On the Energy Detection of Unknown Deterministic Signal over Nakagami Channels with Selection Combining

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Presentation Outline

Motivation

System Model

Detection Problem
  Fading
  Diversity

Novel Derivations
  Fading - Nakagami-$m$
  Selection Combiner - Nakagami-$m$

Numerical Results

Conclusion
Why Spectrum Sensing?

Secondary (unlicensed) users are allowed to utilize spectrum holes opportunistically

- Improves spectrum efficiency
- Spectrum scarcity is resolved to certain extend

Spectrum sensing with Energy detection
Why Energy Detection?

- Primary transmission is modeled as an unknown deterministic signal
- Privacy of primary transmissions is guaranteed
  - Detect just existence of transmissions in the spectrum band of interest
  - Less priori knowledge (Signal shape, duration etc not used)
  - Inherent security to primary communication - not tapped
  - Simple in structure - we will see
  - Detect any shape of signal
- Detect the received energy over a period and decide (sub optimal)
- Performance variation over different wireless environment is important
  - Quantify the performance reduction over Fading
  - Overcome with receiver diversity, user cooperation
Detector Model

![Diagram of energy detection process](image)

**Figure:** Energy Detection

- Detection is a test of binary hypothesis ([2])
  - $H_0$: Primary transmission does not exist (noise) - $\chi^2_{2u}$
  - $H_1$: Primary transmission exist - $\chi^2_{2u}(\epsilon)$
- Selection diversity - Select the branch with highest SNR
  - Simple structure is preserved - minor modification
  - Performance is improved
Detection

- Detection probability ($P_d$) (Conditional) [3]

$$P_d = Q_u\left(\sqrt{2\gamma}, \sqrt{\lambda}\right)$$  \hspace{1cm} (1)

- False alarm probability ($P_f$)

$$P_f = \frac{\Gamma \left(u, \frac{1}{2}\right)}{\Gamma(u)}$$  \hspace{1cm} (2)

- Average detection probability ($\overline{P}_d$)

$$\overline{P}_d = \int_0^\infty Q_u\left(\sqrt{2\gamma}, \sqrt{\lambda}\right)f_\gamma(\gamma) \, d\gamma$$  \hspace{1cm} (3)

- False alarm - Does not depend on SNR $\Rightarrow$ fading

$u$ - Time bandwidth product (integers - by choice)

$Q_u(\cdot, \cdot)$ - $u^{th}$ order Marcum Q-function

$\gamma$ - SNR; $\lambda$ - Energy threshold

$\Gamma(\cdot, \cdot)$ - incomplete gamma function
Detection over Fading Channels

- Existing results over fading channels
  - Rayleigh fading - [3, 4, 5]
  - Rician fading - [3, 5]-\(u = 1\)
  - Nakagami-m fading [3]-integer \(m\), [5]-integral

- Integrals found involving Marcum-Q
  - Product of Marcum-Q, Bessel, exponential and rational of \(\gamma\)
  - Uses limited amount of results available [6, 7]
  - Difficult to evaluate in general

- In this paper: Detection over Nakagami-\(m\) fading - alternative result
  - Use of alternative representations of Marcum-Q function
  - Introduced in [8] for analyzing EGC (our result)
  - Transform the integrals to other forms - relatively tractable
Detection over Diversity Receivers

- Find the conditional detection probability and average over the respective PDF of output SNR
- Available results over Rayleigh branches
  - Selection Combining (SC) [3], [4]
  - Maximal Ratio Combining (MRC) [3], [4]
  - Equal gain diversity combining (EGC) [8] - Nakagami-\(m\)
  - Switch and Stay Combining (SSC) [3]
  - Square-Law Combining Schemes (SLC, SLS) [3]
- All diversity results are limited to Rayleigh fading - except our results in [8]
- In this paper: Selection Combining over Nakagami-\(m\) fading
  - Less complex in implementation
  - Quantify the performance gain/loss over
    - Number of branches \(L\)
    - Other parameters \(m, u, \gamma\)
  - Difficulty of integrals? overcome by transforms using alternate representations
Detection over Nakagami-\(m\) Fading

- **Alternate representation** [9]

\[
Q_u(\sqrt{2\gamma}, \sqrt{\lambda}) = \sum_{n=0}^{\infty} \frac{\gamma^n e^{-\gamma}}{n!} \sum_{k=0}^{n+u-1} \frac{e^{-\frac{\lambda}{2}}}{k!} \left(\frac{\lambda}{2}\right)^k
\]  (4)

- **Avg. detection probability** \(\overline{P}_{d,Nak}\)

\[
\overline{P}_{d,Nak} = \frac{e^{-\frac{\lambda}{2}}}{\Gamma(m)} \left(\frac{m}{\gamma + m}\right)^m \sum_{n=0}^{\infty} \left(\frac{\gamma}{\gamma + m}\right)^n \frac{(n + m - 1)!}{n!} \sum_{k=0}^{n+u-1} \frac{1}{k!} \left(\frac{\lambda}{2}\right)^k
\]

- **Error bound** \(|E_{Nak}|

\[
|E_{Nak}| \leq \left(\frac{m}{\gamma + m}\right)^m \left[ {}_1F_0 \left( m; \frac{\gamma}{\gamma + m} \right) - \sum_{n=0}^{N} \left(\frac{\gamma}{\gamma + m}\right)^n \frac{(m)_n}{n!} \right]
\]

Hypergeometric series - \(pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; x)\)

Pochhammer symbol - \((a)_n\)
Detection over Selection Combining

- Alternate representation [9]

\[ Q_u \left( \sqrt{2\gamma}, \sqrt{\lambda} \right) = 1 - e^{-\left(\gamma + \frac{\lambda}{2}\right)} \sum_{n=u}^{\infty} \left( \frac{\lambda}{2\gamma} \right)^n I_n(\sqrt{2\lambda\gamma}) \] (5)

- Avg. Detection over dual branch any \( m \)

\[
\overline{P}_{d,sc,2} = 1 - \frac{2e^{-\frac{\lambda}{2}}}{m} \frac{\Gamma(2m)}{\Gamma^2(m)} \left( \frac{m}{\gamma + 2m} \right)^{2m} \sum_{n=u}^{\infty} \frac{1}{n!} \left( \frac{\lambda}{2} \right)^n \times \Psi_1 \left( 2m, 1; m + 1, n + 1; \frac{m}{\gamma + 2m}, \frac{\lambda\gamma}{2(\gamma + 2m)} \right) \] (6)

- Error bound | \( E_{sc,2} \) |

\[
| E_{sc,2} | \leq \frac{2e^{-\frac{\lambda}{2}}}{m} \frac{\Gamma(2m)}{\Gamma^2(m)} \left( \frac{m}{\gamma + 2m} \right)^{2m} \left( e^{\frac{\lambda}{2}} - \sum_{n=0}^{N} \frac{1}{n!} \left( \frac{\lambda}{2} \right)^n \right) \times \Psi_1 \left( 2m, 1; m + 1, N + 1; \frac{m}{\gamma + 2m}, \frac{\lambda\gamma}{2(\gamma + 2m)} \right) \] (7)

Horns function - \( \Psi_1 (\alpha, \beta; \gamma, \gamma'; x, y) \)
Detection over Selection Combining cont...

- Avg. Detection over integer \( m \) any number of branches

\[
\overline{P}_{d,sc,L} = 1 - Le^{-\frac{\lambda}{2}} \left( \frac{m}{\gamma} \right)^m \sum_{n=u}^{\infty} \sum_{k=0}^{L-1} \left( \frac{L - 1}{k} \right) (-1)^k \frac{1}{n!} \left( \frac{\lambda}{2} \right)^n \\
\times \sum_{i=0}^{k(m-1)} \frac{\zeta_i (m, k, \gamma) (m)_i}{\beta_L^{(i+m)}} \, _1F_1 \left( i + m; n + 1; \frac{\lambda}{2\beta_L} \right)
\]

- Error bound \(| E_{sc,L} |\)

\[
| E_{sc,L} | \leq L e^{-\frac{\lambda}{2}} \left( \frac{m}{\gamma} \right)^m \left[ e^{\frac{\lambda}{2}} - \sum_{n=0}^{N} \frac{1}{n!} \left( \frac{\lambda}{2} \right)^n \right] \sum_{k=0}^{L-1} \left( \frac{L - 1}{k} \right) \\
\times \sum_{i=0}^{k(m-1)} \frac{\zeta_i (m, k, \gamma) (m)_i}{\beta_L^{(i+m)}} \, _1F_1 \left( i + m; N + 1; \frac{\lambda}{2\beta_L} \right)
\]
False Alarm Probability

- Pick the branch with maximum SNR - \( \text{max}(\gamma_1, \gamma_2, \ldots, \gamma_L) \)
- SNR in each branch - \( \gamma_l = \frac{h^2 E_s}{N_{01}}, \ l = 1, 2, \ldots, L \)
- Select the branch with maximum \( h_l \)
- Under \( H_0 \): Samples from noise (does not matter branch selection)
- The statistics of decision variable is \( \chi^2_{2u} \)
- False alarm probability (tail of \( \chi^2_{2u} \))

\[
\overline{P}_{f,sc,L} = \frac{\Gamma \left( u, \frac{\lambda}{2} \right)}{\Gamma(u)} \tag{9}
\]

- Nothing to average
Convergence of Series

- Truncation error

![Error Bound Graph]

**Figure:** Error Bounds $u = 1, m = 2, SNR = 10 \, dB, L = 2$ and $P_f = 0.01$
**Figure:** Complementary ROC curves over Nakagami-$m$ fading channel ($u = 1, m = 2, SNR = \{0, 10, 15, 20\} \text{ dB}$)
Detector Performance - $m$

Figure: Complementary ROC curves over Nakagami-$m$ fading channel ($u = 1$, $m = \{1, 2, 3, 4\}$, $SNR = 10 \text{ dB}$)
Detector Performance - Branches

Figure: Complementary ROC curves of SC receiver over Nakagami-$m$ fading channel
$(L = \{1, 2, 3, 4\}, u = 2, m = 2, SNR = 10 \, dB)$
Conclusion

- Energy Detection over Fading
  - Nakagami-$m$ Fading
  - Exact detection probability, Alternative expression

- Energy Detection over SC
  - Nakagami-$m$ Fading
  - Exact detection probability

- Use of alternative representation of Marcum-Q function for evaluation of related integral

- Simulation results
  - support the decision variable formulation, derivations

- Helpful in designing and evaluating
  - Cognitive radio, Ultra wide-band
  - Detector threshold for expected false alarm rate and different $u$ and $\bar{\gamma}$
Thank You

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References I


References II

