ISAR Image Formation Based on Minimum Entropy Criterion and Fractional Fourier Transform

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SUMMARY
Conventional radar imaging systems use Fourier transform for image formation, but due to the target’s complicated motion the Doppler spectrum is time-varying and thus the reconstructed image becomes blurred even after applying standard motion compensation algorithms. Therefore, sophisticated algorithms such as polar reformatting are usually employed to produce clear images. Alternatively, Joint Time-Frequency (JTF) analysis can be used for image formation which produces clear image without using polar reformatting algorithm. In this paper, a new JTF-based method is proposed for image formation in inverse synthetic aperture radars (ISAR). This method uses minimum entropy criterion for optimum parameter adjustment of JTF algorithms. Short Time Fourier Transform (STFT) and Fractional Fourier Transform (FrFT) are applied as JTF for time-varying Doppler spectrum analysis. Both the width of Gaussian window of STFT and the order of FrFT, \( \alpha \), are adjusted using minimum entropy as local and total measures. Furthermore, a new statistical parameter, called normalized correlation, is defined for comparison of images reconstructed by different methods. Simulation results show that \( \alpha \)-order FrFT with local adjustment has much better performance than the other methods in this category even in low SNR.

key words: radar imaging system, ISAR, JTF, FrFT, STFT, minimum entropy

1. Introduction

Inverse Synthetic Aperture Radar (ISAR) imaging involves a stationary radar viewing moving targets in order to form 2-D images from them [1]–[3]. The radar image is usually mapped onto range and cross-range planes. The range resolution of a radar image is directly related to the bandwidth of the transmitted radar signal, and the cross-range resolution is achieved by synthetic array processing that coherently combines signals obtained from the sequences of small apertures at different aspect angles to the target [1], [4], [5]. The change of the aspect angle is usually due to the relative motion and rotation between the radar and the target.

ISAR uses Doppler processing to estimate the distribution of the target’s reflectivity. The conventional approach to retrieving Doppler information is the Fourier transform. However, in real-world ISAR, targets are generally engaged complicated maneuvers and so they have both translational and rotational motions. In these cases, the resulting ISAR image, retrieved by Fourier transform, is severely distorted due to the time-varying Doppler spectrum [6]–[8]. To obtain a clear image, many efforts have focused on developing motion compensation algorithms which reduce the effect of nonuniform motion [5], [9], [10]. Standard motion compensation methods reduce the effect of translational motion and are effective for targets with smooth motion, but these methods fail to generate a clear image using Fourier transform for the targets with more complicated motions [1], [3], [5]. In such situations, more complicated algorithms like polar reformatting have been proposed. However, this algorithm has high computational load. Furthermore, it does not result in acceptable images in some situations with complicated motions [2], [3], [10]. In these cases, Joint Time-Frequency (JTF) analysis can be used instead of Fourier transform [2], [6], [8], [11].

In the situations where the Doppler spectrum is time-varying, JTF analysis is an efficient method to resolve the spectrum and to form a clear image. In these methods, standard motion compensation is firstly used without applying sophisticated polar reformatting algorithm [2], [7], [8]. Subsequently, JTF processing is used to analyze the time-varying spectra and retrieve Doppler information to reconstruct a clear ISAR image.

In general, any JTF method with high resolution and low cross-term can be used for image formation. Image formation using STFT**, TFDS*** and AGR**** has been discussed in [12]–[15]. Linear JTF, like STFT, does not have cross-term and uses different types of kernels to analyze a non-stationary signal. Based on the Uncertainty Principle, choosing the type of this kernel and its width is very important [2], [16].

On the other hand, bilinear JTF methods, like WVD***** has better resolution compared with STFT. But WVD produces some cross-terms and so is not proper for ISAR image formation [2], [6], [16]. Some methods like TFDS and Cohen class transforms have been proposed to reduce the effect of cross-terms in bilinear JTF algorithms and are used for ISAR image formation [13], [14], [16].

Recently, Fractional Fourier Transform has been developed as a generalization of Fourier transform [17]–[20]. This is a good Time-Frequency transform that exhibits high resolution property without cross-term effect and therefore is a proper choice for ISAR image formation [11], [21].

Manuscript received November 10, 2008.
Manuscript revised March 7, 2009.

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DOI: 10.1587/transcom.E92.B.2714

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It should be noted that there are some adjustable parameters in all JTF methods (except WVD), which should be properly adjusted and there is no a systematic approach for adjusting them so far.

In this paper, a new approach for ISAR image formation is proposed. In this method, the minimum entropy criterion is used for adjusting the parameters of JTF algorithms like STFT and FrFT\(^\dagger\), while it is conventionally employed for motion compensation. Furthermore, a new parameter, i.e. normalized correlation, is proposed for comparison of some ISAR image formation methods, while conventional comparison of various image formation methods is done without using any statistical parameter. Simulation is used to show that the proposed method has better performance than the other existing methods.

In this paper, the background of radar imaging of moving targets and time-varying behavior of Doppler spectrum is briefly described in Sect. 2. In Sect. 3, the entropy criterion is discussed and by applying this criterion for STFT kernel width adjustment, the ISAR image is reconstructed. Definition and characteristics of \(\alpha\)-order FrFT are introduced in Sect. 4. Furthermore, estimation of the optimum order of FrFT for ISAR Image formation using minimum entropy criterion is proposed in this section. In Sect. 5, normalized correlation parameter for comparison of different image formation methods is given and employed for comparing the performance of investigated methods. Finally, some conclusions are described in Sect. 6.

2. ISAR Imaging

The geometry of radar imaging of a point target is shown in Fig. 1. The distance from the radar to the point scatterer is \(r(t)\), and the distance from the radar to the center of rotation is \(R(t)\). Assume that the point scatterer is located at distance \(l\) from the center of rotation and the initial rotation angle of the point scatterer is \(\theta_0\). If the point target has motion with a radial velocity of \(v\) (m/s), \(R(t) = R(0) - vt\) and rotational motion with an angular rotation rate of \(\omega\), the returned signal at time \(t\) is received with delay equal to \(\tau(t) = 2\pi t/c\) and hence the phase of this signal becomes [1]:

\[
\phi(t) = -2\pi f \tau(t) = \frac{-4\pi f}{c} (R(0) - vt - l \cos(\omega t - \theta_0))
\]  

(1)

where \(f\) and \(c\) are carrier frequency and wave propagation speed, respectively. Note that in Eq. (1), \(l \ll R(t)\), \(\forall t\) has been assumed.

By taking the time derivation of Eq. (1), the Doppler spectrum is obtained as:

\[
f_D(t) = \frac{1}{2\pi} \frac{df(t)}{dt} = \frac{2fv}{c} - \frac{2f}{c} \omega \sin(\omega t - \theta_0)
\]

\[
= (f_D)_0 + (f_D)_\omega
\]  

(2)

As it is seen, Doppler spectrum consists of two terms, one due to the constant velocity of translational motion and another due to the rotational motion. The former is not time dependent and can be removed by using standard motion compensation. However, the latter is time-varying although the rotation rate is constant. This term can be exploited to retrieve Doppler information and to estimate the location of the scatterer.

Based on the above formulation and assuming ideal channel (without noise and any distortion), the returned signal, \(S'_r(t)\), for a point target can be expressed as [2]:

\[
S'_r(t) = \rho \cdot s(t - \tau(t))
\]  

(3)

where \(\rho\) is the reflectivity of the point target and \(s(.)\) denote the transmitted signal.

Considering Eq. (3) for the returned signal from a single point scatterer, the returned signal from the whole target can be represented as the integration of all scatterers in the target [2], [5], [22]:

\[
S'_r(t) = \int_{x,y} \rho(x,y) \cdot s(t - \tau_{x,y}(t))
\]  

(4)

where \(\tau_{x,y}(t)\) is the delay associated with the scatterer located at point \((x, y)\), and \(\rho(x, y)\) is the reflectivity function of the target. The \(x,y\) coordinate origin is assumed to be at the center of rotation. It can be simply shown that [2], [4], [5]:

\[
\tau_{x,y}(t) = \frac{2r_{x,y}(t)}{c} = \frac{2}{c} [R(t) + x \cos(\theta(t)) - y \sin(\theta(t))] \]

(5)

where \(\theta(t)\) is the relative angle between the target and the radar at time \(t\) which in this case becomes:

\[
\theta(t) = \omega t - \theta_0
\]  

(6)

It is assumed that the target has a rigid body and consists of finite number of point scatterers with constant, time-independent reflectivity function, \(\rho(x, y)\). Also, assume that in the pulse time duration, \(s(t) = \text{Re}(\exp(2\pi f \cdot j \cdot f \cdot t))\). Based on these assumptions, the baseband signal received from the target can be written as [2], [6], [22]:

\[
S'_r(t) = \sum_{x,y} \left\{ \rho(x,y) \cdot \exp \left[ -j2\pi \frac{2f}{c} R(t) \right] \times \exp \left[ -j2\pi \frac{2f}{c} (x \cos(\theta(t)) - y \sin(\theta(t))) \right] \right\}
\]

\[
\text{for } \frac{2r_{x,y}(t)}{c} \leq t \leq \frac{2r_{x,y}(t)}{c} + T_p
\]  

(7)

where \(T_p\) is the pulse repetition period.

\(^\dagger\)Fractional Fourier Transform.
By using perfect standard motion compensation, the effect of \( R(t) \) can be eliminated and hence the received signal becomes [2], [5], [22]:

\[
S_r(t) = \sum_{x,y} \rho(x,y) \cdot \exp[-j2\pi(\frac{2f}{c}\cos(\theta(t)) - \frac{2f}{c}\sin(\theta(t))] \quad (8)
\]

where:

\[
\begin{align*}
f_x(t) &= \frac{2f}{c}\cos(\theta(t)) \\
f_y(t) &= \frac{2f}{c}\sin(\theta(t))
\end{align*}
\quad (9)
\]

A radar with stepped-frequency waveform transmits a sequence of \( N \) bursts consisting of \( n \) narrowband pulses. The time duration of each pulse is denoted by \( \tau \). The carrier frequency of each successive pulse in a burst is increased by a constant frequency step \( \Delta f \) [1]–[3]. The total bandwidth of each burst, i.e., \((n - 1) \times \Delta f\), determines the radar range resolution. Furthermore the total number of bursts, \( N \), for a given PRF\(^{\dagger}\), \( n \) and average carrier frequency, affects the radar cross-range resolution [1], [2].

For stepped-frequency radars, \( S_r(t) \) can be written like Eq. (8) and Eq. (9). But \( f \) for each burst should be replaced by \( f_k = f_0 + k \cdot \Delta f; \ k = 0, 1, \ldots, n - 1 \).

To form a radar image, it is assumed that in each pulse repetition period one sample is derived and so we have \( N \times n \) samples for all bursts. These complex samples are organized into a two-dimensional \( N \times n \) array, \( H \), and standard motion compensation is performed on this raw data. Now an \( n \)-point IDFT\(^{\dagger\dagger}\) is performed on each row of standard motion compensated array \( G \):

\[
G_1 = \text{IDFT}_n \{ \text{rows}(G) \} \quad (10)
\]

Therefore, \( N \) range profiles can be obtained and each range profile contains \( n \) range cells. The Fourier imaging approach takes the DFT\(^{\dagger\dagger\dagger}\) of each column of \( G_1 \) to generate \( N \)-point Doppler spectrum as shown in Fig. 2. It should be noted that before this, polar reformatting can be applied.

\[
G_2 = \text{DFT}_N \{ \text{columns}(G_1) \} \quad (11)
\]

Finally, the radar image, \( G_2 \), is the target’s reflectivity mapped on to the range-doppler plane and abs\((G_2)\) can be displayed as a radar image [1]–[5].

On the other hand, proper JTF methods, instead of Fourier transform, can be used to generate Doppler spectrum. In these methods, each \( N \)-point column is analyzed at \( M \) different time to resolve time-varying behavior of the signal.

\[
G_3 = \text{Time–Frequency–Transform}_M \{ \text{columns}(G_1) \} \quad (12)
\]

Hence \( G_3 \) is a \( M \times N \times n \) time-Doppler-range cube, as shown in Fig. 3, and in each particular time, one clear range-doppler image can be extracted [2], [6], [7]. A radar image can be displayed by abs\((G_3)\) and in this approach the polar reformatting is not used.

It should be noted that if the Time-Frequency transform used in Eq. (12) satisfies the marginal condition [2], [16], the Fourier image (without polar reformatting), \( G_2 \), can be obtained by time-averaging of \( G_3 \). In fact, the blurring effect of Fourier image is due to this averaging [2], [6], [16].

3. Parameter Adjustment in JTF Algorithms

Application of minimum entropy criterion has been reported in many references for motion compensation [3], [9]. This paper proposes minimum entropy criterion for image formation, which is one of the contributions of this work. In the following, first the concept of minimum entropy criterion and then the proposed method are explained.
3.1 Minimum Entropy Criterion

A well-known criterion which is used in motion compensation algorithm is minimum entropy [3], [9]. In fact, entropy is used to measure the degree of sharpness, i.e., low entropy means high image focusing.

For a given set \( A = \{a_1, a_2, \ldots \} \) with \( a_i \geq 0 \) and \( \sum a_i = 1 \), entropy of \( A \) is defined as:

\[
H(A) = - \sum_i a_i \ln(a_i)
\]

(13)

Indeed, entropy is defined for a probability distribution, but it can also be defined for any vector (matrix) as a proper function.

For an arbitrary complex vector \( \mathbf{I} = \{I_1, I_2, \ldots \} \), a normalized vector can be defined as:

\[
\tilde{I}_i = \frac{|I_i|^2}{\sum_j |I_j|^2}
\]

(14)

And so:

\[
\forall i : \tilde{I}_i > 0, \sum_i \tilde{I}_i = 1
\]

(15)

Now, entropy can be defined for \( \tilde{I} \) and used to measure the sharpness of \( \tilde{I} \). Likewise, 2-D\(^1\) entropy can be applied to an arbitrary matrix \( \mathbf{B} \) as:

\[
H(\mathbf{B}) = - \sum_{i,j} \frac{|b_{i,j}|^2}{\sum_i |b_{i,j}|^2} \ln \left( \frac{|b_{i,j}|^2}{\sum_{i,j} |b_{i,j}|^2} \right)
\]

(16)

Entropy was first used by Shannon in the information theory [23]. It is shown that entropy is a convex function and has an absolute minimum.

3.2 Application to Image Formation

In this section, the minimum entropy criterion is used for optimizing the parameters involving in image formation of ISAR. It should be noted that Fourier based image formation has no parameter, but JTF methods like STFT, have some parameters.

Linear JTF methods use various kernels to analyze the non-stationary signals [2]. Based on the uncertainty principle, the type and width of this kernel play an important role in the analysis. It is known that Gaussian kernel satisfies the equality in the uncertainty principle and therefore improves the time and frequency resolution in the best manner [2]. Hence, a Gaussian STFT is considered and its kernel width is adjusted by using the minimum entropy criterion to form the best STFT based image.

Minimum entropy criterion is used as total and local measure. In the total case, 2-D entropy function, \( H_{-D} \), is calculated on matrix \( G_3 \) and consequently the optimum kernel width, \( d \), is obtained as follow.

Assume that \( \tilde{G}_3 = [\tilde{a}_{i,j}] \) is the normalized version of one arbitrary chosen \( N \times n \) frame of \( G_3 \) (for example at \( t = M/2 \) for even \( M \)). Like Eq. (14) and Eq. (16), the 2-D entropy function is defined as:

\[
H_{-D}(\cdot) = - \sum_{i=1}^{N} \sum_{j=1}^{n} a_{i,j} \ln(a_{i,j})
\]

(17)

Clearly, \( \tilde{G}_3 \) and hence \( a_{i,j} \) are functions of \( d \). Then for obtaining optimum \( d \), i.e. \( d_{opt} \), the equation \( \frac{\partial H_{-D}}{\partial d} = 0 \) should be solved.

In the local case, 1-D entropy function, \( H_{1-D,j}(\cdot) \), is calculated on each individual normalized column of matrix \( G_3 \) and for each column, kernel width, \( d \), is adjusted separately as bellow:

\[
H_{1-D,j}(\cdot) = - \sum_{i=1}^{N} a_{i,j} \ln(a_{i,j})
\]

(18)

And for obtaining \( d_{opt,j} \), the equation \( \frac{\partial H_{1-D,j}}{\partial d} = 0 \) should be solved.

Because of complicated relationship between \( d \) and \( a_{i,j} \), \( d_{opt} \) and \( d_{opt,j} (j = 1, 2, \ldots, n) \) can be found by search over reasonable range for \( d \).

The simulation has been used for investigating the improvement in image formation due to applying these two methods. In the simulation, the radar is assumed to operate at 9 GHz and transmits stepped-frequency waveform that in each of 512 bursts, 64 stepped frequency are used to cover a total 150 MHz bandwidth and achieve a 1(m) range resolution [1], [2]. The total observation time is 2 seconds and hence \( 0.1(m) \) cross-range resolution is achievable by rotational motion at rate 4.5 degree/sec [1], [2].

An aircraft is simulated in term of its 2-D reflectivity function \( \rho(x, y) \) as a target. The target is assumed to have only rotational motion at the rate of 4.5 degree/sec. Assume that standard motion compensated data, i.e. \( G \), is simulated and after performing IDFT on its rows, it is entered to the JTF algorithm. Figure 4 shows the images formed via FFT without polar reformatting, STFT with local adjustment of kernel width and STFT with total adjustment of kernel width and the original image. Note that \( d_{opt} \) and \( d_{opt,j} (j = 1, 2, \ldots, n) \) have been obtained by direct search.

As can be seen from Fig. 4, the image formed via STFT with local adjustment of the kernel width is better than the others. This can be explained that because of different distance of scatterers from the center of rotation, the time changing of columns spectrum is different and so STFT method with local adjustment of the kernel width can perform better than the other methods. This phenomena has been clarified in Fig. 5. that shows the range dependency of optimum kernel width.

4. Fractional Fourier Transform (FrFT)

In this section, the concept and some important properties of FrFT are firstly explained. After this, the application of

\(^{1}\)2-Dimentioal.
FrFT in radar image formation is described.

4.1 Definition and Some Properties

The FrFT is a generalization of the conventional Fourier transform with some application in quantum mechanics and digital signal processing [17]–[20]. It is not only richer in theory and more flexible in application, but also is not expensive in implementation. FrFT is a powerful tool for the analysis of time-varying signals.

The $\alpha$-order FrFT of a signal $x(t)$ can be described as follows [18], [24]:

$$\text{FrFT}_\alpha\{x(t)\} = F^\alpha(u) = \int x(t) k_\alpha(t, u) dt$$  \hspace{1cm} (19)

where the kernel $k_\alpha(t, u)$ is defined as:

$$k_\alpha(t, u) = A_\alpha \exp\left[j\pi(u^2 \cot \alpha - 2ut \csc \alpha + t^2 \cot \alpha)\right]$$  \hspace{1cm} (20)

where $A_\alpha = \sqrt{1 - j \cot \alpha}$, $\alpha = a\pi/2$ and $u$ is the fractional Fourier domain variable. Performing the $\alpha$-order FrFT rotates the time-frequency (TF) plane anticlockwise around origin by angle $\alpha$. Therefore, FrFT is equivalent to identity transform when $\alpha = 0$, while it is equivalent to the conventional Fourier transform when $\alpha = \pi/2$, and it is 4-periodic respected to $\alpha$.

Furthermore, generalized Parseval’s theorem can be proved for FrFT [17]–[19]:

$$\int h(t)g^*(t)dt = \int H^\alpha(u)G^\alpha(u)du$$  \hspace{1cm} (21)

Linearity, inversibility and unitary are some other fundamental properties of the FrFT [17]–[20].
It should be noted that a discrete FrFT also exists known as DFrFT\(^{†}\). Despite some difficulties in obtaining a closed-form definition of the transform that satisfies all properties of the continuous case, an efficient algorithm for computation of the kernel matrix has been presented by Ozaktas that satisfies the most important properties of the continuous case [18], [24], [25]. Hence, for DFrFT, it is shown:

\[
\hat{f}_a = F_a \cdot f 
\]  
(22)

where \( f \) is the input vector, \( f_a \) is the transformed vector and \( F_a \) is the kernel matrix which is constructed from the DFT kernel [18], [24], [25].

4.2 Application to Radar Image Formation

It was mentioned that FrFT is a linear time-frequency transform. Besides, from Eq. (19) and Eq. (20) FrFT can be considered as a chirp decomposition. Therefore, if ISAR received signal can be approximated as chirp, FrFT can be a proper tool for ISAR image formation.

The radar received signal after standard motion compensation for stepped-frequency waveform can be considered as Eq. (8) and Eq. (9):

\[
S_r(t) = \sum_{x,y} \rho(x,y). \exp[-j2\pi(xf_i(t) - yf_y(t))]
\]

where:

\[
\begin{align*}
    f_i(t) &= \frac{2f_k}{c} \cos(\theta(t)) \\
    f_y(t) &= \frac{2f_k}{c} \sin(\theta(t))
\end{align*}
\]

Based on Eq. (8), for fixed value of \( x, y \) (one special scatterer), the instantaneous frequency, \( f_i(t) \), becomes:

\[
\frac{df_i(t)}{dt} = \frac{2f_k}{c} \cdot \frac{d\varphi_i(t)}{dt} = \frac{2f_k}{c} \cdot [x \sin(\theta(t)) + y \cos(\theta(t))]
\]

(23)

Where \( \varphi_i(t) \) is the instantaneous phase for fixed value of \( x, y \). Assuming \( \theta(t) = \theta_0 + \omega \cdot t \) and \( \theta(t) \ll 1 \) (rad) we have:

\[
\begin{align*}
    \cos(\theta(t)) &\approx 1 \\
    \sin(\theta(t)) &\approx \theta(t)
\end{align*}
\]

The instantaneous frequency can be written as:

\[
\begin{align*}
    f_i(t) &= \frac{2f_k}{c} \omega \cdot (x\theta_0 + x\omega \cdot t + y) \\
    &= \frac{2f_k}{c} \omega \cdot (x\theta_0 + y) + \frac{2f_k}{c} x\omega^2 \cdot t
\end{align*}
\]

(24)

Hence, this instantaneous frequency shows that the reflected signal of each scatterer can be considered as a chirp pulse even for linear assumption for \( \theta(t) \). Therefore, \( S_r(t) \) can be thought as the sum of these chirp pulses for all scatterers.

By sampling \( S_r(t) \) at \( t = (l + mn)T_p, l = 0, 1, \ldots, n - 1; m = 0, 1, \ldots, N - 1 \), the 2-D \( N \times N \) complex standard motion compensated array, \( G \), is organized. The cross-range profile sampled at \( t = mnT_p; m = 0, 1, \ldots, N - 1 \) becomes:

\[
S_r(m) = \sum_{x,y} \rho(x,y) \exp \left\{ -j\frac{4\pi}{c} \cdot \left[ x\cos(\theta(mnT_p)) - y\sin(\theta(mnT_p)) \right] \right\}
\]

(25)

And a typical scatterer instantaneous frequency in discrete variable, \( m \), can be written as:

\[
\frac{2f_k}{\omega} \cdot (mnT_p) + \frac{2f_k}{c} x\omega^2 \cdot (x\theta_0 + y)
\]

(26)

In [11], it has been shown that the sum of some discrete chirp pulses can be approximated as a discrete chirp and hence well analyzed by \( \alpha \)-order FrFT:

\[
G_4 = FrFT^\alpha\{column(G_1)\}
\]

(27)

The parameter of FrFT, i.e. \( \alpha \), should be adjusted for Doppler processing. We propose to use minimum Entropy criterion for parameter adjustment like in the previous section. Similar to those for STFT, the minimum entropy criterion can be employed as a local criterion that is a 1-D function, \( H_{\alpha-DFrFT}^\alpha \) imposed on each column \( G_4 \), or can be used as a total criterion that is a 2-D function, \( H_{\alpha-DFrFT}^\alpha \) imposed on all the columns of the matrix \( G_4 \). Assume that \( G_4 = [b_{ij}] \) is the normalized version of \( G_4 \) like Eq. (17). \( H_{\alpha-DFrFT}^\alpha \) can be defined \(^{\dagger}\)Discrete FrFT.
as follow:

\[ H'_{z-D}(\cdot) = - \sum_{j=1}^{N} \sum_{i=1}^{N} b'_{ij} \ln(b'_{ij}) \] (28)

And for obtaining \( \alpha_{\text{opt}} \), the equation \( \frac{\partial H'_{z-D}}{\partial \alpha} = 0 \) should be solved.

The same equation can be written for local case like Eq. (18):

\[ H'_{z-D,j}(\cdot) = - \sum_{i=1}^{N} b_{ij} \ln(b_{ij}) \] (29)

Then for obtaining \( \alpha_{\text{opt},j} \), the equation \( \frac{\partial H'_{z-D,j}}{\partial \alpha} = 0 \) should be solved.

In this situation, \( \alpha_{\text{opt},j} \) and \( \alpha_{\text{opt}} \) can be found by search over \(-\pi\) to \(\pi\) because of periodicity of FrFT respected to \(\alpha\).

Again, simulation is used for comparing this approach with some other existing methods. The simulation parameters are the same as those of Sect. 3. It is expected that local optimization of FrFT parameter is better than its total optimization. This can be confirmed from Fig. 6. Also, in Fig. 7, the range dependency of optimum order of FrFT, \( \alpha_{\text{opt},j} \), has been shown to clarify the effect of proposed method. Note that this method only requires standard motion compensation and adjustment of FrFT order, \(\alpha\), by using minimum entropy criterion. In other words, it does not need any estimation of motion parameters. Besides, the optimum order can always be found because of the convexity of the entropy function.

5. Normalized Correlation Parameter

Different methods have been suggested for radar image formation in the literatures [1], [2], [5]. Many reconstructed images obtained from different methods have minor difference, and this can be hardly recognized by human eyes. In the simulations, shown in Sects. 3 and 4, the samples have been generated based on 2-D reflectivity function \( \rho(x, y) \). Therefore normalized correlation parameter can be defined for comparison of reconstructed image with the original one. Clearly, this parameter can be used only in the theory and simulation (for comparison of some methods), but it is not applicable in the real ISAR, because the real reflectivity function \( \rho(x, y) \) is not accessible.

Assume that pixel values of the original image, i.e. \( \rho(x, y) \), is denoted by matrix \( I_{\text{org}} \) and for the reconstructed image by matrix \( I_{\text{isar}} \). So, we use normalized correlation between original and reconstructed images, \( I_{\text{org}} \) and \( I_{\text{isar}} \), to compare them.

Because of remained error of motion compensation algorithm, the reconstructed image, \( I_{\text{isar}} \), may be shifted in the plane and therefore the following definition of \( R \) may be preferred:

\[
R = \max_{p,q} \left\{ \frac{1}{\sqrt{E_{\text{isar}} E_{\text{org}}}} \sum_{i,j} I_{\text{org}}(i,j) \cdot I_{\text{isar}}(i-p,j-q) \right\} \] (30)

Where, \( p, q \) are the range and cross-range shift parameters, and \( E_{\text{isar}}, E_{\text{org}} \) are the energy of \( I_{\text{isar}} \) and \( I_{\text{org}} \), respectively. Simulation results for rotational rate of 4.5 deg/s (total rotated angle=9 degree) are shown in Table 1.

Table 1 shows that there is much differences between FrFT and STFT in the local adjustment (FrFT is much better) and also FrFT with local adjustment is the best method in this category.

Figure 8 shows the normalized correlation values cal-

<table>
<thead>
<tr>
<th>Method</th>
<th>Average normalized correlation ((R))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fourier transform</td>
<td>0.3856</td>
</tr>
<tr>
<td>STFT with local adjustment of kernel</td>
<td>0.5915</td>
</tr>
<tr>
<td>STFT with total adjustment of kernel</td>
<td>0.5219</td>
</tr>
<tr>
<td>FrFT with local adjustment of (\alpha)-order</td>
<td>0.9381</td>
</tr>
<tr>
<td>FrFT with total adjustment of (\alpha)-order</td>
<td>0.4469</td>
</tr>
</tbody>
</table>
calculated for various total rotated angle. As can be seen, the performance of all investigated methods is decreasing when the total rotated angle is increasing, while the proposed method, FrFT with local adjustment of $\alpha$, shows relatively the same performance for a large range of total rotated angle. Besides, FrFT with local adjustment of $\alpha$ is the best method among the investigated methods.

In real ISAR, the baseband received signal may be severely affected with noise. If $r(t)$ be the actual baseband received signal and $n(t)$ be the additive white Gaussian noise, we have:

$$r(t) = S_r(t) + n(t)$$  \hspace{1cm} (31)

Therefore, for studying the effect of noise on the performance, the simulation parameters similar to those of Sect. 3 are considered, and the additive complex white Gaussian noise (AWGN) with various powers is added to the baseband signal $S_r(t)$. Figure 9 shows the variation of $R$ against the SNR at a constant rotational rate (4.5 deg/s). It is shown that noise degrades the performance of all methods, but the proposed method still demonstrates reliable results even at low SNR.

6. Conclusion

In this paper, the minimum entropy criterion has been proposed for optimum parameter adjustment in image formation methods, while it has been conventionally employed for motion compensation. The width of Gaussian kernel of STFT and the order of FrFT, $\alpha$, have been adjusted using this criterion in the local and total cases. Also, normalized correlation parameter has been used to permit the comparison of various reconstructed images. Simulation results illustrate that FrFT method with local order adjustment has better performance than the other methods (like Fourier based and STFT based methods). Moreover, it has been shown that this method has high reliability even at low SNR.

Acknowledgments

The authors would like to thank Dr. H.R. Karshenas for paper review and useful notes.

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