Novel Blind Carrier Frequency Offset Estimation for OFDM System with Multiple Antennas

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Abstract—In this paper, we address the problem of carrier frequency offset (CFO) estimation for Orthogonal Frequency Division Multiplexing (OFDM) systems with multiple antennas. The received signal can be denoted as a trilinear model, then the trilinear decomposition-based CFO estimation algorithm is proposed. Comparing to both ESPRIT method and the cyclostationarity (CS) approach, the algorithm that we presented has improved CFO estimation performance. Furthermore, our proposed algorithm can even work in condition of no virtual carrier. Simulation results illustrate performance of this algorithm.

Index Terms—Carrier frequency offset (CFO), OFDM, multiple antennas, trilinear model.

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) represents an efficient technique distinguished for high-speed digital transmission over multipath fading channels. It is well known that OFDM provides a fast-developing technology for the next generation of mobile communications, however, beside the inherent defects such as time-synchronous error and inter-carrier interference within OFDM, high sensitivity to carrier frequency offset (CFO) has been widely recognized as its considerable weakness. In general, CFO estimation algorithm for OFDM can be divided into two categories: the non-blind CFO estimation methods (based on pilots [1], training sequence [2] or the cyclic prefix [3]) and blind CFO estimation methods (MUSIC [4] and ESPRIT [5], kurtosis-based and constant modulus-based estimator [6], [7], as well as cyclostationarity-based approach [8]). OFDM system with multiple-antenna receiver offers receive diversity to overcome fading [9] and has been applied to many communication systems such as digital video broadcasting for handheld and future mobile wireless communications [10], etc. Similar as the single-input-single-output OFDM systems, the one with multiple-antenna diversity receivers are also sensitive to carrier synchronization errors. Hence, the CFO between transmitter/receiver requires significant necessity to be estimated and compensated so as to ensure the subcarrier orthogonality.

In contrast to [1]–[3], blind CFO estimation methods improve bandwidth efficiency. Some subspace-based technique such as MUSIC and ESPRIT method [4], [5] have high accurate CFO estimation, but their necessity of virtual carriers leads to spectrum inefficiency. According to the previous blind methods, [6] assumes non-Gaussian sources, and exploits kurtosis to measure non-Gaussianity, while [7] and [8] assume other special properties of the sources, e.g., constant-modulus and cyclostationarity.

We develop the trilinear model [11] for OFDM systems with multiple antennas, and present the trilinear decomposition-based CFO estimation algorithm in Section II and Section III, respectively. Section IV illustrates the numerical results. Section V summarizes our conclusions.

Notation: $(\cdot)^{*}, (\cdot)^{T}, (\cdot)^{H}, (\cdot)^{i}, (\cdot)^{-1}$ and $\|\cdot\|_F$ denote the complex conjugation, transpose, conjugate-transpose, inverse, pseudo-inverse operations and Forbenius norm, respectively. diag$(\cdot)$ stands for diagonal matrix whose diagonal is the vector $\cdot$ ; $\min(\cdot)$ get minimum elements of an array; $I_P$ denotes a $P \times P$ identity matrix; $I_{N \times 1}$ is an $N \times 1$ vector of ones.

II. DATA MODEL

We consider the uplink of a single-input-single-output (SIMO) OFDM system with $J$ receive antennas. Assume that each antenna is affected by the same CFO. Among the $N$ subcarriers in this OFDM system, $P$ subcarriers are used for data transmission while the rest $N - P$ ones are virtual carriers. The cyclic prefix (CP) of $L$ sampling intervals, where $L$ is chosen to exceed the maximum delay spread, has been adopted. After inserting CP, the output signal is transmitted throughout multipath fading channel.

Define $H_i(n) = \sum_{l=0}^{L-1} h_i(l) e^{-j2\pi ml/N}$ as channel frequency response for the $n$th subcarrier corresponding to the $i$th antenna, where $\{h_i(l)\}_{l=0}^{L-1}$ is discrete-time channel impulse response. The frequency domain channel vector for the $i$th receive antenna is $h_i = [H_i(1), H_i(2), ..., H_i(P)]^T$. Hence, the frequency domain channel matrix for the multiple antenna receivers is shown

$$H = [h_1, h_2, \ldots, h_J]^T \in \mathbb{C}^{1 \times P} \quad (1)$$

After removing CP, the output signal of the $i$th antenna can be denoted as

$$x_i(k) = E_F \text{diag}(h_i) s(k) e^{j2\pi \Delta_f (k-1)(N+L)} \quad (2)$$

where $s(k) = [s_1(k), s_2(k), \ldots, s_P(k)]^T$ stands for the $k$th block, $\Delta_f$ is CFO, $E_F = \text{diag}(1,e^{j2\pi \Delta_f}, \ldots, e^{j2\pi (N-1)\Delta_f}) \in \mathbb{C}^{P \times N}$ represents the CFO matrix, and $F_P \in \mathbb{C}^{N \times P}$ comprises the first $P$ columns of the inverse discrete Fourier transform matrix. We assume that the channel parameters are constant for $K$ blocks, and the source...
matrix can be defined as $S = [s(1), s(2), ..., s(K)]^T \in K \times P$. Define $X_i = [x_i(1), x_i(2), ..., x_i(K)]$, and hence we have

$$X_i = A diag(h_i) B^T = A D_i(H) B^T, \quad i = 1, ..., I \tag{3}$$

where

$$B = diag\{e^{2\pi i f(N+L)}, ..., e^{2\pi i f(K-1)(N+L)}\} S \in K \times P,$$

$D_i(\cdot)$ is to extract the $i$th row of its matrix and construct a diagonal matrix out of it. The Vandermonde matrix $A$ is expressed as Eq. (4). (See next page.) In the presence of noise, the received signal model becomes $X_i = A D_i(H) B^T + W_i, i = 1, ..., I$, where $W_i$ is the received noise corresponding to the $i$th antenna. The signal in (3) is denoted as the trilinear model [11]

$$x_{n,k,i} = \sum_{p=1}^{P} \alpha_{n,p} b_{k,p} h_{i,p}, \quad n = 1, ..., N, k = 1, ..., K, i = 1, ..., I \tag{5}$$

where $h_{i,p}$ stands for the $(i, p)$ element of the matrix $H$, and similarly for the others. Eq. (3) can be considered as slicing the trilinear model in a series of matrix along the antenna direction. Meanwhile, the symmetry of the trilinear model in (5) allows other matrix system rearrangement, for which we have

$$Y_n = B D_n(A) H^T, \quad n = 1, ..., N, \quad Z_k = H D_k(B) A^T, \quad k = 1, ..., K.$$

We link the problem of CFO estimation with trilinear decomposition and derive a trilinear decomposition-based CFO estimation algorithm, which is proposed and discussed in detail in the following sections.

III. TRILINEAR DECOMPOSITION ALGORITHM FOR CFO ESTIMATION

In this section, we primarily use trilinear decomposition to estimate the matrix $A$, and then obtain CFO estimation with the least square (LS) principle.

A. Trilinear Decomposition

Trilinear Alternating Least Square (TALS) algorithm is the common data detection method for trilinear model [11]. The principle of TALS can be adopted to fit low rank trilinear models on the basis of noisy observations. We concisely show the basic idea behind TALS for three major steps: 1) Update one matrix each time using LS, which is conditioned on previously obtained estimates for the remaining matrices; 2) Proceed to update the other matrices; 3) Repeat until convergence of the LS cost function. TALS algorithm is discussed in detail as follows:

The signal in (3) is also represented as

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_I \end{bmatrix} = \begin{bmatrix} A D_1(H) \\ A D_2(H) \\ \vdots \\ A D_I(H) \end{bmatrix} B^T = [H \circ A] B^T \tag{6}$$

where $H \circ A$ stands for Khatri–Rao product. LS fitting is

$$\min_{A,H,B} \left\| \tilde{X} - [H \circ A] B^T \right\|_F,$$

where $\tilde{X}$ is the noisy signal. LS update for $B$ is

$$\tilde{B}^T = \left[ \tilde{H} \circ \hat{A} \right]^T \tilde{X} \tag{7}$$

where $\hat{A}$ and $\tilde{H}$ are the previously obtained estimates of $A$ and $H$, respectively.

Similarly, from the second way of slices: $Y_n = B D_n(A) H^T, \quad n = 1, 2, ..., N$, in which we have

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_N \end{bmatrix} = \begin{bmatrix} B D_1(A) \\ B D_2(A) \\ \vdots \\ B D_N(A) \end{bmatrix} H^T = [A \circ B] H^T \tag{8}$$

According to (8), LS fitting is

$$\min_{A,H,B} \left\| \tilde{Y} - [A \circ B] H^T \right\|_F,$$

where $\tilde{Y}$ is the noisy signal. Then LS update for $H$ is given by

$$\tilde{H}^T = \left[ \hat{A} \circ \tilde{B} \right]^T \tilde{Y} \tag{9}$$

where $\hat{A}$ and $\tilde{B}$ denote previously obtained estimates of $A$ and $B$, respectively.

Finally, from the third way of slices: $Z_k = H D_k(B) A^T, \quad k = 1, 2, ..., K$, LS update for $A$ is shown as

$$\tilde{A} = \left[ \tilde{B} \circ \hat{H} \right]^T \tilde{Z} \tag{10}$$

where $\tilde{H}$ and $\tilde{B}$ denote previously obtained estimates of $H$ and $B$, respectively, and $\tilde{Z}$ stands for the noisy signal.

According to (7), (9) and (10), the matrices $B, H$ and $A$ are updated with the conditioned LS, respectively. For zero-mean white Gaussian noise, TALS yields maximum likelihood (ML) estimates provided that the global minimum has been achieved [12]. The major shortcomings of TALS algorithm lie on the occasional slowness of the convergence process [13] though TALS is quite easy to implement and guaranteed to converge, initialized randomly, or initialized by eigen-decomposition method to accelerate convergence. In this paper, we use the COMFAC algorithm [14] for trilinear decomposition. COMFAC algorithm is essentially a fast implementation of TALS which speeds up the LS fitting. As for the improvement, COMFAC compresses the three-way data into a smaller three-way data. After fitting the model in the condensed space, the solution can be recovered to the original space within a few TALS steps. Normally, smaller TALS steps are sufficient for this refinement stage because the recovered model is close to the LS solution.

B. Identifiability

**Theorem 1** [15]: $Y_n = B D_n(A) H^T, \quad n = 1, ..., N$, where $A \in N \times P$, $H \in I \times P$, $B \in K \times P$. Consider that matrices are full-$k$-rank [11] and the matrix $A$ is with Vandermonde characteristic, if

$$\min (P, K) + \min (N + \min (I, P), 2P) \geq 2P + 2 \tag{11}$$

then $A$, $H$ and $B$ are unique up to permutation and scaling of columns, that is to say, any other matrix $\tilde{A}$, $\tilde{H}$ and $\tilde{B}$ that
construct $Y_n$, $n = 1, \ldots, N$, can be related to $A$, $H$ and $B$ through $\hat{A} = A \hat{\Omega} \hat{\Delta}_1$, $\hat{H} = H \hat{\Delta}_1$, $\hat{B} = B \hat{\Delta}_1$ where $\hat{\Omega}$ is a permutation matrix, and $\Delta_1, \Delta_2, \Delta_3$ note for the diagonal scaling matrices satisfying $\Delta_1 \Delta_2 \Delta_3 = I_P$.

Generally we have $P > I$, then Eq. (11) becomes $\min(P, K) + \min(N + I, 2P) \geq 2P + 2$. When $N + I > 2P$ and $K < P$, the identifiable condition is rewritten as $2 \leq K \leq P$. Hence, our proposed algorithm has the capacity to support small samples. When $K > P$ and $P \geq 2$, the identifiable condition is $N + I > P + 2$.

For the received noisy signal, we use trilinear decomposition to get the estimated matrix $\hat{A} = A \hat{\Omega} \hat{\Delta}_1 + N$, where $N$ is the noise matrix. Within trilinear decomposition, permutation ambiguity and scale ambiguity are inherent. Notably, the scale ambiguity can be resolved easily, and the existence of permutation ambiguity cannot affect the CFO estimation performance.

C. CFO Estimation

As discussed in Section II, the $p$th column of the matrix $A$ is $a_p(\Delta f) = \begin{bmatrix} e^{j2\pi(\frac{p}{N}+\Delta f)} & \ldots & e^{j2\pi((N-1)(\frac{p}{N}+\Delta f)} \end{bmatrix}^T$, from which we denote

$$\hat{g} = \text{imag}(\ln(a_p(\Delta f)))$$

(12)

where $\ln(.)$ is natural logarithm, imag(.) is to get imaginary part of a complex number.

$$\hat{g} = \begin{bmatrix} 0, 2\pi \left(\frac{p-1}{N} + \Delta f\right), \ldots, 2\pi (N-1) \left(\frac{p-1}{N} + \Delta f\right) \end{bmatrix}^T$$

(13)

where $q = [0, 2\pi, \ldots, 2\pi (N-1)]^T$, then LS principle is adopted to estimate CFO $\Delta f$. $\hat{a}_l$ (the $l$th column of the matrix $\hat{A}$) is divided by $\hat{a}_{l,1}$ (the first element of the vector $\hat{a}_l$), by which the scale ambiguity has also been resolved, and thereafter $\hat{g}$ can be obtained within the process according to Eq. (12). Now that we use LS principle to estimate CFO.

Define LS fitting as $\hat{Q} = \hat{g}$, where $Q = [1_{N \times 1}, q]$, $c = [c_0, f_p]^T$, and $\hat{f}_p$ is the estimated value of $\frac{p-1}{N} + \Delta f$, $p \in \{1, 2, \ldots, P\}$. Finally, we have the LS solution for $c$

$$\begin{bmatrix} \hat{c}_0 \\ \hat{f}_p \end{bmatrix} = Q^T \hat{g}$$

(13)

Similarly, we can estimate $\hat{f}_p$, $p = 1, 2, \ldots, P$. Hence, the CFO estimation can be obtained via

$$\hat{f} = \frac{1}{P} \left( \sum_{p=1}^P \hat{f}_p \right)$$

(14)

Notably, the performance of CFO estimation cannot be affected by permutation ambiguity because of the sum operation $\sum_{p=1}^P \hat{f}_p$ in CFO estimation.

In contrast to ESPRIT, our algorithm has a heavier computational load. ESPRIT requires $O(KN^2 + N^3 + 2(N-1)P^2 + 3P^3)$. In our algorithm, the complexity of each TALS iteration is $O(3P^3 + 3PNIK + P^2(IN + IK + NK + I + N + K))$, and the scores of TALS iterations are required for trilinear decomposition with COMFAC algorithm.

Notably, the trilinear decomposition-based algorithm cannot be appropriate for SISO-OFDM systems. When there is only one antenna in the receiver, or just $I = 1$, the signal becomes $X = AD_t(H)B^T = AS$, where $S = D_t(H)B^T$. We consider $X = AS$ as the bilinear model, where the matrix $A$ is with constant modulus. It is generally known that bilinear decomposition lacks uniqueness in normal conditions; however, when under constant modulus constraints, bilinear decomposition can be unique [16], [17].

IV. SIMULATION RESULTS

In most cases of our simulations, we consider the SIMO-OFDM system containing $N = 32$ subcarriers and a CP of $L = 8$. CFO $\Delta \omega = 2\pi \Delta f$ is fixed at $0.3\omega$, where $\omega = 2\pi / N$ represents the subcarrier spacing. Now that we have the received signal $\hat{X}_i = AD_t(H)B^T + W_i (i = 1, \ldots, I)$ in the presence of noise, where $W_i$ is the additive Gaussian white noise matrix. Define SNR

$$SNR = 10 \log_{10} \frac{\sum_{i=1}^I \|A\hat{D}_i(H)B^T\|^2_F}{\sum_{i=1}^I \|W_i\|^2_F}$$

(15)

To quantify the performance of the CFO estimation, the mean-square error (MSE) are estimated from 1000 Monte Carlo tests. We hereby define MSE as $MSE = \frac{1}{1000} \sum_{m=1}^{1000} \left( \hat{\Delta}^m - \Delta f \right)^2$, where $\hat{\Delta}^m$ is the estimated CFO of the $m$th Monte Carlo test, and $\Delta f$ is the perfect CFO.

The channel between the transmitter and the $i$th antenna receiver is characterized by $h_i(t) = \sum_{l=0}^{L_m-1} \rho_l \delta(t - \tau_i)$, where $L_m$ is the number of multipaths, $\rho_l$ and $\tau_i$ are complex gain and delay of the $l$th path, respectively. The channel is modeled to consist of 4 independent Rayleigh fading taps with exponential power delay profile. In each Monte Carlo test, all the channel parameters are randomly generated. We consider a lower mobility case, where the normalized Doppler frequency is $f_dT = 0.002$, with $f_d$ the maximum Doppler frequency shift and $T$ OFDM symbol duration.

We firstly investigate the convergence performance of our proposed algorithm in this simulation. The sum of squared residuals (SSR) in the trilinear fitting should be defined as $SSR = \sum_{n=1}^N \sum_{k=1}^K \sum_{l=1}^L \left[ \tilde{x}_{n,k,i} - \sum_{p=1}^P \tilde{a}_{n,p}\tilde{b}_{k,p}\hat{h}_{i,p} \right]^2$, where $\tilde{x}_{n,k,i}$ is the noisy data. Define $DSSR = SSR_i - SSR_0$, where $SSR_i$ is the SSR of the $i$th iteration, $SSR_0$ is the SSR in the convergence condition. Fig. 1 presents the algorithmic
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In sum, the advantages of our proposed algorithm can be summarized as follows: comparing to ESPRIT method and CS approach, it has better performance (see Figs. 2–3); as a blind method, it requires neither channel knowledge nor statistical characteristics; moreover, our method can still work without virtual carriers. The disadvantages of our method simply lie on two aspects: for one thing, the computational complexity is heavier than ESPRIT; for another, when $I = 1$ (only one antenna in the receiver) the signal should be considered as the bilinear model, for which our algorithm cannot work. The proposed algorithm can be suitable for vehicular channel environment with normalized Doppler frequency of $f_d T \leq 0.02$.

V. CONCLUSIONS

We link the CFO estimation problem of multiple antenna OFDM system to the trilinear model, and then derive a novel blind trilinear decomposition-based CFO estimation algorithm that requires neither pilot nor training sequence. Simulation results reveal that our method is much better than ESPRIT and CS approach. Furthermore, our algorithm utilizes no knowledge of constant modulus or statistical characteristic, and even works without virtual carriers.

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