The reflection of nonlinear irregular surface water waves

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A B S T R A C T

This paper concerns the reflection of irregular surface water waves from an impermeable vertical wall and investigates the similarities between this case and the direct, head-on collision of two identical wave groups travelling in opposite directions. A new set of experimental measurements is presented and compared with fully nonlinear numerical predictions based upon a multiple-flux boundary element method (BEM). Comparisons concern both the spatial and the temporal water surface elevations in the vicinity of the focused wave event; the latter occurring at the location of the wall. Linear and second-order irregular wave theories, commonly adopted as the basis for design solutions, are shown to significantly under-predict both the wave steepness and the maximum crest elevation in the vicinity of the wall. In contrast, the fully nonlinear numerical predictions are shown to be in very good agreement with the experimental measurements; the present results having been achieved without the need for smoothing, filtering or regridding.

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1. Introduction

Building on the initial success of Longuet-Higgins and Cokelet [1], many authors have sought to provide accurate descriptions of surface gravity waves based upon either a boundary integral or a boundary element formulation. Indeed, the popularity of these approaches, involving a cumulative and sustained research effort, is demonstrated by the fact that the numerical wave tank is commonly adopted as a test case for intermodel comparisons. Whenever such calculations are undertaken in the physical domain, the imposition of the lateral boundaries at the upstream and downstream ends, involves the introduction of geometric discontinuities or corners. In the context of the boundary element method (BEM) these discontinuities are notoriously difficult to model, particularly where they involve the intersection between the oscillating water surface and an impermeable boundary. They define what is commonly referred to as the “corner problem” and provide a contribution to the occurrence of saw-tooth instabilities; the latter limiting the application of many BEM solutions.

In practical applications, particularly those relevant to the coastal and offshore engineering industries, the imposition of lateral boundary conditions often represents a key part of the problem to be solved. First, the boundaries themselves may represent a critical point of interest. Examples of this include the reflection of incident waves from a sea wall or the excitation of resonant modes within a confined harbour. Second, boundaries are frequently introduced within a computational domain to take advantage of symmetries and hence reduce the required computational effort. Third, even in those circumstances where the boundaries are (physically) far removed from the key points of interest, perhaps specified by an open or radiation condition, the nature of a BEM solution is such that any errors arising at the discontinuity will not remain local to the corner area. Indeed, they will spread outwards to “contaminate” the entire domain; their rate of spread being significantly faster than that associated with spurious reflections.

In previous studies corner problems have been tackled using either discontinuous elements or double nodes; the latter being more commonly applied in the context of wave modelling [2,3]. However, irrespective of which technique is employed, descriptions of the largest most nonlinear waves, which are also those most pertinent to engineering design, can only be achieved with significant smoothing and filtering of the evolving water surface elevation. Furthermore, if calculations are undertaken in a Lagrangian frame of reference with the surface nodes free to move both vertically and horizontally, the occurrence of significant wave-induced drift causes additional difficulties at the input boundary. These are associated with the stretching of surface elements, involving a consequent loss of accuracy and further compounding the local corner problem. Such difficulties are typically overcome by regridding, but this implies an additional level of filtering.

In many practical applications the smoothing, filtering and regridding noted above does not detract from the success of the modelling procedure; BEM formulations provide many valuable descriptions. However, recent research concerning both the evolution of extreme waves and their interaction with structures and/or vessels has highlighted the importance of unexpected
energy transfers. For example Johannessen and Swan [4,5] and, more recently, Gibson and Swan [6] have shown that the evolution of freak or rogue waves may be associated with a local and rapid broadening of the wave spectrum, involving a movement of wave energy into the high-frequency tail. Unrelated work [7] has also shown that when a single surface-piercing column is subject to steep incident waves, part of the scattered or diffracted wave field is associated with a circulation of fluid about the column at the instantaneous water surface and involves the scattering of unexpectedly high-frequency components. These effects cannot be predicted by existing diffraction solutions, involve significant force components up to and beyond the fifth harmonic of the incident waves, and are believed to be relevant to the onset of “ringing” or transient structural deflections that have adversely affected some large volume offshore structures.

In considering these effects, concerns have been raised that where solution procedures are critically dependent upon the applied filtering, important contributions may be lost [8,9]; not least because the extent of the energy transfers cannot be known a priori. With this in mind, we have sought to provide a fully nonlinear wave model, ultimately capable of describing overturning waves, that has no smoothing, filtering or regridding. To realise this goal, two initial steps need to be undertaken: (i) the corner problem needs to be fully resolved and (ii) the success of the model in respect of steep waves interacting with the lateral boundary needs to be clearly demonstrated. The initial step has been addressed by several authors with a recent contribution by Hague and Swan [8], who implemented the multiple-flux technique of Brebbia and Dominguez [10] in a numerical wave tank. The second step is the subject of the present paper and will contrast multiple-flux BEM calculations with a new series of laboratory observations concerning the evolution of large (and steep) waves in an irregular wavefield and their interaction with a vertical boundary.

To quantify the success of the BEM solution, comparisons will be made between numerical predictions and experimental observations relating to two test cases. Both involve large individual or focused wave events arising as part of a transient wave group in an irregular or random wave field. The related test cases are denoted as:

(A) Wave reflections: Involving the interaction between a wave group and an impermeable vertical wall.

(B) Wave-wave interactions: Involving the interactions (or head-on collision) of two wave groups travelling in opposite directions.

In undertaking numerical and experimental studies of both cases, comparisons between (A) and (B) and between the numerical and experimental results provide a clear opportunity to investigate the success of the model, both in terms of the effects included and what it omits.

The paper continues in Section 2 with a brief review of related work. A summary of the BEM formulation, concentrating on the method of multiple fluxes, is given in Section 3. Details of the laboratory observations are presented in Section 4; while Section 5 provides comparisons between the measured data and the numerical predictions. Section 6 outlines some concluding remarks and comments on the wider implications of the results achieved.

2. Previous work

Several authors have considered the interaction of solitary waves with vertical walls. For example, Maxworthy [11] undertook an experimental study of both solitary wave reflection and solitary wave–wave interaction, concluding that the maximum water surface elevation was smaller in the former due to viscous and surface wetting effects on the vertical wall. He also showed that the maximum run-up was more than twice the individual solitary wave amplitude; a result that was subsequently predicted by Su and Mirie [12] using a third-order perturbation expansion. More recently, Cooker et al. [13] investigated similar interactions using the boundary integral method outlined by Dold and Peregrine [14] and determined that the maximum run-up for strongly nonlinear solitary waves is greater than three-times the individual solitary wave amplitude.

Maxworthy [11] also observed a phase shift that was independent of the incident wave amplitude. However, using a Fourier series solution of the Euler equations, Fenton and Rienecker [15] demonstrated that the phase shift is spatially dependent; suggesting that Maxworthy [11] placed his wave gauges too close to the wall. To overcome this spatial dependence, Cooker et al. [13] proposed a different means of determining the phase changes due to reflection based upon the wall residence time; the latter defining the duration over which the largest wave crest is attached to the vertical wall. Having noted that the wall residence time decreases as the wave amplitude increases, Cooker et al. [13] re-analysed the original cine film taken by Maxworthy [11] and showed good agreement between the boundary integral predictions and the experimental observations.

In respect of irregular wave reflection, the main focus of earlier work has been on the dynamic water pressure and the corresponding hydrodynamic force exerted on vertical walls. For example, Mallayachari and Sundar [16] undertook an experimental study of the reflection of regular and irregular waves from an impermeable vertical wall and, in respect of the latter, showed that the spectral densities of the dynamic pressure, \( p(t) \), at depths below the still water level (SWL) agree with linear theory based on small amplitude wave theory. However, at SWL the peak of the spectral density of \( p(t) \) is over-estimated by up to 48%. Furthermore, Mallayachari and Sundar [16] calculated the coherence function for the measured water surface elevation and the dynamic pressures. The coherence between \( \eta(t) \) and \( p(t) \) was shown to be excellent for the majority of the frequency components, but decreased for the low- and high-frequency ends of the input spectrum. Whilst the present study does not investigate the dynamic water pressure, it is clear that to predict \( p(t) \) with confidence it is essential to accurately model \( \eta(t) \). In the present paper it will be shown that the fully nonlinear BEM is capable of accurately predicting both the temporal and spatial water surface profiles, thereby highlighting the potential for reliable pressure predictions.

3. Boundary element method (BEM)

Assuming the fluid is incompressible and inviscid and the flow irrotational, mass continuity is defined by Laplace’s equation, \( \nabla^2 \phi = 0 \), where \( \phi(x, z, t) \) is the velocity potential. Within the spatial domain Green’s function, \( G(p, q) = -\frac{1}{2\pi} \ln|r(p, q)|\), where \( r(p, q) = |x_p - x_q| \) with \( x_p \) and \( x_q \) the source and evaluation points on the boundary, respectively, defines a fundamental solution of Laplace’s equation. Applying Green’s second identity reduces the dimensionality by one and results in the well-known boundary integral equation (BIE)

\[
C(p)\phi(p) + \int_f \phi(q) \frac{\partial G(p, q)}{\partial n} d\Gamma(q) = \int_f G(p, q) \frac{\partial \phi(q)}{\partial n} d\Gamma(q), \tag{1}
\]

where \( \mathbf{n} \) is the unit outward normal, \( \Gamma \) defines the boundary of the domain and \( C(p) \) is a function of the position of the source on the
boundary; the latter being calculated using a rigid mode technique [17].

In describing a numerical wave tank, the model utilises mixed boundary conditions, consisting of Neumann conditions (prescribed \( \partial \phi / \partial n \)) on the bed, left and right boundaries \( (\Gamma_b, \Gamma_l, \Gamma_r) \), and a Dirichlet boundary condition (prescribed \( \phi \)) on the water surface \( (\Gamma_s) \). Further details of the computational domain and the notation employed are given in Fig. 1.

Within the present study two types of computational domain are utilised corresponding to the wave reflection case and the wave–wave interaction case outlined in Section 1. The essential difference between these two domains lies in the boundary conditions applied at the right boundary, \( \Gamma_r \). Considering Fig. 1 and taking each boundary in turn, the following description applies:

(a) The left boundary, \( \Gamma_l \), is defined as a semi-Lagrangian input boundary on which the analytical velocities corresponding to irregular waves are prescribed following the second-order solution of Sharma and Dean [18]. Along this boundary the nodes are free to move vertically, but not horizontally.

(b) On the bed, \( \Gamma_b \), a zero flux condition is imposed, such that \( \partial \phi / \partial n = 0 \).

(c) On the right boundary, \( \Gamma_r \), a zero flux condition is applied to model the impermeable vertical wall present in the wave reflection domain. As far as the wave–wave interaction domain is concerned, the right boundary is treated as a semi-Lagrangian input in the exact same manner as the left boundary, \( \Gamma_l \) described in (a).

(d) Finally, on the water surface, \( \Gamma_s \), a velocity potential of \( \phi = 0 \) is initially prescribed to model still water.

It has already been noted that when a BEM approach is applied in physical space, the corners of the domain represent geometrical discontinuities and that these create certain difficulties, commonly referred to as the corner problem. Traditionally, BEM-based wave models have overcome this hurdle using double-nodes. This method places two nodes in exactly the same position at the corner, each node being assigned to one direction of the unit outward normal for the determination of the potential flux, \( \partial \phi / \partial n \). The BIE (1) is then solved and compatibility conditions introduced to remove any discontinuities between the overlapping nodes [19]. In contrast, the multiple-flux technique of Brebbia and Dominguez [10] specifies only one node at a corner, but considers all of the fluxes associated with that location. As a result, no information is lost and the necessity to impose compatibility conditions is removed. In considering the wave reflection simulation, an accurate treatment of the corner problem is essential as the principal area of interest (the water–wall interface) is defined by a geometrical discontinuity. Throughout the present study the multiple-flux formulation of Brebbia and Dominguez [10] as outlined in Hague and Swan [8] is adopted.

In order to evaluate the BIE (1), the boundary of the fluid domain is discretised into \( M \) isoparametric quadratic elements, resulting in \( N \) nodes [17]. The discretised version of the BIE is numerically integrated by Gaussian quadrature, resulting in a linear system of equations

\[
H\Phi = G\Phi_b. 
\]

where \( H \) (size \( N \times N \)) and \( G \) (size \( N \times 3M \)) are coefficient matrices and \( \Phi \) (size \( N \times 1 \)) and \( \Phi_b \) (size \( 3M \times 1 \)) are the column vectors of all the \( \phi \) and \( \partial \phi / \partial n \) variables, respectively. After applying the mixed boundary conditions, the unknown values are transferred to the left-hand side resulting in a linear system of equations

\[
Ax = b, 
\]

where \( A \) is the influence matrix, \( x \) contains all the unknown variables and \( b \) is the vector determined by the matrix–vector product of the known quantities. The unknown vector, \( x \), is then determined using the GMRES iterative solver [20] with a Jacobi preconditioner [21,22], reducing the computational effort from \( O(N^3) \) to \( O(N^2) \).

A semi-Lagrangian framework is used throughout the present simulations, allowing the nodes to move vertically but not horizontally. The free surface is defined by both the kinematic free surface boundary condition (KFSBC)

\[
\frac{\delta \eta}{\delta t} = \frac{\partial \phi}{\partial z} - \frac{\partial \phi}{\partial x} \frac{\partial \eta}{\partial x}, 
\]

and the dynamic free surface boundary condition (DFSBC)

\[
\frac{\delta \phi}{\delta t} = \frac{\partial \eta}{\partial t} \left( \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial z} \right) - \frac{\partial \phi}{\partial x} \frac{\partial \eta}{\partial x} - \frac{\partial \phi}{\partial z} \frac{\partial \eta}{\partial z}, 
\]

where \( \eta \) is the water surface elevation and \( \delta / \delta t \) denotes a time derivative in the semi-Lagrangian frame. With the right-hand side of both (4) and (5) independent of time, they can be treated as ordinary differential equations and time marched to obtain values of \( \eta \) and \( \phi \) at the next time step. This mixed Eulerian–Lagrangian time marching is undertaken using the fourth-order predictor–corrector method of Adams–Bashforth–Moulton. As this method requires information from three previous time steps, it is necessary to kick-start the model using three steps of a fourth-order Runge–Kutta integration scheme; the latter being a more computationally intensive single step method.

4. Laboratory apparatus and measurement programme

The experimental measurements were undertaken in a glass-walled wave flume located in the Hydrodynamics Laboratory in the Department of Civil and Environmental Engineering at Imperial College London. The wave flume is 27 m long, 0.3 m wide, with an operating water depth of \( d = 0.7 \) m. A numerically controlled, bottom-hinged, flap-type wave maker capable of generating and absorbing irregular waves with frequencies in the range \( 0.3 \) Hz \( \leq f \leq 3 \) Hz is located at each end of the wave flume. A schematic diagram indicating the overall layout of the wave flume is given in Fig. 2.

In accordance with the discussion given in Section 1, two experimental set-ups were adopted. One involved the generation of irregular wave records from each end of the wave flume, the phasing of the wave components adjusted so that in each case a transient focused wave group arises at the mid-point of the wave flume. These focused wave groups are characterised by the

![Fig. 1. Schematic of the BEM domain.](image-url)
summation or constructive interference of wave crests at one point in space (the mid-point of the wave flume) and time, and defined in terms of their linear amplitude sum: \( A = \sum_{n=1}^{N} a_n \), where \( a_n \) is the amplitude of the \( n \)th frequency component and \( N \) is the total number of components. The other set-up involved placing a vertical wall at the centre or mid-point of the wave flume and generating the same transient focused wave group, but in this case using only one wave paddle. The wall was rigid and impermeable, having been constructed from 10 mm thick perspex, supported on a metal frame and sealed to the side walls and bed of the wave flume using silicon sealant. If the wall perfectly reflects all of the incident wave energy, and if the wave motion and wave interactions follow idealised potential theory, it is clear that the water surface elevations, \( \eta(t) \), measured both on the wall and on the wave side of the wall will correspond closely to the first set-up involving the interaction of the two wave groups propagating in opposite directions. A schematic representation of these two set-ups is provided in Fig. 2.

Throughout the experimental study, time-histories of the water surface elevation, \( \eta(t) \), were recorded using surface-piercing, resistance wave gauges. Previous studies have shown that such measurements are accurate to within \( \pm 0.5 \) mm and causing no significant disturbance of the incident or ambient wave field. In each experimental run, a control wave gauge was placed 2.85 m from wave paddle 1 (Fig. 2), allowing the repeatability of the incident waves to be monitored. In both experimental set-ups, the spatial water surface profile, \( \eta(x) \), was measured using an array of 16 wave gauges; individual gauges being equally spaced 20 mm apart. By positioning this array at three locations, and repeating the measurements, data were recorded from 10 to 650 mm away from the mid-point of the wave flume, which corresponds to the location of the vertical wall in the second set-up. In all cases there was some overlap in the positioning of the wave gauges, allowing a second check of the repeatability of the generated waves. In addition, a single wave gauge was always located at the mid-point of the wave flume. With the wall in place, this wave gauge was built into the wall with its measuring surface flush with the front surface of the wall. In this way the gauge is able to measure the run-up on the wall. Information concerning the layout and location of the wave gauges is also given in Fig. 2.

5. Discussion of results

Within the present study all of the focused wave cases correspond to a simplified (or idealised) top hat spectrum. In such cases the underlying linear wave components, representing both the waves generated by the wave paddles and those introduced into the computational domain, are of equal amplitude and equally spaced within the period domain. Wave groups of this form are convenient to generate in a laboratory context and have been the subject of previous investigations by Baldock et al. [23] and Johannessen and Swan [4,5]. Indeed, the specific wave spectra investigated herein relate to their case B, with 28 wave components generated within the period range \( 0.8 \leq T_1 \leq 1.2 \) s. This corresponds to a relatively narrow-banded wave spectrum, but one in which it has previously been shown that the nonlinear wave–wave interactions can become very significant. As a result, it provides a challenging test case for the boundary element formulation.

The majority of the data presented within this section relates to three wave groups for which the linear amplitude sum is defined by \( A = 12, 30 \) and 46 mm, respectively. With the wave steepness defined in terms of a central wave number, \( k_c \), these three wave groups correspond to \( A_{k_c} = 0.05, 0.12 \) and 0.19, respectively. In earlier work, Baldock et al. [23] identified the limiting, or breaking, condition for uni-directional wave groups based upon an identical top-hat spectrum to be \( A_{k_c} \approx 0.24 \). It therefore follows that in terms of the present tests, the largest incident wave groups will be strongly nonlinear. However, when they interact with the vertical wall, or an identical wave group travelling in the opposite direction, the maximum crest elevation will be at least doubled leading to a highly nonlinear event. Once again, this points to the challenging nature of the test conditions.

If the comparisons outlined in Section 4 are to be realised, three key criteria must be met:

1. The underlying linear wave components generated at each wave paddle must be precisely controlled, both in terms of the amplitude of the components and their absolute phase; the latter being necessary to ensure a perfectly focused wave group. In the context of the present study, the control of the wave paddles, and hence the wave generation, is essential both in terms of producing the required wave groups and in defining the input to the BEM calculations thereby ensuring consistent comparisons. To achieve the required control, each wave paddle was calibrated independently using the procedures outlined by Masterton and Swan [24]. The success of this approach is demonstrated in Fig. 3. This concerns the smallest (\( A = 12 \) mm) linear wave group and contrasts the measured data with the required linear target. Figs. 3(a), (b) and (c) present the time-history of the water surface elevation, \( \eta(t) \), the amplitude spectrum, \( a(f) \), and the phase angle, \( \phi(f) \); where \( f \) is the wave frequency measured in Hz. These data relate to conditions at the focal location \( (x = 0) \) and concerns waves generated from paddle 1 (see Fig. 2). Figs. 3(d)–(f) provide a similar sequence of plots relating to wave paddle 2. In both cases excellent agreement is achieved between the linear target and the measured data; the maximum absolute discrepancy in the water surface elevation being 0.52 and 0.67 mm for wave paddles 1 and 2, respectively.
(ii) Given the nature of the experimental study, particularly the need to undertake repeated measurements to assemble a spatial description of the water surface elevation, $\eta(x)$, the wave conditions must be entirely repeatable. Figs. 4(a) and (b) concern a wave group with $A = 8$ mm (completed as part of the calibration process) and contrast three consecutive runs of paddles 1 and 2. In both cases the wave profiles are highly repeatable; the maximum difference between the three runs being 0.98 and 0.40 mm, respectively.

(iii) To achieve consistent comparisons between the wave reflection case (with the wall present) and the wave–wave interaction case (involving two colliding wave groups), the incident waves generated by the two paddles must be identical. Furthermore, if the BEM calculations are to be comparable, they must also involve identical incident waves. Fig. 5 addresses this point, providing comparisons between the wave groups generated by paddles 1 and 2 and that calculated within the BEM solution. Three comparisons are provided relating to the three input amplitudes: (a) $A = 12$ mm, (b) $A = 30$ mm and (c) $A = 46$ mm. In the latter two cases, the nonlinear shift in the focal time has been removed to facilitate the comparisons. In each case the agreement is good, with maximum discrepancies in the vicinity of the focal time ($-0.5 \leq t \leq 0.5$ s) of the order of 0.6 mm (5.1%), 1.2 mm (3.7%) and 3.7 mm (6.4%) for the $A = 12$, 30 and 46 mm cases, respectively; the bracketed percentages defining the maximum discrepancy as a proportion of the focused crest amplitude.

In comparing the wave reflection and wave–wave interaction cases there is also the additional requirement that each individual wave group should focus at the centre of the wave flume, or at the location of the vertical wall if it is present. In the linear wave group ($A = 12$ mm) this follows directly from the phasing of the wave components at the input boundary; the entire motion being governed by linear processes. However, as the nonlinear wave groups evolve towards a focusing event local and rapid energy transfers, involving a movement of energy to the higher frequencies [6], lead to a change in the phase velocity and hence a shifting in the focus position and the focal time relative to the linearly predicted values [23]. Consequently, for the nonlinear wave cases ($A = 30$ and 46 mm) the initial phasing of each wave component was adjusted iteratively until the focus position was located at the centre of the wave flume. This process was undertaken independently for both the laboratory wave paddles and for the BEM calculations; the required results typically achieved after three iterations.

With these key criteria established, two computational domains were utilised to simulate the experimental set-ups outlined in Section 4. The wave reflection domain (set-up (A)) was initially defined within the region $-13.0 m \leq x \leq 0.0 m$ and $-0.7 m \leq z \leq 0.0 m$, with $dx = 0.05 m$ and $dz = 0.0125 m$ producing 316 elements and 632 nodes. In contrast, the wave–wave interaction domain (set-up (B)) was initially defined within the region $-13.0 m \leq x \leq 13.0 m$ and $-0.7 m \leq z \leq 0.0 m$ with $dx = dz = 0.05 m$ resulting in 534 elements and 1068 nodes.

Fig. 6 concerns a spatial description of the water surface elevation, $\eta(x)$, at several instances in time; the data describing...
the evolution of the wave fields up to and including the occurrence of the largest wave crest at $t = 0$. The subplots ((a), (b) and (c)) address the three wave cases $A = 12, 30$ and $46$ mm, respectively. In each case comparisons are made between all the available data, including laboratory observations and numerical predictions relating to the two set-ups: (A) the wave reflection case and (B) the wave–wave interaction case. To quantify the agreement between these data, Table 1 addresses each of the six possible intercomparisons, providing a measure of both the maximum and the rms error, both expressed as a percentage of the focused crest elevation.

To complement these data, Fig. 7 provides a time-history of the water surface elevation, $\eta(t)$, at several spatial locations. Once again, the three wave cases are considered in parts (a)–(c), and comparisons are provided between all the available data. Based upon these results Table 2 provides an alternative measure of the maximum and rms error, again expressed as a percentage of the focused crest elevation, for each of the six intercomparisons.

Taken as a whole, the data presented in Figs. 6 and 7 show remarkably good agreement in both space and time. In some comparisons the agreement is near-perfect, demonstrating the success of the BEM formulation and, in particular, the treatment of
the corner nodes. In others, the discrepancy between the measured and predicted data is somewhat larger. This is consistent with earlier research [11] and hints at the importance of fluid properties, notably viscosity, not included within the BEM formulation. This is a shortcoming of the BEM and is in contrast to other models, such as the volume of fluid (VoF) [25] and smoothed particle hydrodynamics (SPH) [26,27], which can simulate these viscous effects. However, even in these cases the discrepancy is not perhaps as large as might have been anticipated; the BEM formulation providing an effective upper-bound for engineering design/analysis.

Taking each set of comparisons in turn, the following detailed implications can be drawn. First, comparisons between the two boundary element solutions, one describing the wave reflection case and the other the wave–wave interaction case, show remarkable agreement with maximum relative errors less than 0.7%. Given the proximity of the highest wave case (A = 46 mm) to its limiting behaviour, involving wave breaking and the inevitable break-up of the water surface (see additional comments on nonlinearity given below), this agreement demonstrates the success of the BEM formulation. In judging these results it is also important to note that no smoothing, filtering, or regridding of any kind has been adopted.

Second, comparisons between the BEM results and the laboratory observations of the wave–wave interaction case, again show very good agreement; the maximum relative errors being comparable to the more straightforward wave cases considered in Fig. 5. It is interesting to note that in these comparisons the largest error is not associated with the description of the focal event, at \( x = t = 0 \), but occurs at some distance from the wall after the occurrence of the largest crest. This is particularly noticeable in Fig. 7(c), curve (11) for \( t > 0.4 \). However, the reason for this remains unclear.

The third set of comparisons concern the laboratory observations of the wave reflection case. It is in this case that the BEM calculations show the largest departures from the laboratory observations; the maximum relative error corresponding to almost 12% of the focused crest elevation. Although this value occurs in the largest, near-breaking, wave case (A = 46 mm), comparable discrepancies also arise in the other two (smaller) wave cases. In all three examples the largest discrepancies arise in the description of the extreme crest elevation evolving at the wall. However, additional comparisons between the two experimental cases confirm that the maximum crest elevation observed at the wall is consistently lower than that arising in the wave–wave interaction case; the latter case involving the collision of two identical wave groups travelling in opposite directions rather than the imposition of a vertical wall.

The explanation for this difference lies in the effect of the viscous shear stresses arising within the boundary layer on the surface of the vertical wall. The work done in moving the fluid

![Fig. 6](image_url) Spatial water surface elevation, \( \eta(x) \), of the experimental measurements and numerical predictions for (A) the wave reflection case and (B) the wave–wave interaction case during the run-up process. (a) A = 12 mm, (b) A = 30 mm and (c) A = 46 mm wave cases. --- BEM set-up (A), + BEM set-up (B), ⋄ experimental set-up (A) and □ experimental set-up (B). The times of the various profiles are (1) 0 s, (2) 0.117 s, (3) 0.125 s, (4) 0.141 s, (5) 0.180 s, (6) 0.203 s, (7) 0.242 s, (8) 0.258 s, (9) 0.305 s, (10) 0.313 s, (11) 0.391 s, (12) 0.398 s, and (13) 0.406 s. (Note: \( t = 0 \) is the time at maximum run-up.)

### Table 1

<table>
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<th>A (mm)</th>
<th>Data set</th>
<th>Max. relative error (% of ( \eta_{\text{max}} ))</th>
<th>rms relative error (% of ( \eta_{\text{max}} ))</th>
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</table>
against these stresses represents an additional dissipative mechanism, limiting the rise of the maximum water surface elevation. These stresses only develop in the presence of the wall, due to the imposition of the no-slip condition, and are not (and cannot) be incorporated in the BEM calculations. Hence the difference in the observed and predicted behaviour.

In terms of engineering applications, the single most important aspect of the present tests concerns the prediction of the maximum water surface elevation occurring at the vertical boundary or wall. In describing random or irregular sea states, design practice commonly adopts either linear random wave theory or a second-order wave model [18]. Indeed, there is a widespread and misguided view amongst designers that in describing the water surface elevation very little of practical significance arises above second order. For example, neither the International Standards Organisation (ISO) or the American Petroleum Institute (API) design guidance notes for offshore structures (both fixed and floating) make mention of effects beyond second order. Furthermore, there is also a view that reflection (from a vertical wall) is essentially a linear process; the total surface elevation being given by the linear sum of the incident and reflected wave components. The upshot of these two views is that the maximum crest elevation is commonly taken as twice the predicted linear incident crest elevation. The present tests help evaluate the effect of the approximation and, in so doing, also allow the nonlinearity of the wave field to be investigated.

Fig. 8 considers the wave–wave interaction case, comparing the measured data and the BEM predictions with both linear and second-order wave models. Subplots (a), (b) and (c) address the three wave cases, and (b) the wave–wave interaction case. (a) A = 12 mm, (b) A = 30 mm and (c) A = 46 mm wave cases. Subplot (a) represents the experimental set-up (B), subplot (b) represents the BEM set-up (B), and subplot (c) represents the BEM set-up (A) and the experimental set-up (B). The spatial location of the various time-histories are (1) 0 mm, (2) 170 mm, (3) 190 mm, (4) 210 mm, (5) 270 mm, (6) 310 mm, (7) 350 mm, (8) 390 mm, (9) 430 mm, (10) 450 mm, (11) 650 mm from the wall. (Note: all the profiles have had their focus time shifted to \( t = 0 \) s to facilitate the comparisons.)

![Graph](image)

Fig. 7. Time-histories of the water surface elevation, \( \eta(t) \), of the experimental measurements and numerical predictions for (A) the wave reflection case and (B) the wave–wave interaction case. (a) \( A = 12 \) mm, (b) \( A = 30 \) mm and (c) \( A = 46 \) mm wave cases. --- BEM set-up (A), + BEM set-up (B), * experimental set-up (A) and - experimental set-up (B). The spatial location of the various time-histories are (1) 0 mm, (2) 170 mm, (3) 190 mm, (4) 210 mm, (5) 270 mm, (6) 310 mm, (7) 350 mm, (8) 390 mm, (9) 430 mm, (10) 450 mm, (11) 650 mm from the wall. (Note: all the profiles have had their focus time shifted to \( t = 0 \) s to facilitate the comparisons.)

<table>
<thead>
<tr>
<th>A (mm)</th>
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<th>rms relative error (% of ( \eta_{\text{max}} ))</th>
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Table 2

Maximum and rms relative errors between the experimental measurements (Exp.) and numerical predictions (BEM) of the time history of the water surface elevation, \( \eta(t) \) (shown in Fig. 7) for (A) the wave reflection case and (B) the wave–wave interaction case.
of solitary wave run-up [13], which showed that for strongly nonlinear waves the maximum wave crest elevation is more than three times the individual wave amplitude. In considering the present results it is clear that in predicting the maximum crest elevation a rigorous treatment of the full nonlinearity of the interacting wave field is more important than the small viscous dissipation that arises due to run-up on the physical boundary.

6. Concluding remarks

The present study has considered the run-up and reflection of a single focused wave group on an impermeable vertical wall and contrasted the results with the head-on collision of two identical wave groups travelling in opposite directions. New experimental measurements have been presented of both the wave reflection and the wave–wave interaction cases and the results compared to fully nonlinear numerical calculations based upon a multiple-flux boundary element model. These comparisons illustrate that by accurately addressing the corner problem with multiple fluxes, as outlined by Brebbia and Dominguez [10], very good agreement with the laboratory data can be achieved. Furthermore, there is no need for smoothing, filtering or regridding thereby avoiding the uncertainties associated with the possible removal or redistribution of actual energy distributions in highly nonlinear wave groups.

Comparisons between the two laboratory set-ups and the BEM predictions allow the influence of the viscous shear stresses acting
on the vertical boundary to be addressed. Although this can be readily observed in terms of the reduced maximum run-up, this effect is small in comparison to the nonlinear increase arising due to the wave–wave interactions. Indeed, in the steepest irregular wave groups the run-up is shown to be more than three times the crest elevation of the incident wave group. Run-up of this magnitude is critically important for design and can only be predicted on the basis of a fully nonlinear wave model such as the BEM formulation applied herein.

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References