Nonlinear scattering of non-breaking waves by a submerged horizontal plate: Experiments and simulations

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Abstract

Brossard et al. (2009. Coastal Engineering 56, 11–22), hereinafter referred to as BPBD) presented an experimental study of the higher harmonics induced by a submerged horizontal plate and a submerged rectangular step. In this paper we present a numerical study of the wave scattering by a submerged thin plate as described in BPBD. A fully nonlinear numerical wave tank based on a desingularized boundary integral equation method (DBIEM) was developed for the interactions between non-breaking waves and a fixed 2D body. We also present a new set of experimental data on regular waves interacting with a submerged thin plate with different submergences, with the aim of removing the influences of multiple reflections between the wave-maker and submerged structures in a wave flume. We compare our numerical simulations with our experiments as well as the experiments of BPBD. The comparison shows that DBIEM can predict satisfactorily the multiple wave reflections caused by the thin plate as well as the main features of the generation and transmission of higher harmonic waves.

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1. Introduction

Breakwaters are marine structures used to reduce the wave energy transmitted to their lee sides. There is a rich literature in the traditional breakwaters such as rubble mound breakwaters, caisson breakwaters, and vertical wall breakwaters. For the purpose of reducing the construction cost and maintaining the eco-environmental system in harbors, recently new types of breakwaters have received considerable attention. One type of the new breakwaters is the slotted/pile breakwaters, which reduce the energy of transmitted waves by turbulent jet flows through the gaps between two adjacent cylinders/piles (see for example Mei et al., 1974; Isaacson et al., 1999; Huang, 2007; Huang and Ghidaoui, 2007). Another new type of breakwaters is in the form of horizontal plates, which can be a single submerged plate, a single plate fixed at the mean water surface, a horizontal twin-plate with the top plate fixed at the mean water surface, or a multi-plate breakwater. In contrast to the slotted/pile breakwaters, horizontal plate structures reduce the transmitted wave energy by controlling the wave breaking and the phase change of the scattered waves over the plate. From the velocity measurements carried with Ultrasonic 3D probe, (Graw, 1992) found strong pulsating flow opposite to the wave direction. Aiming at the potential combination of submerged plate breakwaters with wave energy converters, Carter et al. (2006), using N–S equation with VOF method for free surface, numerically studied the reverse flow beneath a submerged plate due to wave action.

Linear wave scattering by submerged plates has been studied extensively in the past. Dick (1968) probably was the first to measure the characteristics of regular wave scattering by a submerged plate. For 2D linear wave scattering by objects of rectangular cross section, it is possible to use eigen-function expansion methods to find analytical solutions. Siew and Hurley (1977) analyzed the scattering of long waves by a submerged plate, and showed that both reflection and transmission coefficients vary with the ratio of the plate length and the wave length in the shallower water above the plate. Patarapanich (1984a, 1984b) and Patarapanich and Fatt (1989) studied the various aspects of linear reflection and transmission of regular and irregular waves by a submerged horizontal plate, both experimentally and by a finite element method (FEM). Liu and Iskandarani (1991) studied the scattering of short-wave groups by a submerged plate. Cheong and Patarapanich (1992) studied, both theoretically and experimentally, double-plate submerged breakwater. Cheong et al. (1996) compared the eigen-function expansion solutions with those obtained by FEM. Using the eigen-function expansion method, Wang and Qiang (1999) investigate the reflection and transmission of regular waves over a group of

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submerged horizontal plates. Rey et al. (2002) studied the effects of currents on the linear scattering of regular waves by a submerged horizontal plate. More recently, Usha and Gayathri (2005) examined the twin-plate breakwater and (Wang et al., 2006) studied the transmission and reflection of regular waves by a multi-layer breakwater.

The study of the nonlinear scattering by submerged horizontal plate is relatively scarce. Brossard and Chagdali (2001) reported their experimental study of higher harmonic generation by regular waves over a submerged horizontal plate. Their experiments were conducted in a relatively small wave flume (9.5 m long and 0.3 m wide) equipped with a simple flap-type wave generator. The reflection coefficient of the beach ranged from 2% to 10% for the wave frequencies in their experiments. Moving wave probes were used to separate the reflected waves from incident waves, as well as free waves from phase-locked waves. They find that the transfer of energy from the fundamental waves to higher harmonic waves is significant for cases of small submergences. Recently, Brossard et al. (2009) re-examined the nonlinear wave scattering by a submerged plate and a submerged rectangular step, focusing on the second higher harmonic free waves downstream of the submerged object. The experiments were carried out in the same wave flume as in Brossard and Chagdali (2001). They find that the plate is more efficient than the step at dissipating wave energy through wave breaking. They also provided detailed information on the phase changes of first and second order waves over the plate. On the theoretical side, Massel (1983), using an eigen-function expansion method, analytically investigated the weakly nonlinear wave scattering by a submerged rectangular step, but no theoretical analysis of nonlinear wave scattering by a submerged thin plate is reported in the literature. Because of the considerable complexity involved in the analytical analysis of nonlinear wave scattering, recent research on this topic prefers to use nonlinear numerical wave tanks (NWT).

Numerical wave tanks can be categorized into two groups: viscous NWT based on N–S equations and idealized NWT based on potential theory. Generally speaking, viscous NWT based on N–S equations can have the potential to provide much better solutions at a cost of long computation time and higher requirements on other computer resources. One the other hand, idealized NWTs based on potential theory do not directly model the vortices generated by viscous effects, but it can provide solutions within a reasonable time on regular computers. For a recent survey of various numerical methods for water waves, readers can refer to Lin (2008).

Most of the idealized NWT are based on boundary integral equations (BIE), which are solved traditionally by boundary element methods (BEM) (see Ohyama and Nadaoka, 1994 for an application of BEM in the study of nonlinear wave scattering by rectangular steps). Longuet-Higgins and Cokelet (1976) proposed a mixed Eulerian–Lagrangian (MEL) formulation to track the free surface of steep waves. This method was further explored by many authors in developing numerical wave tanks. To avoid the singular integration in traditional BEM, a desingularized boundary integral equation method (DBIEM) was proposed by Cao et al. (1991) because of the simplicity in DBIEM coding, the method has attracted many authors to use it to solve many potential flow problems. Beck (1994) reviewed the state-of-the-art of DBIEM at that time. Kim et al. (1998) studied the fully nonlinear wave interaction with a fixed 3D body. Celebi (2001) extended the DBIEM to wave interactions with fixed 3D body in the presence of currents. Zhang et al. (2006, 2007) studied the wave scattering by topography using DBIEM with MEL formulation. Zhang and Robert (2008) examined the wave interactions with 3D floating bodies with the nonlinear free surface condition specified at the still water level. Experience in using DBIEM to study wave-structure interactions shows that the desingularization distance can greatly affect numerical accuracy (Cao et al., 1991; Lalli, 1997). Theoretically, DBIEM cannot be used for plates of zero thickness (i.e., the desingularization distance is zero). No attempt has been found in the published work to simulate, using DBIEM, the nonlinear wave scattering by submerged thin plate.

Recently, vortices generated at the edges of a submerged rectangular step has caught the attention of several published papers (Ting and Kim, 1994 for regular waves; Chang et al., 2001; Huang and Dong, 2001; and Lin, 2006, for solitary waves; etc.). Previous research shows that the vortex motion is more powerful at the lee side and that the nonlinear effects and separation loss are important only in the transmission processes. For submerged thin plates, however, the flows above and beneath the plate will meet in the vicinity of the two ends of the plate, possibly resulting in smaller vortices when compared with the vortices generated by a step of the same length and submergence, i.e. the wave energy dissipation due to vortex shedding should be generally weaker for a submerged plate than for a submerged rectangular step. Indeed, Brossard et al. (2009), from their experiments, did noticed that the vortices at the edges of the plate might not be responsible for the generation of the free harmonic waves in the lee side of the plate, suggesting that it is possible to use potential flow theory to study the generation of higher harmonic waves induced by the nonlinear wave scattering by submerged thin plates.

Previous research has shown that the flow upstream of the submerged obstacle can be predicted well by potential flow theory, the question now is: to what extent can an idealized NWT simulate the essential features in the nonlinear transmission processes? Using a modified DBIEM code reported in Liu et al. (2008), the present study explores the capacity of the idealized NWT in simulating the nonlinear wave scattering by submerged thin plate. Numerical results are first compared with the experiments of Brossard et al. (2009). As the influences of the multiple reflections between the wave maker and the submerged obstacles cannot be easily assessed from the results reported in Brossard et al. (2009), we conducted a new set of experiments in a relatively long wave flume, which was equipped with a computer-controlled, piston-type wave generator (HR Wallingford). The higher harmonic free waves and phase-locked waves are separated by using two fixed wave probes in our experiments. The focus of this study is on the numerical simulation of the nonlinear transmission processes. The limitations of DBIEM will also be discussed briefly based on our numerical experiments.

2. Mathematical formulation and numerical method

Referring to Fig. 1, let the horizontal coordinate, $x$, be pointing in the direction of wave propagation with its origin located at the damping layer, wave-maker, model, and damping layer.
center of the horizontal plate, and the vertical coordinate, \( z \), be pointing upward with its origin located at the still water level. The velocity components in the \( x \) and \( z \) directions are \( u \) and \( w \), respectively. The water depth, the length and immersion depth of the plate are denoted by \( D \), \( B \), and \( h_1 \), respectively. To analyze the wave transformation and interaction with the submerged structure, six wave gages are arranged at locations presented in Table 1. These locations were chosen based on test wave elevation measured at these locations.

### 2.1. Mixed Eulerian–Lagrangian formulation (MELF)

For irrotational flows in an incompressible fluid, the velocity potential \( \phi \) is governed by Laplace equation

\[
\nabla^2 \phi = 0, \quad \nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial z} \right)
\]

At the impermeable boundaries (bottom, surfaces of the submerged plate), the no-flux condition requires

\[
(\nabla \phi) \cdot \vec{n} = 0
\]

where \( \vec{n} \) is the unit vector normal to the impermeable boundary at point \( \vec{x} = (x,z) \).

At the free surface, \( z = \eta(x,t) \), the kinematic and dynamic boundary conditions are expressed in Lagrangian form as follows:

\[
\frac{d\vec{x}}{dt} = \nabla \phi, \quad \frac{d\phi}{dt} = \frac{1}{2} \nabla \phi \cdot \nabla \phi - g\eta
\]

Water waves can be generated by adding an artificial pressure term to Eq. (4) (see Clamond et al., 2005). To ensure that waves are out-going at the two ends of the wave flume, non-reflective boundary conditions are imposed by adding artificial damping terms to both Eqs. (3) and (4) (see Zhang et al., 2006). The implementations of non-reflective boundary conditions and the generation of water waves in the numerical wave tank are given in Appendix A.

### 2.2. Boundary value problem

The Lagrangian boundary conditions (3) and (4) can be integrated numerically to find the position of the moving boundary and the velocity potential on the free surface. At time \( t_n+1 = t_n + \Delta t \), the new position of the free surface, \( \vec{x}_m^{n+1} \), and velocity potential on it, \( \phi_m^{n+1} \), can be calculated from the known \( (\vec{x}_m^n, \phi_m^n) \) at time \( t_n \) by

\[
\vec{x}_m^{n+1} = \vec{x}_m^n + \Delta t \vec{z}(t_n, \Delta t, \vec{x}_m^n), \quad m = 1, \ldots, N, \quad (5)
\]

\[
\phi_m^{n+1} = \phi_m^n + \Delta t \vec{z}(t_n, \Delta t, \phi_m^n), \quad m = 1, \ldots, N, \quad (6)
\]

where the subscript \( m \) is the index of the \( m \)th node on the boundary and the operator \( \vec{z}(\cdot) \) can be determined by a numerical integrator. In this study, we used the RK4 integrator.

Now Eq. (1), the Neumann boundary condition (2), and Dirichlet boundary condition (6) form a boundary value problem (BVP) for velocity potential \( \phi_m^{n+1} \). In the following, the BVP will be solved by a desingularized boundary integral equation method (DBIEM), as described in Section 2.3.

### 2.3. DBIEM for boundary value problem

In DBIEM, the velocity potential \( \phi(x,t) \) at a point \( \vec{x}_m \) inside the computational domain is expressed as an integral of a simple Rankine source distribution over an imaginary source boundary \( \Omega \).

\[
\phi(\vec{x}_m, t) = \int_\Omega \sigma(\vec{x}_m, \vec{x}_d) G(\vec{x}_m - \vec{x}_d) d\Omega.
\]

where \( \sigma(\vec{x}_m, t) \) is the strength of the Rankine source at the point \( \vec{x}_d \) on \( \Omega \) at time \( t \). The Rankine source distribution for a two-dimensional problem is given by

\[
G(\vec{x}_m - \vec{x}_d) = \ln |\vec{x}_m - \vec{x}_d| + \ln |\vec{x}_m + 2\vec{k} + \vec{x}_d|,
\]

where \( \vec{k} \) is the unit vector in the vertical direction and the second term on the right hand side is the image of the first term (Rankine image source) about the horizontal bottom at \( z = -D \). The use of the Rankine image source makes it unnecessary to place Rankine sources along the bottom boundary. Our numerical experiments showed that the use of the image source could improve the computational efficiency by 50–100%.

### Table 1

Locations of the wave gages.

<table>
<thead>
<tr>
<th>Coordinate on x-axis</th>
<th>Distance from wave-maker</th>
<th>Distance from beach</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G_1^a )</td>
<td>-3.75</td>
<td>11.48</td>
</tr>
<tr>
<td>( G_2^a )</td>
<td>-3.65</td>
<td>11.58</td>
</tr>
<tr>
<td>( G_3^a )</td>
<td>-3.50</td>
<td>11.73</td>
</tr>
<tr>
<td>( G_4^a )</td>
<td>-0.20</td>
<td>15.03</td>
</tr>
<tr>
<td>( G_5^a )</td>
<td>0.20</td>
<td>15.43</td>
</tr>
<tr>
<td>( G_6^a )</td>
<td>3.50</td>
<td>18.73</td>
</tr>
<tr>
<td>Wave-maker</td>
<td>-15.23</td>
<td>-</td>
</tr>
<tr>
<td>Beach toe</td>
<td>9.77</td>
<td>25.00</td>
</tr>
</tbody>
</table>

The unit of the distance is meter (m).

\( ^a \) Locations of \( G_1 \) for \( T_w = 0.8, 0.9, \) and 1.0 s.

\( ^b \) Locations of \( G_2 \) for \( T_w = 1.1, 1.2, 1.3, 1.4, \) and 1.5 s.
grid throughout the simulation. Numerical smoothing was also used six time steps to remove the so-called saw-tooth instability. Some implementation details of our DBIEM code were given in Liu et al. (2008).

3. Description of the experiment

3.1. Experimental setup

The experiments were conducted in a wave flume located in the Hydraulics Laboratory at the Nanyang Technological University, Singapore. The glass-walled wave flume was 32 m in total length, 0.55 m in width and 0.60 m in depth. A piston type wave-maker (HR Wallingford) was placed at one end of the flume, and a 1:10 beach (covered with a layer of 3 cm-thick porous mat) was placed at the other end to minimize wave reflections. Reflection coefficients of the beach are <0.05 for all wave conditions examined in this study. One horizontal Perspex plate was placed at the middle section of the flume. The horizontal plate, which occupied the entire width of the flume, was 0.6 m long and 0.01 m thick. The horizontal distance of the center of the plate was 15.23 m away from the wave generator. Fig. 3 shows a sketch of the experimental setup.

In all experiments, data sampling rate was 50 Hz and the total time for data acquisition was 60 s. The time series of surface elevation were recorded by six wave gages, which are labeled as $G_i$, $i = 1, \ldots, 6$, whose locations are presented in Table 1. Based on the time series of surface elevation, the reflection and transmission coefficients, phase difference between wave gage $G_3$ and $G_6$, the free and phase-locked higher harmonic transmitted waves were analyzed. In this study, both numerical simulations and experiments focus on non-breaking wave.

3.2. Test wave conditions

In our experiments, the still water depth was fixed at 0.3 m. Two submergences, $h_1 = 0.15 \text{ m}$ and $h_1 = 0.1 \text{ m}$, were studied. Eight wave periods ($T_w = 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4$, and $1.5 \text{ s}$) combined with six target wave heights ($H = 0.02, 0.03, 0.04, 0.05, 0.06, \text{ and } 0.07 \text{ m}$) were examined in the experiments. The instantaneous surface elevations were measured by the resistance-type wave gages, with a resolution of 0.1 mm. Six wave gages, which are labeled as $G_i$, $i = 1, \ldots, 6$ in Fig. 3, were used.

3.3. Multiple wave reflections in the wave flume

Multiple wave reflections exist in the wave flume. The time series of the surface elevations recorded by gages $G_1$ and $G_5$ are shown in Fig. 4, where several stages of multiple reflections can be identified on each of the wave record.

There are four stages in region of $x < -B/2$, as shown in Fig. 4(a). The leading wave in the wave front reaches $G_1$ at about $t = 6 \text{ s}$. Within the wave front, the wave amplitude first increase with time and then decrease to a constant if no reflection is present in the wave flume (see Longuet-Higgins, 1974). After the wave front passes $G_1$, it will be reflected by the front end and the rear end of the plate. At $t \approx 13.5 \text{ s}$, the reflected leading wave in the wave front reaches $G_1$, and at about $t \approx 22.5 \text{ s}$ the tail of the wave front leaves $G_1$. Before the leading wave front is reflected back from the wave maker, the wave amplitude at $G_1$ remain constant theoretically. The distance between the wave maker and $G_1$ is 11.5 m and the wave phase velocity is 1.43 m/s, thus it needs about 16 wave period for the reflected leading wave front to pass
There are also four stages in the region $x > B/2$, as shown in Fig. 4(b). After the wave front passes the plate, the leading wave will reach $G_5$ at $t \approx 12 \text{s}$, and the tail of the wave front will leave $G_5$ at $t \approx 22 \text{s}$. Between $t = 22$ and $t = 26 \text{s}$, the recorded waves are affected by the free and phase-locked waves induced by the wave front, which defines the second stage on the wave record shown in 4(b). For the third stage the leading wave front has not yet been reflected back from the beach to $G_5$, thus the wave amplitude will remain constant. The distance between the toe of the breach and $G_5$ is about 6.3 m, thus it needs about nine wave period for the leading wave front to pass $G_5$ again. The influence of the reflection of the leading wave front by the plate is small and will not affect too much the recorded waves at $G_5$.

To separate the reflected waves from the incident waves, we use the wave records taken by $G_1$ and $G_2$. For all the wave periods examined in this study, there are 6–12 waves at the third stage, which are free of the multiple wave reflections. To separate the free waves from the phase-locked waves, we use the wave records taken by $G_5$ and $G_6$. Again, there are 4–8 waves at the third stage, which are free of the influences of the multiple wave reflections. In this study, 3 waves at the third stage were used to perform the wave separations through curve fitting. Therefore, the method we used to separate waves can avoid the possible influences of multiple wave reflections in the flume.

### 3.4. Data analysis

#### 3.4.1. Curve fitting

In order to determine the amplitudes and phases of component waves, we chose three waves from the third stage on a wave record to perform the data fitting. The surface elevation measured by each wave gage located at $x = x_a$ is fitted with the following expression:

$$
\eta(t, x_a) = \sum_{j=1}^{3} (a_j(x_a) \cos(j \omega t + \psi_j(x_a))),
$$

where $\omega$ is the angular frequency of the fundamental waves, $a_j(x_a)$ and $\psi_j(x_a)$ are the amplitude and the phase angle of the $j$th harmonic waves, respectively. Fig. 5 is examples showing the measured data and the corresponding fitting curve for $G_1$ and $G_5$. It is found that the higher harmonic components in the lee side of the plate are much larger than those in the weather side of the plate. In this study, the second-order analysis is focused on the higher harmonic waves in the transmitted waves.

#### 3.4.2. Phase change between two points above the plate

Between two points $x = x_a$ and $x = x_a + \Delta x$, the phase difference of the $j$th harmonic waves can be calculated by $\Delta \psi_j = \psi_j(x_a + \Delta x) - \psi_j(x_a)$, which is defined within $0$ and $2\pi$ for our experiments. In this study, we focus on the phase change of the fundamental waves and the second harmonic waves over the plate.

![Fig. 3. Sketch of experimental setup.](image-url)
3.4.3. Separation of reflected waves from incident waves
At the front end of the plate, waves measured at a point in space are a superposition of the incident waves and the reflected waves. From the surface elevations measured by wave gages \( G_1 \) and \( G_2 \), the reflected waves can be separated from the incident waves by the two-point method of Goda and Suzuki (1976). The reflection coefficient is defined as

\[ R = \frac{a_T}{a_I}, \quad (13) \]

where \( a_T \) and \( a_I \) are the amplitudes of reflected waves and incident waves, respectively. The reflections of the higher order phase-locked waves are weak and not investigated in this study.

3.4.4. Separation of free waves from phase-locked waves
After waves pass the submerged horizontal plate, higher harmonic waves generated by nonlinear wave–wave interactions in the shallow water over the plate will leave the plate leeward as free waves. In the lee side of the plate, the higher harmonic waves contain both free waves and phase-locked waves. We used two fixed wave gages \( (G_2 \) and \( G_6 \)) to measure the surface elevations at two points, \( \Delta x \) apart.

The separation of the free waves from the locked waves is done using the method given in Grue (1992). The surface elevation measured at \( x \) is written as

\[ \eta(t, x) = a_T \cos(kx - \omega t + \phi(x)) + \sum_{n=2}^{\infty} a_T^{(n)} \cos(nkx - n\omega t + \psi_n(x)) \]

\[ + \sum_{n=2}^{\infty} a_f^{(n)} \cos(k_n x - n\omega t + \psi_n(x)), \quad (14) \]

where \( a_T \) is the amplitude of the transmitted waves with the fundamental frequency; \( a_T^{(n)} \) and \( a_f^{(n)} \) are the amplitudes of the \( n \)th order locked waves and free waves, respectively; \( \psi_n \) is the phase angle of the \( n \)th harmonic free waves. Assuming that all waves propagate in one direction, the free waves can be separated from the locked waves using the wave records obtained at \( x = x_0 \) and \( x = x_0 + \Delta x \). The amplitudes of the free waves and locked waves are given by

\[ a_f^{(n)} = \frac{\hat{\eta}_f(x_0) - \hat{\eta}_f(x_0 + \Delta x) e^{i n k \Delta x}}{\sinh(nk \Delta x)}, \quad n = 2, 3, 4, \ldots, \quad (15) \]

\[ a_T^{(n)} = \frac{\hat{\eta}_T(x_0) - \hat{\eta}_T(x_0 + \Delta x) e^{i n k \Delta x}}{\sinh(nk \Delta x)}, \quad n = 2, 3, 4, \ldots, \quad (16) \]

where \( \hat{\eta}_f(x) \) is obtained by Fourier transform of

\[ \hat{\eta}_f(x) = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \eta(x, t)e^{-i\omega t} dt. \quad (17) \]

See Grue (1992) for details.

3.4.5. Definitions of transmission coefficients
The amplitudes of various transmitted wave components are normalized by the amplitude of the incident waves. We used transmission coefficients to represent these dimensionless amplitudes. The transmission coefficients for the fundamental waves and the higher harmonic free and locked waves are defined, respectively, by

\[ T = \frac{a_T}{a_I}, \quad T_f^{(n)} = \frac{a_f^{(n)}}{a_I}, \quad T_l^{(n)} = \frac{a_T^{(n)}}{a_I}. \quad (18) \]

An theoretical expression for \( T_f^{(2)} \) is available. For the second order stokes waves propagating in one direction, it is well known that the amplitude ratio of the second harmonic waves to the fundamental waves is

\[ \gamma = \frac{kh \cosh(kh)(2 + \cos(2kh))}{\sinh^3(kh)}. \quad (19) \]

where \( H \) is the height of the fundamental waves (see, e.g., Dean and Dalrymple, 1992). With Eq. (19), we can rewrite the definition of \( T_f^{(2)} \) as

\[ T_f^{(2)} = \frac{H_f^{(2)}}{H_I} = \frac{H_f^{(2)}}{H_T}, \quad (20) \]

where \( H_I \) and \( H_T \) are the height of the incident and transmitted waves, respectively. \( H_f^{(2)} \) is the hight of the second order locked waves. After realizing that \( H_I \) is the \( H \) in Eq. (19) and \( H_T = TH_I \), Eq. (20) can be rewritten as

\[ T_f^{(2)} = \frac{H_f^{(2)}}{H_I} = \frac{H_f^{(2)}}{H_T}, \quad (21) \]

\[ \frac{a_T}{a_I} = \frac{H_f^{(2)}}{H_I} = \frac{H_f^{(2)}}{H_T}. \quad (22) \]

The wave record used to perform the Fourier transform is the fitting curve given by Eq. (12).
Table 2

<table>
<thead>
<tr>
<th>Locations</th>
<th>Grid size ΔΩ</th>
<th>Desingularization distance t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free surface</td>
<td>min(L/125, B/30)</td>
<td>t = L = πΔΩ/L^6</td>
</tr>
<tr>
<td>Plate surfaces</td>
<td>δ/6</td>
<td>t = 1.5ΔΩ</td>
</tr>
<tr>
<td>Plate ends</td>
<td>δ/π/80</td>
<td>t = 1.5ΔΩ</td>
</tr>
<tr>
<td>Tank ends</td>
<td>D/30</td>
<td>t = L = πΔΩ/L^6</td>
</tr>
</tbody>
</table>

It is obvious that \( T_2^{(1)} \) depends on the wave transmission coefficient \( T \). Two limits of \( T_2^{(1)} \) are given by

\[
k h \to 0 \quad \text{as} \quad \frac{3kh_1}{8(kh)^2},
\]

and

\[
k h \to \infty \quad \text{as} \quad \frac{2kh_1}{4}.
\]

4. Results and discussion

In this section, we first compare the simulated multiple wave reflections, phase changes of waves above the plate, reflection and transmission coefficients with those obtained from our experiments. At the end of this section, comparison is made between DBIEM simulations and the experiments of Brossard et al. (2009).

4.1. Grid sizes and desingularization distances

For all numerical simulations, the dimensional time step \( \Delta t = 0.01T_w \), where \( T_w \) is the wave period. Non-uniform grids are used in our simulations. On the free surface, we use either \( \Delta \Omega = L/125 \) (where \( L \) is the wave length) or \( \Delta \Omega < B/30 \), whichever is smaller. On the horizontal surfaces of the plate, \( \Delta \Omega = δ/6 \), where \( δ \) is the plate thickness. On the two ends of the numerical wave tank \( \Delta \Omega = D/30 \), where \( D \) is the water depth. On the two ends of the thin plate, \( \Delta \Omega = πδ/80 \). The desingularization distances along the boundaries of the computational domain are summarized in Table 2.

4.2. Simulated multiple reflections

When incident waves arrive at the plate, partial wave energy of incident waves will be reflected by the front edge of the plate and the rest will continue to propagate as transmitted waves in the water above the plate (ignoring the viscous energy dissipation). The transmitted waves will be partially reflected back again by the rear edge of the plate, resulting in a partial standing wave pattern in the water above the plate.

Fig. 6 shows a typical example of the simulated surface elevations at \( G_1 \) and \( G_5 \). The wave period is \( T_w = 1.1 \) s and wave height is \( H = 0.02 \) m in this example. The submergence of the plate is \( h_1 = 0.15 \) m. In comparison with Fig. 4, it can be seen that the numerical simulation captures the main features of the waves scattered by the plate, especially the multiple wave reflections in both the weather side and lee side of the plate. The number of waves contained in the wave front simulated by DBIEM is slightly different from that in the experiment. This is because the waves were generated with different ramp functions. As non-reflective boundary conditions were used at the two ends of the computational domain, the simulated wave surface elevations at \( G_1 \) and \( G_5 \) do not have the stage 4.

4.3. Phase change of fundamental waves above the plate

A typical comparison between the simulated and measured phase differences between two points above the plate are shown in Fig. 7 as functions of the relative plate width \( B/L_s \), where \( L_s \) is the length of the fundamental waves in water above the plate. The phase change of fundamental waves in water above the plate is simulated well by DBIEM. For the second harmonic waves, noticeable scatter is observed in both the measured and simulated \( Δψ_2 \), but reasonable agreement still can be found between the simulated and measured \( Δψ_2 \). Similar large scatter in the measured \( Δψ_2 \) has also been observed by Brossard et al. (2009). The scatter in \( Δψ_2 \) shows the difficulty in extracting the second order quantities from both the numerical simulations and the physical experiments.

Fig. 6. Samples of simulated surface elevation: (a) for wave gage \( G_1 \) and (b) for wave gage \( G_5 \).
4.4. Reflection and transmission of fundamental waves

Fig. 8 show the measured and simulated reflection and transmission coefficients of the fundamental waves for \( h_1 = 0.15 \) m and \( H_1 = 0.04 \) m. The reflection coefficients obtained by DBIEM agree very well with measurements. This is what we expected as the viscous effects are important only in the lee side of the plate (Ting and Kim, 1994) and the flow in the weather side of the plate can be treated as irrotational one. The transmission coefficients calculated by DBIEM are generally \( \approx 10\% \) greater than those measured, but the general trend of the variation of the transmission coefficient with \( B/L_s \) can still be captured by the DBIEM simulation.

It has been shown, by the long wave theory (see, e.g., Mei, 1989), that when \( B/L_s = 0.5 \), the phase difference between the two edges of a submerged step is \( \pi \). This is the resonant condition termed by Brossard et al. (2009). Studies have shown that the evanescent waves may slightly modify the resonant condition predicted by long wave theory (Devillard et al., 1988). In our experiments, the phase change between the two ends of the plate is \( \frac{1}{2} \) times of \( \Delta \psi_1 \) given in Fig. 7(a), where \( \Delta \psi_1 \) is the phase change between \( x = \pm 20 \) cm. The two ends of the plate were located at \( x = \pm 30 \) cm in our experiments. It can be seen from Fig. 8 that both the measured and computed \( R \) take their maximum at \( B/L_s \approx 0.48 \) and both the measured and computed \( T \) take their minimums at \( B/L_s \approx 0.44 \). We believe the difference from the classical \( B/L_s = 0.5 \) is associated with the evanescent waves that were not considered in the long wave approximation.

A comparison of Figs. 9 and 10 illustrates the effects of the plate submergence on the wave reflection and transmission. The reflection coefficients decrease with increasing submergence depth \( h_s \), while the transmission coefficients increase with decreasing plate submergence. These results are expected as in the limit of infinite submergence there will be no wave reflection; all wave energy can be transmitted to the other side of the plate. Both the maximum of the reflection coefficient and the minimum of the transmission coefficient still occur at \( B/L_s \approx 0.5 \). The reflection coefficient can be as large as \( R \approx 0.7 \) for \( h_1 = 0.1 \) m and \( H_1 = 0.02 \) m, demonstrating that a submerged thin plate can be one good candidate for the potential low cost breakwaters.

4.5. Free and phase-locked second harmonic waves

Nonlinear wave–wave interactions in the water above the plate will generate higher harmonic waves; these harmonic waves are phase-locked waves before they leave the plate. When they enter the deeper water in the lee side of the plate, these higher harmonic waves would become free waves. As the beach reflection coefficients were \( <5\% \) in our experiments, the free waves were found to be very small in the weather side of the
plate. Therefore, the discussion on the free and locked waves will focus on the transmitted waves in the lee side of the plate.

The amplitudes of the computed free waves and locked waves are shown in Figs. 11 and 12 and compared with those measured in our experiments. The second harmonic waves can be predicted with quite good accuracy by DBIEM simulations, suggesting that the second order waves are not affected by the viscous dissipation. In all the cases studied here, the locked waves are in general much smaller than the free waves, except for very short or very long waves.

Fig. 11 shows the measured and simulated second harmonic free and locked waves for the submergence \( h_1 = 15 \text{ cm} \) and two wave heights. In the limit of very short waves \( (B/L_s \to \infty) \), waves scattered by the deep submerged plate will disappear, so are the free waves; while in the limit of zero-length plate or very long waves \( (B/L_s \to 0) \), waves scattered by the zero-length submerged plate will disappear. For the submergence \( h_1 = 0.15 \text{ cm} \), the amplitude of the free waves takes its maximum at \( B/L_s = 0.4 \). Both the amplitudes of the free waves and the locked waves increase with increasing amplitude of the incident waves. Recall the theoretical expression, Eq. (21), the existence of the locked waves is not affected by the presence of the submerged plate. The theoretical curve of \( T_{21}^{(3)} \) is also shown in Fig. 11 for comparison. Both the computed and measured \( T_{21}^{(3)} \) agree well with that given by Eq. (21), which indicates the accuracy of the wave separation in our physical and numerical experiments. It can also be seen from Figs. 11 and 10 and 8 that the minimum of \( T_{21}^{(3)} \) is controlled by the minimum of the transmission coefficient \( T \).

The effects of the submergence are shown by comparing Figs. 11(a) and 12. In general, the amplitude of free waves decreases with increasing submergence, which again is expected: in the deep submergence limit the free waves will disappear. Again, both the computed and measured \( T_{21}^{(3)} \) agree well with that given by Eq. (21), and the minimum of \( T_{21}^{(3)} \) is controlled by the minimum of the transmission coefficient \( T \). The shallower submergence moves the maximum amplitude of second harmonic free waves to \( B/L_s = 0.65 \), which agrees with the experiments of Brossard et al. (2009): both our experimental results and those of Brossard et al. (2009) show that the resonant condition for the free waves is different from that for the fundamental waves. Brossard et al. (2009) tried to use the phase change of the second harmonic waves in the water above the plate to explain the resonant condition for the free waves. However, it is difficult to assess the influences of the evanescent waves on the determination of the phase change using the wave records taken right at the edges of the plate.

4.6. Computed velocity fields

The velocity fields also were computed from the velocity potentials on the boundaries. Fig. 13 shows an example of the
Computed velocity fields for \( h_1 = 0.15 \text{ m}, \quad T_w = 1.3 \text{ s} \) and \( H_f = 0.04 \text{ m} \). The time \( t = t_0 \) is chosen such that the water surface elevation at \( x = 15.70 \text{ m} \) is zero.

As expected, the calculated maximum vertical velocity on the free surface occurs at \( z = 0 \) and the maximum horizontal velocity on the free surface occurs at either the wave crest or wave trough. As potential flow theory cannot predict the flow separation and vortex shedding, the computed velocity fields in the vicinity of the two ends of the plate are expected to be different from the real flow fields (e.g., Batchelor, 2000); but away from the two ends, potential flow theory can still provide a reasonable description of the velocity field. It can be observed from Fig. 13 that the flow beneath the plate is more or less uniform horizontally, except in small regions next to the two ends where the influence of the evanescent waves is significant. The locations of zero velocity occur on the upper surface of the plate at \( x = 15.66 \text{ m} \) for \( t = t_0 \), 15.42 m for \( t = t_0 + T_w/4 \), 15.67 m for \( t = t_0 + T_w/2 \), and 15.39 m for \( t = t_0 + 3T_w/4 \).

Yu and Dong (2001) and Carter et al. (2006), using a viscous numerical wave tank based on Navier–Stokes equations, found a mean circulation pattern around the submerged plate, which confirmed the experimental findings of Murakami et al. (1992) (as quoted in Carter et al., 2006). However, our results, obtained by the DBIEM wave tank for inviscid fluids, does not show such mean circulation pattern, which agree with the results obtained by the linear BEM in Carter et al. (2006), suggesting that the mean circulation around the submerged plate is possibly due to the viscous streaming induced by water waves.

Second harmonic waves are generated in the shallow water region above the plate and propagate into the deep region behind the plate (see Massel, 1983 for example). The difference between the measured and calculated transmission coefficients is related to the dissipation of wave energy, which, in the problem studied here, might be due mainly to the vortex shedding from the plate. The noticeable discrepancy found in the calculated and measured transmission coefficients of the fundamental waves can possibly be explained by the following hypothesis: in the experiments, the frequency of vortex shedding and the strength of the vortices in the turbulent wake are controlled by the fundamental waves; as a result, only the energy in the transmitted fundamental waves is significantly dissipated into turbulence. This hypothesis needs to be further tested by more experimental studies.

### 4.7. Comparison with Brossard et al. (2009)

In Brossard et al. (2009), the thickness of the plate was 0.002 m, the water depth was fixed at \( D = 0.2 \text{ m} \), and the wave height was fixed at \( H_f = 0.02 \text{ m} \). In view of Eq. (8), the source is singular when the thickness of the plate approaches zero. From our numerical experiments, we found that DBIEM would result in noticeable numerical error when the thickness of the plate was much < 0.01 m. Therefore, when simulating the experiments of Brossard et al. (2009), the thickness of the plate was increased from 0.002 to 0.01 m. The increase in thickness is not expected to change the characteristics of wave scattering for a not-so-shallow submergence (see Appendix C for two examples). Results for four plate submergences were reported in Brossard et al. (2009), we report here the comparisons with their experimental results for the largest submergence \( h_1 = 0.1 \text{ m} \).

Fig. 14 compares the reflection and transmission coefficients computed by DBIEM with those measured by Brossard et al. (2009). The noticeable scatter in the reflection coefficients measured by Brossard et al. (2009) is believed to be partly due to the multiple reflections between the wave maker and the beach. The reflection coefficients computed by DBIEM agree reasonably well with those reported in Brossard et al. (2009).

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\(^{5}\) In this limit, the imaginary boundary for the Rankine sources will coincide with the physical boundary.
Fig. 15(a) compares the phase differences computed by DBIEM with those measured by Brossard et al. (2009) for the fundamental waves $\Delta \psi_1$ as well as for the second harmonic free waves $\Delta \psi_2$. For an easy comparison, we followed in this section the convention of Brossard et al. (2009) and defined the phase change between $\psi_0$ and $\psi$. The computed phase changes of the fundamental waves $\Delta \psi_1$ agree reasonably with those measured. The slight difference might be due to the exclusion of the evanescent waves in the determination of the phase angle at the two ends of the plate. Even though both the measured and computed phase changes for the second harmonic waves show certain degree of scatters, the trend of the phase change $\Delta \psi_2$ can still be captured well by DBIEM simulations.

Fig. 15(b) is a comparison of the free waves reported in Brossard et al. (2009) and those computed by DBIEM. Phase-locked waves are not reported in Brossard et al. (2009). The free waves $\psi_1$ are reproduced well by DBIEM simulations, suggesting that over-prediction of the transmission coefficient $T$ in Fig. 14(b) is due mainly to the viscous loss through skin friction and the vortex shedding.

It is worth mentioning the limitations of DBIEM when used for wave-structure interactions. We found from our numerical experiments that DBIEM might not be a good choice for problems involving very thin plates: large error may occur when the Rankine source is very close to the physical boundary. Also, for near-breaking waves, the plunger tip may need certain special treatment in order to avoid large numerical errors. For very large thick plates, the elasticity of the plate needs to be considered. After introducing the equation governing the elastic response of the plate to the mathematical formation, the method used here can be extended to study the effects of the elastic responses of the plate on the nonlinear wave scattering.

5. Concluding remarks

In this study, the desingularized boundary integral equation method (DBIEM) with Rankine image source is used to investigate the nonlinear wave scattering by a submerged thin plate. A section of the wave record free of multiple reflections is selected for each wave record to analyze the wave scattering. The numerical results
The authors thank three reviewers whose valuable comments have greatly improved the quality of the manuscript. This work was funded partially by Ministry of Education, Singapore, through Project NTU-SUG 3/07. The writers would like to thank Mr. Adi Kurniawan for proof reading the manuscript. Technicians at the Hydraulics Modeling Laboratory and the Construction Laboratory in the School of Civil and Environmental Engineering, NTU, are acknowledged for their assistance in preparing the breakwater models used in the experiments.

Appendix A. Wave generation in NWT and non-reflective boundary conditions at the two ends

To generate waves in the numerical wave tank, we prescribe an oscillating pressure distribution at free surface to simulate a pneumatic wave maker. At two ends of the computational domain, numerical damping layers adjacent to the non-reflective boundary are used to reduce the wave reflections from the artificial boundaries. The position of the pneumatic wave maker and the damping layers are shown in Fig. 1.

The implementations of the pneumatic wave maker and the damping layer in simulation are achieved by modifying the kinematic and dynamic free surface boundary conditions (Eqs. (3) and (4));

\[
\frac{dx}{dt} = \nabla \phi - \omega \nu(x) \tilde{x},
\]

\[
\frac{d\phi}{dt} = \frac{1}{2} \nabla \cdot \nabla \phi - \nu \nabla (x) \phi - P_c(x, t),
\]

where \( \nu(x) \) and \( P_c(x, t) \) are artificial viscosity and pressure distribution, respectively. The following quadratic artificial viscosity function used by Zhang et al. (2006) is adopted in this study.

\[
\nu(x) = \begin{cases} 
0, & -6L < x < 6L, \\
\left(\frac{x + 6L}{2L}\right)^2, & -8L < x < -6L, \\
\left(\frac{x - 6L}{2L}\right)^2, & 6L < x < 8L,
\end{cases}
\]

where \( L \) is the wave length.

To generate the regular waves in the numerical wave tank, the artificial pressure distribution used by Clamond et al. (2005) is adopted.

\[
P_c(x, t) = \left(\frac{g A}{2 \pi} \sinh(2kD) \right) e^{-\left(\frac{(x-x_0)^2}{2\ell^2}\right)} \sin(\omega t),
\]

where \( k = 2\pi/L \) is the wavenumber, \( A \) the wave amplitude, \( D \) the water depth, and \( \ell = 2.73 \). The position of the numerical wave-maker is denoted by \( x_0 \), for which we take \( x_0 = -5.5L \) in this study. Note that \( P_c \) decays exponentially with \( (x-x_0)^2/L^2 \).

Appendix B. Considerations on grid sizes and desingularization distances

In DBIEM, numerical errors are controlled by two factors: (i) the distance between two adjacent sources (i.e., the grid size) and (ii) the distance between the Rankine source and the physical boundary, i.e., the desingularization distance. There is no theoretical expression available for \( \ell \). Therefore, for a given problem, \( \ell \) is in practice related to the grid size \( \Delta \Omega \) by certain empirical relationships.

Fig. 15. A comparison of DBIEM with EXP (Brossard et al., 2009): (a) phase angle change and (b) the second order free waves. Submergence \( h_i = 0.10 \text{m} \) and wave height \( h_s = 0.02 \text{m} \).
B.1. Flows without submerged objects

For free surface flows without submerged objects in a deep water flow field, the desingularization distance \(\ell\) is determined by the grid size \(\Delta \Omega\) and a typical length scale \(L\), which characterizes the spatial variations of the water surface. For such flows, Cao et al. (1991) proposed the following empirical formula for selecting \(\ell\):

\[
\ell = \frac{\alpha}{L} \left(\frac{\Delta \Omega}{L}\right)^{\beta},
\]

where \(\alpha\) and \(\beta\) are empirical parameters. This empirical relationship is adopted in this study for Rankine sources on water surface and the two ends of the wave tank where no other length scales are relevant.

Previous studies have suggested that \(\beta = 0.5\), which is adopted in this study. Note that the parameter \(\alpha\) depends on the choice of the length scale \(L\). Suppose that we fix \(\beta\) and take \(\alpha = \alpha_1\) for \(L = L_1\) and \(\alpha = \alpha_2\) for \(L = L_2\), we have

\[
\frac{\alpha_1}{\alpha_2} = \left(\frac{L_1}{L_2}\right)^{-1}. \tag{29}
\]

In our study, we choose the wave length as the length scale \(L\) and require that there are at least 125 Rankine sources within one wave length in this study. Our numerical experiments showed that \(\alpha = 0.21\) could give results in good agreement with experiments.

B.2. Flows with a submerged plate

When there is a submerged plate in the flow field, in addition to the wave length, both the thickness, \(\delta\), and the length of the submerged plate, \(B\), will affect the selection of the grid size \(\Delta \Omega\) and the desingularization distance \(\ell\).

There will be a partial standing wave pattern in the water above the submerged plate. For very short waves, the grid size should be determined by the wave length; for very long waves, the grid size should be determined by the length of the plate. Our numerical experiments showed that

\[
\Delta \Omega = \min(L/125, B/30) \tag{30}
\]

produce results in good agreement with experiments, i.e., when \(L / B > 25 / 6\) we choose \(\Delta \Omega = L / 125\), otherwise we choose \(\Delta \Omega = B / 30\). Just like for free surface flows without submerged objects, we adopt Eq. (28) with \(\alpha = 0.21\) and \(\beta = 0.5\) for the Rankine sources of the free water surface and the two ends of the wave tank.

On the upper or lower surface of the submerged plate, using Eq. (28) with Eq. (30) will lead to \(\ell > \delta\), which should be avoided in DBIEM. The maximum desingularization distance is \(\ell = \delta / 3\) if we assume that the errors associated with the distance between two adjacent sources are of the same order of magnitude as those associated with the distance between the physical boundary and the Rankine source. In our simulations we chose \(\ell = \delta / 4\) to reduce the possible influences of the Rankine sources for upper surface on the flows beneath the lower surface of the plate, and vice versa. The grid size \(\Delta \Omega = \delta / 6\) was chosen for both the upper and lower surfaces, which gave \(\ell / \Delta \Omega = 1.5\).

### Table 3

<table>
<thead>
<tr>
<th>(\ell / \Delta \Omega)</th>
<th>1.8</th>
<th>1.5</th>
<th>1.0</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R)</td>
<td>0.266</td>
<td>0.279</td>
<td>0.204</td>
<td>0.024</td>
</tr>
</tbody>
</table>

Anticipating relative large velocity gradients at the two ends of the plate, 40 Rankine sources were used at each end. The distance between the two adjacent Rankine sources can be estimated by \(\theta / 4 - \ell\) with \(\theta = \pi / 40\), which also means that the grid size on the physical boundary is \(\Delta \Omega = \pi \delta / 80\). To reduce the errors associated with the distance between two adjacent Rankine sources, \(\ell < \delta / 2\) is recommended. We write \(\ell = \delta / (2 \gamma)\) with \(\gamma = \Omega / (\pi \gamma\)\), which gives \(\ell / \Delta \Omega = 40 / (\pi \gamma\)\). For \(\gamma = 10\), we have \(\ell / \Delta \Omega = 1.27\); for \(\gamma = 8\), we have \(\ell / \Delta \Omega = 1.59\). To be consistent with the expression for \(\ell\) used for the lower and upper surfaces of the plate, we took \(\ell = 1.5 \Delta \Omega\) in this study.

For Rankine sources on the two surfaces and two ends of the plate, we have tested \(\ell / \Delta \Omega = 0.5, 1.0, 1.5,\) and 1.8 for the submergence \(h_1 = 0.15\) m and waves of the period 0.9 s and the height 0.04 m. The measured reflection coefficient is \(R = 0.294\). The reflection coefficients calculated with different values of \(\ell / \Delta \Omega\) are listed in Table (3). It can seen that the results obtained with \(\ell / \Delta \Omega = 1.5\) agrees with that measured better than other choices of \(\ell / \Delta \Omega\).

### Appendix C. Effects of plate thickness on reflection coefficients

Two numerical examples are provided here to show the effects of the thin plate thickness on reflection coefficients. The numerical results are compared with our measurements reported in this paper and the those reported in Brossard et al. (2009).

- **Case 1**: This case provides a comparison with our experiments shown in Fig. 8. The thickness and length of the plate are 10 and 600 mm, respectively. The submergence is 150 mm and the water depth is 300 mm. The incident wave height and period are 40 mm and 0.9 s, respectively, giving \(B / L_o = 0.628\). The measured reflection coefficient is 0.294. In the numerical experiments, five values of the plate thickness are examined here: \(\delta = 5, 7.5, 10, 15,\) and 20 mm.

- **Case 2**: This case provides a comparison with Brossard et al. (2009) shown in Fig. 14. The thickness and length of the plate are 2 and 250 mm, respectively. The submergence is 100 mm and water depth is 200 mm. The incident wave height and period are 20 mm and 0.575 s, respectively, giving \(B / L_o = 0.549\). The measured reflection coefficient is 0.150. In the numerical experiments, eight values of the plate thickness are examined here: \(\delta = 2.5, 3, 4, 5, 7.5, 10, 15,\) and 20 mm.

![Fig. 16. Effects of the plate thickness on reflection coefficients.](image-url)
Note that the length of the plate for the case 2 is just \( \frac{L}{2} \) of that for the case 1. For the same \( B/L \sim 0.6 \), the relative submergence is \( h_1/L = 0.239 \) for the case 2 and \( h_1/L \approx 0.15 \) for the case 1. It is expected that the effects of the plate thickness on the reflection coefficient are weaker for the case 2 than for the case 1.

The calculated and measured reflection coefficients are shown in Fig. 16. For the case 1, the best agreement between the calculated and the measured reflection coefficients are obtained with \( \delta = 10 \text{mm} \), which is also the actual plate thickness; while for case 2, the effects of the plate thickness on the reflection coefficient is not significant. This observation agrees with our analysis based on the values of \( h_1/L \).

For the case 2, even though the calculated reflection coefficients for the eight plate thicknesses all are very close to the measured one, but the computation time increases with decreasing plate thickness since more iterations are needed to solve the system of algebraic equations when the system is not well-conditioned. As a compromise between accuracy and the computation time, we chose \( \delta = 10 \text{mm} \) for the case 2, which is the same as that for the case 1.

References


