = 5 \) GHz, \( BW = 800 \) KHz and \( \Delta f = 25 \) KHz in this paper. It is approximately estimated that \( e^{-j2\pi f_c(n-1)(n-1)\Delta t} e^{-j2\pi f_c(n-1)\sin \theta_k} \approx 1 \) and Eq.(2) becomes

\[
h_{k,n}(t) = e^{-j\pi (n-1)\sin \theta_k} h_{k,n}, n = 1, 2, \ldots, N
\]

According to Eq.(3),

\[
\begin{bmatrix}
h_{k,1,n} \\
h_{k,2,n} \\
\vdots \\
h_{k,N,n}
\end{bmatrix} = \begin{bmatrix}
1 \\
e^{-j\pi \sin \theta_1} \\
e^{-2j\pi \sin \theta_2} \\
\vdots \\
e^{-j\pi (N-1)\sin \theta_N}
\end{bmatrix} h_{k,n}, n = 1, 2, \ldots, N
\]

Although each subcarrier has a different frequency, different subcarriers of one user have the same array response, which was also assumed in [8]-[14]). Considering the subcarriers signals of one user pass through the same channel, different subcarriers of one user towards a receive antenna have the same channel response. That is to say, \( h_{k,n} = h_{i,k}, n = 1, 2, \ldots, N \), where \( h_{i,k} \) is the channel response between the \( k \)th user and the \( n \)th antenna.

We assume \( K \) users impinging array antenna, and the received signal of the \( i \)th antenna is

\[
X_i = \sum_{k=1}^{K} D_{k} h_{k} = \sum_{k=1}^{K} F^H e_k b_k^H h_{k} = F^H C \text{diag}(h_{1,i}, h_{2,i}, \ldots, h_{K,i}) B^T, \quad i = 1, 2, \ldots, I
\]

(4)

where \( C = [c_1, c_2, \ldots, c_K] \in \mathbb{C}^{L \times K} \) is the spread matrix, \( B = [b_1, b_2, \ldots, b_K] \in \mathbb{C}^{L \times K} \) is the source matrix.

Define channel matrix \( H \)

\[
H = \begin{bmatrix}
h_{1,1} & h_{1,2} & \cdots & h_{1,K} \\
h_{2,1} & h_{2,2} & \cdots & h_{2,K} \\
\vdots & \vdots & \ddots & \vdots \\
h_{I,1} & h_{I,2} & \cdots & h_{I,K}
\end{bmatrix} \in \mathbb{C}^{L \times K}, \quad \text{and}
\]

\[
h_{i,k} = h_{i,k} e^{-j\pi (i-1)\sin \theta_k}
\]

(5)

Eq.(5) is also denoted as \( H = A \Phi \), where \( \Phi = \text{diag}(h_{1}, h_{2}, \ldots, h_{K}) \), \( A = [a(\theta_1), a(\theta_2), \ldots, a(\theta_K)] \) is direction matrix, and \( a(\theta_k) = [1, e^{-j\pi \sin \theta_k}, \ldots, e^{-j\pi (I-1)\sin \theta_k}]^T \).

Eq.(4) is also denoted as

\[
X_i = SD_i(H)B^T, \quad i = 1, 2, \ldots, I
\]

(6)

where \( S = F^H C \in \mathbb{C}^{N \times K} \), \( D_i(.) \) is to extract the \( i \)th row of its matrix and construct a diagonal matrix out of it. In the presence of noise, the received signal model becomes \( \tilde{X}_i = SD_i(H)B^T + V_i \), where \( V_i \) is the received noise corresponding to the \( i \)th antenna. Besides, Eq.(6) can be regarded as trilinear model [15], which also has another matrix system rearrangement way, such as \( Y_i = HZ_i(B)B^T = A \Phi(D_i(B))S^T, \quad i = 1, 2, \ldots, I \), in which the matrix \( A \) is with Vandermonde characteristic, and then ESPRIT algorithm [16] can be used for DOA estimation.

III. MULTI-INVARIENCE ESPRIT-BASED DOA ESTIMATION ALGORITHM

According to (6), we form the following matrix,

\[
X = \begin{bmatrix}
X_1 \\
X_2 \\
\vdots \\
X_I
\end{bmatrix} = \begin{bmatrix}
SD_1(H) \\
SD_2(H) \\
\vdots \\
SD_I(H)
\end{bmatrix} \begin{bmatrix}
B^T \\
\vdots \\
B^T
\end{bmatrix} = \begin{bmatrix}
S \\
S \Psi \\
\vdots \\
S \Psi^{I-1}
\end{bmatrix}
\]

(7)

where \( B = \Phi \), \( \Psi = \text{diag}(e^{-j\pi \sin \theta_1}, e^{-j\pi \sin \theta_2}, \ldots, e^{-j\pi \sin \theta_K}) \in \mathbb{C}^{K \times K} \) is the rotation matrix. According to the multi-invariance characteristic of the signal in (7), we use multi-invariance ESPRIT [17] to estimate DOAs. For Eq.(7), \( R = XX^H \). We denote the matrix containing the eigenvectors \( f_{k,i}^{n} \) associated with the \( K \) largest eigenvalues of \( R \) by \( E \)

\[
E = \begin{bmatrix}
S \\
S \Psi \\
\vdots \\
S \Psi^{I-1}
\end{bmatrix}
\]

(8)

where \( T \) is a \( K \times K \) full-rank matrix. According to (8), we define \( E_1 \) and \( E_2 \),

\[
E_1 = \begin{bmatrix}
S \\
S \Psi \\
\vdots \\
S \Psi^{I-2}
\end{bmatrix} \quad T \quad E_2 = \begin{bmatrix}
S \Psi \\
\vdots \\
S \Psi^{I-2}
\end{bmatrix}
\]

According to Eq.(9),

\[
E_2 = \begin{bmatrix}
S \\
S \Psi \\
\vdots \\
S \Psi^{I-2}
\end{bmatrix} \quad T^H \quad \Psi = \begin{bmatrix}
S \Psi \\
\vdots \\
S \Psi^{I-2}
\end{bmatrix} \quad T^H \Sigma = E_1 T^H \Psi T
\]

(10)

Define \( \Omega = T^H \Psi T \). Eq.(10) becomes \( E_2 = E_1 \Omega \), and then \( \Omega = E_1^H E_2 \). Because \( \Omega \) has the same eigenvalues as \( \Psi \), we use eigenvalue decomposition for \( \Omega \) to get \( e^{-j\pi \sin \theta_k} \), \( k = 1, 2, \ldots, K \), and then estimate DOA \( \theta_k \), \( k = 1, 2, \ldots, K \).

It should be pointed out that ESPRIT algorithm only works well for \( K < I \), where \( K, I \) are the number of users and antennas, respectively. Our proposed algorithm has no this constrain. Exploiting multiple invariance characteristic in Eq.(9) and Eq.(10), it is easy to determine the maximum number of users \( K_{\text{max}} \), which our proposed algorithm can detect. It is clear that \( K_{\text{max}} = N (I - 1), \forall I \geq 2 \) (in general, \( N > I \)) when the matrix
IV. ERROR ANALYSIS

This section aims at analyzing error, including array gain error, array phase error, model error and mutual coupling. Considering array gain error and array phase error, the direction matrix becomes \( \hat{A} = A + \partial A \), where \( \partial A \) is the random array error matrix. Then Eq.(8) becomes

\[
\hat{E} = \begin{bmatrix}
SD_1(A + \partial A) \\
SD_2(A + \partial A) \\
\vdots \\
SD_{I-1}(A + \partial A) \\
SD_I(A + \partial A)
\end{bmatrix}^T
\]

And we get \( \hat{E}_1 \) and \( \hat{E}_2 \) followed by

\[
\hat{E}_1 = \begin{bmatrix}
SD_1(A + \partial A) \\
SD_2(A + \partial A) \\
\vdots \\
SD_{I-1}(A + \partial A) \\
SD_I(A + \partial A)
\end{bmatrix} = [A_1 + \partial A_1] \Psi^T
\]

\[
\hat{E}_2 = \begin{bmatrix}
SD_1(A) \\
SD_2(A) \\
\vdots \\
SD_{I-1}(A) \\
SD_I(A)
\end{bmatrix} = [\partial A_1] \Psi^T
\]

where

\[
A_1 = \begin{bmatrix}
S \\
M \\
\vdots \\
\Psi \\
\Psi^{l-2} \\
\Psi^{l-1} \\
\Psi^T
\end{bmatrix} \quad \partial A_1 = \begin{bmatrix}
SD_1(\partial A) \\
SD_2(\partial A) \\
\vdots \\
SD_{I-1}(\partial A) \\
SD_I(\partial A)
\end{bmatrix}
\]

According to the first-order approximation for \([A_1 + \partial A_1]^T\), we get \( \hat{\Omega} \),

\[
\hat{\Omega} = \Psi^T \begin{bmatrix}
\hat{P}_1 + \hat{p}_1 \\
\hat{P}_2 + \hat{p}_2 \\
\vdots \\
\hat{P}_I + \hat{p}_I
\end{bmatrix}
\]

The \( k \)th eigenvalue of \( \hat{\Omega} \) is \( \hat{\rho}_k = p_k + \partial p_k \), where \( \partial p_k = p_k e_i^T \hat{A}_1(\partial A_2 - \partial A_1)e_i \), and \( e_i \) is a unit vector, in which the \( i \)th element is 1, and others are zeros. Using a first order Taylor series expansion, the variance of DOA estimation is shown

\[
E[\hat{\theta}_i^2] = \frac{1}{2} \left[ \frac{\lambda}{2 \pi d \cos \theta_i} \right]^2 \left[ E[|\hat{p}_i|^2] - \text{real} \{E[(\hat{p}_i)^*(\hat{p}_i^*)] \} \right]
\]

For the existence of mutual coupling, the direction matrix should be modeled with \( \hat{A} = ZA \), where \( Z \) is an \( I \times I \) mutual coupling matrix. The mutual coupling coefficients between two elements that are far enough from each other can be approximated as zero, and the coupling between any two equally spaced sensors appears the same [18]. For a uniform linear array, mutual coupling matrix can be model as a banded symmetric Toeplitz matrix \( Z \) given by

\[
\begin{cases}
\hat{z}_{ik} = z_{ijkl}, & 1 \leq i, k \leq I \\
\hat{z}_{ij} = 1, & 1 \leq i \leq I
\end{cases}
\]

where \( \hat{z}_{ik} \) is the \((i,k)\) element of the matrix \( Z \).

\[
\hat{S} = ZA = A + \partial A,
\]

where \( \partial A = (Z-I)A \), \( I \) is an \( I \times I \) identity matrix. As is mentioned above, we take the same method to analyze its error.

For multichannel modulation signals, when the subcarrier differs, the array response varies. Yet, we assume different subcarriers of one user have the same response for array antenna. In this paper, the channel response between the \( n \)th subcarrier of \( k \)th user and the \( i \)th antenna is approximated by \( \exp(-j\pi(i-1)\sin \theta_i)h_{ikn} \), and the channel response error is \( \exp(-j\pi(i-1)\sin \theta_i)(1-\exp(-j2\pi(i-1)(n-1)\lambda/\pi)\sin \theta_i)h_{ikn} \), which results in array model error. The direction matrix with this array model error is \( \hat{\Omega} = \hat{Z}A + \partial A \), where \( \partial A \) is considered as the model error. When dealing with model error, we also take the same method similarly to analyze this kind of array gain error and array phase error.

V. SIMULATION RESULTS

Let the received noisy signal \( \hat{X}_i = SD_i(H)B^T + V_i, i = 1, 2, \ldots, I \), where \( V_i \) is additive Gaussian (AWGN) matrix. And we define SNR

\[
\text{SNR} = 10 \log_{10} \frac{\sum_{i=1}^{I} \|SD_i(H)B^T\|^2}{\sum_{i=1}^{I} \|V_i\|_F^2} \text{ dB}
\]

The MC-CDMA receiver is equipped with 8-element uniform-linear-array (ULA). Mutual coupling in the antenna array is neglected in this simulation. We hereby adopt BPSK symbols as the transmitted data, which are spread by Walsh–Hadamard sequences. The number of subcarriers or spread gain is 32. Note that \( L \) is the number of snapshots, and \( K \) is the number of users.

There are three MC-CDMA signals impinging on the uniform linear array at \( \theta_1 = 10^\circ, \theta_2 = 20^\circ, \theta_3 = 30^\circ \), respectively. We compare our proposed algorithm with ESPRIT algorithm.

Define \( RMSE = \sqrt{\frac{1}{1000} \sum_{m=1}^{1000} (\theta_m - \hat{\theta}_m)^2} \), where \( \theta_m, \theta_n \) are the perfect DOA and the estimated DOA of the \( m \)th Monte Carlo trial, respectively. Fig. 2 presents DOA estimation performance for \( K = 3 \) and \( L = 50 \). From Fig. 2, we find our proposed algorithm works well. Fig. 3 shows DOA = 10° estimation performance comparison with \( K = 3 \) and \( L = 50 \). From Fig. 3, we find that our proposed algorithm has much better DOA estimation performance than ESPRIT algorithm, and the performance of Multi invariance-ESPRIT is very close to the Cramer-Rao bound (CRB).

Fig. 4 presents DOA = 30° estimation performance with \( K = 3 \) and different \( L \). From Fig. 4, we find that DOA estimation...
performance of our proposed algorithm is improved with $L$ increasing.

Fig. 5 shows DOA = 20° estimation performance with $L = 50$ and different $K$. From Fig.5, we find that DOA estimation performance of our proposed algorithm is improved with $K$ decreasing.

Array antennas contain $I = 8$ elements in this simulation, and ESPRIT algorithm can identify $K = 7$ sources. When $K \geq I$, (I, $K$ are the number of sources and antennas, respectively), ESPRIT algorithm fails to work. Our proposed algorithm can identify 32 source signals in this simulation. Suppose there are 30 signals impinging on a uniform linear array at $\theta = [0°; 3°; 87°]$, respectively. Fig. 6 shows DOA estimation performance with $K = 30$ and $L = 100$. From Fig. 6 we conclude that our proposed algorithm works well in condition of larger user-number. We also find that DOA estimation performance is improved with the value of DOA decreasing.

Practically, the array response error caused by multi-carrier modulation brings about some negative influence on our proposed DOA estimation. This kind of influence is quite little and can even be neglected when $\Delta f \ll f_c$; as $\Delta f / f_c$ rises, the influence that was introduced also gradually worsens and renders a decreasing DOA estimation performance.

VI. CONCLUSIONS

Multi-invariance ESPRIT-based DOA estimation for MC-CDMA system with antenna array is proposed. This algorithm has improved DOA estimation performance, and can identify more DOAs than ESPRIT method. Furthermore, our algorithm enables DOA estimation of a large number of impinging waves.

ACKNOWLEDGMENT

This work is supported by China NSF Grants (60801052), Ph. D. programs foundation of ministry of education of China (No.

Fig.2. DOA estimation performance with $K = 3$ and $L = 50$

Fig.3. DOA 10° estimation performance comparison with $K = 3$ and $L = 50$

Fig.4. DOA 30° estimation performance with $K = 3$ and different $L$

Fig.5. DOA 20° estimation performance with $L = 50$ and different $K$

Fig.6. DOA estimation performance with $K = 30$ and $L = 100$
This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.

200802871056) and Jiangsu NSF Grants (BK2007192). The authors wish to thank the anonymous reviewers for their valuable suggestions on improving this paper. The authors are also grateful to the Associate Editor Dr. Tomohiko Taniguchi for contacting with this manuscript.

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