Nonlinear parameter estimation in water-saturated sandy sediment with difference frequency acoustic wave

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Abstract

Difference frequency acoustic wave from nonlinear interaction of two primary acoustic waves at frequencies of 76 and 114 kHz was utilized with a parametric acoustic array theory to estimate the nonlinearity parameter of water-saturated sandy sediment. Such nonlinearity parameter can be used as background information for the nonlinear acoustic investigation of bottom or sub-bottom profiling in the ocean sandy sediments. Because of its lower attenuation the difference frequency acoustic wave method can be usefully applied to estimate the nonlinearity parameter of ocean sediment in the ocean as well as under laboratory conditions. The nonlinearity parameter \( \beta \) for the water-saturated sandy sediment used as a reference in this study was estimated as \( \beta = 80.5 \pm 5.1 \) at the difference frequency of 38 kHz. It was agreed very well with that estimated at the difference frequency of 67 kHz, when two primary frequencies were 137 and 204 kHz. The estimated nonlinearity parameter of water-saturated sandy sediment in this study was also compared and analyzed with those estimated in previously published literatures. It was suggested that the difference frequency wave method used to estimate the nonlinearity parameter of water-saturated sandy sediment can be employed as a good method to estimate the nonlinearity parameters of fluid-like granular media.

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1. Introduction

Granular media, such as soils, rocks, and gassy sediments, exhibit high nonlinearities of two to three orders of magnitude greater than that of water [1–4]. These nonlinearities are due to the structural inhomogeneities of the granular media such as defects, cracks and discontinuities, which can be used in medium characterization. Such nonlinearities easily generate nonlinear acoustic waves that can be usefully utilized in seismology, geophysics, and geophysical oceanography [3,5–7]. Especially, applications of nonlinear acoustic waves in geophysical oceanography have received much attention because of the practical importance in oil field prospecting and ecological monitoring in the ocean sediments [1,7,8].

Some of ocean sediments contain a lot of bubbles and show strong nonlinear responses to acoustic waves. These nonlinear acoustic responses are mainly attributable to high nonlinearities of bubbles in the ocean sediments, which can be usefully employed in estimation of the gas void fraction in gassy sediments [7,8]. As the background information for nonlinear acoustic experiments in gassy sediments, nonlinear acoustic properties in water-saturated sediments should be investigated. Even though the nonlinearity parameters of water-saturated sediments are much lower than those of gassy sediments [7,8], they can provide important and useful background information for the nonlinear acoustic investigation of bottom or sub-bottom profiling in the ocean [9–11].

When a primary acoustic wave with finite-amplitude propagates in a medium, nonlinear acoustic waves, such as subharmonic, ultraharmonic, second harmonic, higher harmonic, sum and difference frequency acoustic waves, can be easily generated due to the nonlinearity of the medium [3,6–9,12–14]. Among these nonlinear acoustic waves, the second harmonic and difference frequency waves are usually used to estimate the nonlinearity parameter of the medium [3,6,13,14]. The second harmonic acoustic wave has been usefully used to estimate the nonlinearity parameters of fluid, solid, and biological media under laboratory conditions [6,13,14]. However, it is difficult to estimate the nonlinearity parameter of the ocean sediment in the ocean because of its high attenuation. The difference frequency wave has not been widely used to estimate the nonlinearity parameter of a medium even it has lower attenuation and narrower directivity [15]. These characteristics of the difference frequency wave can be employed to estimate the nonlinearity parameter of the ocean sediments in the ocean as well as under laboratory conditions [7–9].
The present study is aimed at determining the nonlinearity parameter of water-saturated sandy sediment with a difference frequency wave method. A pulse transmission technique is used to generate the difference frequency wave in water-saturated sandy sediment. The parametric acoustic array theory described in the following section is applied to estimate the nonlinearity parameter of water-saturated sandy sediment.

2. Parametric acoustic array theory

Nonlinear wave equation known as the Westervelt equation can well describe nonlinearity, attenuation and diffraction for acoustic wave propagation in fluid-like nonlinear media. It is written as [16]

\[ \left( \nabla^2 - \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} \right) p + \delta \frac{\partial^2 p}{\partial t^2} = - \frac{\beta}{\rho_0 c_0^3} \frac{\partial^2 p}{\partial t^2}, \]

(1)

where \( p \) is the acoustic pressure, \( \delta \) is the diffusivity associated with sound absorption, which is related with the attenuation of acoustic wave, \( \beta \) is the nonlinearity parameter of the medium, \( c_0 \) and \( \rho_0 \) are the sound speed and the density of the medium, respectively. Since the sediment is generally an unconsolidated granular medium, it can be considered as a fluid-like sediment [7,8,17–20]. For this case, Eq. (1) can be applied to describe the propagation of nonlinear acoustic waves in the sediment.

If the attenuation of acoustic wave is considered in the solution of Eq. (1) and two primary waves are incident on a sediment layer with thickness \( l \) in water as shown in Fig. 1, Eq. (1) can be expressed for propagation of the difference frequency wave as the form of [21]

\[ \nabla^2 p_d - \frac{1}{c_0^2} \frac{\partial^2 p_d}{\partial t^2} = -Q, \]

(2)

where \( Q = -\frac{\rho_0^2}{\rho_w^2} p_1 p_2 \) is the virtual source term of the difference frequency wave field in the sediment layer, \( \omega_{d} = \omega_1 - \omega_2 \) is the angular frequency of the difference frequency wave, \( \omega_{1,2} \) are the angular frequencies of two primary waves, \( c_w \) is the sound speed of water, \( c_s \) and \( \rho_s = \rho_w (1 - \sigma) \) are the sound speed and the density of sediment, \( \sigma \) is the porosity of sediment, \( \rho_w \) and \( \rho_s \) are the densities of sediment grain and water, \( p_1,2 \) and \( p_d \) are the acoustic pressures of primary and difference frequency waves, respectively. When an observation point of difference frequency wave is located at a far-field distance of the primary wave as shown in Fig. 1, the solution of Eq. (2) for difference frequency wave radiated from an element of sediment volume \( dV \) to a far-field point \( O \) can be written as

\[ p_d = \frac{-i \rho_0^2}{4 \pi \rho_w c_s^2} \int \frac{T e^{i \mathbf{k}_d \mathbf{r} - \mathbf{e}^{i \mathbf{k}_d \mathbf{r}}} p_1 p_2 dV}{R}, \]

(3)

where

\[ k_d R = k_{d0} R_w + k_{d1} R_s \]

\[ = k_{d0} \frac{R \cos \phi - (d + l - z)}{\cos \phi} + k_{d1} \frac{(d + l - z)}{\cos \phi}, \]

(4)

\[ \chi_{d0} R = \chi_{d0} R_w + \chi_{d1} R_s \]

\[ = \chi_{d0} \frac{R \cos \phi - (d + l - z)}{\cos \phi} + \chi_{d1} \frac{(d + l - z)}{\cos \phi}, \]

(5)

\[ R \approx R_0 (z - d) \cos \theta, \]

(6)

\[ T' = \frac{2 \rho_w c_w \cos \phi}{\rho_w c_s \sqrt{1 - \frac{(c_w/c_s)^2 \sin^2 \phi + \rho_w c_w \cos \phi}}}, \]

(7)

\( R_w \) and \( R_s \) are the propagation distances of difference frequency waves, \( k_{d0w/ds} = \omega_{d}/(c_w/c_s) \) and \( \chi_{d0w/ds} \) are the wave numbers and attenuation coefficients of difference frequency waves in water and the sediment layer, respectively, and \( R_0 \) is the distance from the sediment layer to the far-field point \( O \), \( d \) is a distance from the primary acoustic source to the sediment layer, \( z \) is the distance from the primary acoustic source to the element of sediment volume \( dV \), \( T' \) is the transmission coefficient of difference frequency wave propagating from the sediment layer to water, \( \phi \) is a radiation angle of the difference frequency wave and it can be approximately the same as another angle \( \theta \). If we assume that two primary waves are collimated plane waves radiated by a circular piston of radius \( a \), the acoustic pressure fields of two primary waves in the sediment layer located at distance \( z (d \leq z \leq d + l) \) can be expressed as follows:

\[ p_{1,2} = A_{1,2} T e^{i \mathbf{k}_{1,2} (z-d)} e^{i (\mathbf{r}_{1,2} - \mathbf{r}_d)}, \]

(8)

where \( A_{1,2} \) are the incident acoustic pressure amplitudes of the primary acoustic waves at the sediment layer, \( T = 2 \rho_w c_w / (\rho_w c_s + \rho_s c_w) \) is the pressure transmission coefficient of the primary wave from water to the sediment layer, \( \chi_{1,2s} \) and \( k_{1,2s} = \omega_{1,2s}/c_s \) are the atten-
Table 1
Grain size distribution in water-saturated sandy sediment.

<table>
<thead>
<tr>
<th>Grain diameter (μm)</th>
<th>Distribution (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>354–500</td>
<td>78.2</td>
</tr>
<tr>
<td>250–354</td>
<td>18.5</td>
</tr>
<tr>
<td>&lt;250</td>
<td>3.3</td>
</tr>
</tbody>
</table>

Theoretical transmission loss [22] of the primary waves is 0.65 dB at the interface between water and sediment layer. Therefore, the reincident acoustic pressure fields of the reflected primary waves from the interface between water and sediment layer can be ignored in Eq. (8). By substituting Eq. (8) into Eq. (3), the pressure field of the difference frequency wave can be given as

\[ p_d(R_0, \theta) = \frac{-A_1 A_2 \omega_0^2 T^2 \beta \rho_0 c_s^2}{4 \pi R_0 \rho_c c_s^2} \times\int_{-1}^{1} \int_{0}^{2\pi} e^{i(\beta_1 + \beta_2)} e^{i (R_0 + d \cos \theta - d)} d\theta dz \]

(9)

where

\[ B_1 = \omega_0 \left( \frac{d + l}{\cos \theta} \right) + \omega_d \left( R_0 + d \cos \theta - d \right), \]

(10)

\[ B_2 = k_d \left( \frac{d}{\cos \theta} - d \right) + k_d \left( R_0 + d \cos \theta - d \right), \]

(11)

\[ B_3 = \omega_0 + \omega_0 \left( \frac{1}{\cos \theta} - \cos \theta \right), \]

(12)

\[ B_4 = k_d \left( \frac{1}{\cos \theta} - \cos \theta \right) - k_d \left( \frac{1}{\cos \theta} - 1 \right), \]

(13)

Since the attenuation coefficient of the acoustic wave in water is very small compared with that in the sediment layer [23], the attenuation coefficient \( \omega_d \) of the difference frequency wave in Eqs. (10) and (12) can be negligible. The nonlinearity parameter \( \beta \) of the sediment layer in Eq. (9) can be simply expressed for \( \theta = 0 \) as follows:

\[ \beta = \frac{4 \rho_c c_s^2 \rho p_0 e^4 \omega_d^2}{A_1 A_2 T^2 \alpha^2 \omega_s^2 \left[ 1 - e^{-\omega_s} \right]} \]

(14)

where \( \omega_s = \omega_1 + \omega_2 - \omega_d \) is the combined attenuation coefficient of primary and difference waves in the sediment layer.

3. Material and methods

3.1. Water-saturated sandy sediment

Water-saturated sandy sediment was prepared under laboratory conditions. In order to remove micro-bubbles in it, the sediment was boiled for 1 hour in water and then was packed into an acrylic box with very thin films. The grain density and the porosity of water-saturated sandy sediment were 2559 ± 52 kg/m³ and 0.41 ± 0.01, respectively. They were the mean values of 10 sediment samples. The grain density \( \rho_c \) and porosity \( \sigma \) of water-saturated sandy sediment are defined as follows:

\[ \rho_c = \frac{M_G}{V_G}, \]

(15)

\[ \sigma = \frac{V_w}{V_G + V_w}, \]

(16)

where \( M_G \) and \( V_G \) are the mass and volume of sediment grains, respectively, and \( V_w \) is the volume of interstitial pore water in water-saturated sandy sediment. The grain size distribution for water-saturated sandy sediment is shown in Table 1.

3.2. Experimental measurements

A schematic diagram of the experimental setup for acoustic measurements of water-saturated sandy sediment is shown in Fig. 2. The dimension of the anechoic water tank in Fig. 2 was 1520 × 750 × 600 mm. The anechoic water tank was degassed.
using the fiberflow hollow fiber degassing system (Mintech Nos. 93024-232, and 93024-233). The temperature in the water tank was maintained between 20 °C and 25 °C. Two arbitrary waveform generators (Agilent 33250A) and a power amplifier (Amplifier Research 75A 250) were used to drive a transmitter with a diameter of 80 mm. The transmitter was the acoustic transducer with the main resonance frequency of 114 kHz and the second resonance frequency of 76 kHz, as shown in Fig. 3. Thus, it was simultaneously driven at two primary frequencies of 76 and 114 kHz, in order to generate the difference frequency wave at 38 kHz in water-saturated sandy sediment. The water-saturated sandy sediment was located at a distance of 120 mm from the transmitter, which was the far-field distance of the primary wave at a frequency of 114 kHz. As seen in Fig. 2, the waveguide with a diameter of 100 mm was installed to minimize the diffraction of the primary waves in the water tank. It was made from acrylic material, with its inner wall lined with anechoic materials.

The signals transmitted through the water-saturated sandy sediment at two primary frequencies were tone burst sinusoidal signals with a pulse duration of 500 μs and repetition time of 500 ms. Two primary tone burst sinusoidal signals were mutually synchronized through a signal trigger function of arbitrary waveform generators before they were amplified through the power amplifier. The pulse duration was determined for enough nonlinear interaction of two primary acoustic waves in the sediment. For primary waves of 76 kHz and 114 kHz, incident acoustic pressure amplitudes on the sediment were 48.9 kPa and 67.0 kPa, respectively. The incident pressure amplitudes were measured by a hydrophone (B&K 8103). The transmitted signals through the water-saturated sandy sediment in Fig. 2 were also received by the hydrophone. The hydrophone had an omni-directional receiving beam pattern and a constant receiving sensitivity –211.3 dB re 1 V/μPa within ±2 dB between 1 Hz and 150 kHz. The hydrophone was located at a distance of 400 mm from the transmitter to satisfy the far-field condition of the primary waves. The hydrophone was located using a positioning system. The received signals were acquired using a 500 MHz digital storage oscilloscope (LeCroy LT342) and stored on a computer for off-line analysis.

3.3. Measurements of attenuation coefficient and sound speed

A pulse transmission technique can be used to measure the attenuation coefficients of two primary waves in water-saturated sandy sediment [24]. For this technique, the pulse length needs to be as short as possible to measure the precise attenuation coefficients. For the measurements of attenuation coefficients of acoustic waves over the frequency range from 30 to 160 kHz a broadband spherical transducer (Gearing&Watson D70, 30 mm diameter) with short pulse length and a hydrophone (B&K 8103) were used. The spherical transducer was located at distance of 50 mm in front of water-saturated sandy sediment. This distance was determined by considering the wavelength of acoustic wave at 30 kHz. To minimize diffraction effect of acoustic wave due to edges of the sediment box, the hydrophone (B&K 8103) was located at distance of 10 mm in back of water-saturated sandy sediment. In the present study, the measured pulse length was about 15 μs. For the measurements of attenuation coefficients of acoustic waves over the frequency range from 160 to 300 kHz a pair of broadband transducers (TKS IMA, 25.4 mm diameter) with a center frequency of 200 kHz and short pulse length was used. The measured pulse length was about 24 μs. The pair of transducers was aligned facing each other and was separated by a distance of 150 mm, which was beyond the far-field distance.

Fig. 4a and b show the time signals transmitted without and with the water-saturated sandy sediment in water. The speed of sound and the attenuation coefficients were measured by using the fiberflow hollow fiber degassing system (Mintech Nos. 93024-232, and 93024-233). The temperature in the water tank was maintained between 20 °C and 25 °C. Two arbitrary waveform generators (Agilent 33250A) and a power amplifier (Amplifier Research 75A 250) were used to drive a transmitter with a diameter of 80 mm. The transmitter was the acoustic transducer with the main resonance frequency of 114 kHz and the second resonance frequency of 76 kHz.

**Fig. 3.** Transmitting response of the transmitter measured through the frequency sweep of the arbitrary waveform generator between 20 and 200 kHz. The main resonance frequency of the transmitter was 114 kHz. The second resonance frequency was 76 kHz.

**Fig. 4.** The time signals transmitted with and without water-saturated sandy sediment for (a) a broadband spherical transducer and (b) a pair of transducers with center frequency 200 kHz. The time signals were used to measure the attenuation coefficients of the difference frequency and two primary acoustic waves at 38, 76 and 114 kHz.
sound in the sediment was determined by the following equation [24]

\[ c_s = \frac{l}{C_0 D t}; \tag{17} \]

where \( l \) is the thickness of water-saturated sandy sediment, \( D_t \) is the difference between the arrival times of the received signals with and without the sediment in water, and \( C_w \) is the temperature-dependent speed of sound in water. The measured sound speed in water-saturated sandy sediment was 1697 ± 14 m/s.

A fast Fourier transform (FFT) was used to obtain the power spectra of the time signals in Fig. 4a and b. The signal loss was obtained by subtracting the power spectrum of the signal through the water-saturated sandy sediment from that of the reference signal through water only. Dividing the signal loss by 8.686 and the thickness of water-saturated sandy sediment and compensating the transmission loss due to the reflection of the acoustic wave at the interface between water and the sediment, the attenuation coefficients were obtained in a unit of Np/m. The formula was as follows [25]

\[ \alpha = [\ln(T_p) + \ln(A_w/A_s)]/l, \tag{18} \]

\[ T_p = \frac{4Z_wZ_s}{(Z_w + Z_s)^2}, \tag{19} \]

where \( T_p \) is the power transmission coefficient of the acoustic wave, \( Z_w = \rho_w C_w \) and \( Z_s = \rho_s C_s \) are the impedances in water and the water-saturated sandy sediment, \( l \) is the thickness of the sediment, \( A_w \) and \( A_s \) are amplitudes of signal spectra received without and with the sediment in water, respectively. Fig. 5 shows the measured attenuation coefficients as a function of frequency in water-saturated sandy sediment. Table 2 shows the mean value of the attenuation coefficients of two primary and difference frequency waves in water-saturated sandy sediment.

<table>
<thead>
<tr>
<th>Frequency (kHz)</th>
<th>Attenuation coefficient of water-saturated sandy sediment (Np/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>38</td>
<td>1.96 ± 0.19</td>
</tr>
<tr>
<td>76</td>
<td>2.84 ± 0.37</td>
</tr>
<tr>
<td>114</td>
<td>3.69 ± 0.21</td>
</tr>
</tbody>
</table>

4. Results and discussion

Fig. 6a and b show the frequency spectra of the signals received at a distance of 400 mm from the transmitter without and with the water-saturated sandy sediment in the anechoic water tank. Two primary waves of 76 kHz and 114 kHz were simultaneously amplified through the power amplifier and were emitted at the transmitter. Therefore, the difference frequency component at 38 kHz in Fig. 6a could be generated as a result of the nonlinear interaction of two primary waves due to the nonlinearities of the power amplifier and the transmitter as well as water. However, the difference frequency component in Fig. 6b was clearly generated by the nonlinearity of water-saturated sandy sediment. The pressure level at the difference frequency component in the sediment was about 7.6 dB higher than that in water. The sum frequency component at 190 kHz in Fig. 6b was also observed due to the nonlinearity of water-saturated sandy sediment. The pressure level at the sum frequency component in the sediment was about 8.7 dB higher than that in water.

Fig. 7 shows the pressure levels of the difference frequency wave at 38 kHz generated in water-saturated sandy sediment as a function of the receiving distance of hydrophone (\( R_0 \)). Nonlinearity parameter of water-saturated sandy sediment can be estimated with Eq. (14) by substituting the pressure amplitude of the difference frequency wave given in Fig. 7. Table 3 shows the values of
the estimated nonlinearity parameter for receiving distances. The mean value of the estimated nonlinearity parameter from Table 3 was $\beta = 80.5 \pm 5.1$. The input parameters in Eq. (14) are listed in Table 4.

In order to reconfirm the estimated nonlinearity parameter of water-saturated sandy sediment, other transmitter (NEC/TOKIN TGM50, 70 mm diameter) and hydrophone (Reson TC4038) were used in the experimental setup of Fig. 2. The transmitter was simultaneously driven at two primary frequencies of 137 and 204 kHz to generate the difference frequency wave at 67 kHz in water-saturated sandy sediment. The sediment was located at a distance of 170 mm from the transmitter. The distance was the far-field distance of the primary acoustic wave at the frequency of 204 kHz. The hydrophone had an omni-directional receiving beam pattern and a constant receiving sensitivity of 204 kHz. The hydrophone had an omni-directional receiving beam pattern and a constant receiving sensitivity of 204 kHz.

The distance was the water-saturated sandy sediment. The sediment was located at a distance of 500 mm from the transmitter. The distance was the far-field distance of two primary acoustic waves. Fig. 8 shows the pressure levels of the difference frequency wave at 67 kHz, generated in the sediment as a function of the receiving distance of the hydrophone ($R_0$). Then, the nonlinearity parameter of water-saturated sandy sediment estimated from Eq. (14) was listed in Table 5 as a function of the receiving distance. Its mean value was $\beta = 88.6 \pm 8.7$. It agrees very well with that estimated at the difference frequency of 38 kHz. The input parameters in Eq. (14) are listed in Table 6. The attenuation coefficients of the two primary and difference frequency waves were determined with the extrapolated line in Fig. 5.

Bjorno [26] has carried out an experiment using the thermodynamic method [27] to estimate the nonlinearity parameters of water-saturated sandy sediments. The estimated nonlinearity parameters were between 5.8 and 6.9. These values were obtained assuming the water-saturated sandy sediments as fluid-like media.

Since the sediment has practically a frame structure, Hovem [28] has indicated that Bjorno’s results need to be complemented considering the frame of sediment. For this reason, Hovem [28] has developed a theoretical model to estimate the nonlinearity parameter considering the nonlinear effect due to the frame of sediment, using Biot’s theory for acoustic wave propagation in a fluid-saturated porous media [29,30]. The model results for the nonlinearity parameter of water-saturated sandy sediment was around $\beta = 10$. It was very different from our estimated value in this study. The sand grains can be considered as a kind of small rocks. In this case, the nonlinearity of water-saturated sandy sediment can be more affected by the intergrain contacts than the frame of sediment [1,4,31]. Since Hovem’s model did not consider these nonlinear acoustic sources, the estimated nonlinearity parameter of water-saturated sandy sediment with Hovem’s model appears to be underestimated.

Donskoy et al. [2] have developed a theoretical model for finite-amplitude wave propagation in a fluid-saturated porous medium, using the nonlinear dynamic equation [32]. This model showed that the nonlinearity parameter of a sediment could become from $10^2$ to $10^3$ as a function of porosity. For the porosity of water-saturated sandy sediment used in this study, the nonlinearity parameter predicted from this model was about $\beta = 1300$. It was a very high value compared with our estimated value. Donskoy et al. [2] have referred many experimental literatures [5,6,33–35] relating to high nonlinearities of rocks and soils to support their theoretical model, but they have not referred the literatures with respect to high nonlinearities of water-saturated sediments. In the present, high nonlinearity of the sediment has only been observed for gassy

![Graph](image)

**Fig. 7.** Pressure level of the difference frequency wave at a frequency of 38 kHz in water-saturated sandy sediment as a function of receiving distance of hydrophone ($R_0$).

**Table 3**

<table>
<thead>
<tr>
<th>Receiving distance, $R_0$ (mm)</th>
<th>Value of nonlinearity parameter of water-saturated sandy sediment, $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>280</td>
<td>78.4</td>
</tr>
<tr>
<td>290</td>
<td>83.1</td>
</tr>
<tr>
<td>300</td>
<td>83.9</td>
</tr>
<tr>
<td>310</td>
<td>74.1</td>
</tr>
<tr>
<td>320</td>
<td>81.3</td>
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<tr>
<td>330</td>
<td>76.3</td>
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<tr>
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<td>350</td>
<td>79.7</td>
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<tr>
<td>360</td>
<td>74.2</td>
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<tr>
<td>370</td>
<td>82.5</td>
</tr>
<tr>
<td>380</td>
<td>88.9</td>
</tr>
<tr>
<td>390</td>
<td>78.9</td>
</tr>
<tr>
<td>400</td>
<td>85.5</td>
</tr>
<tr>
<td>410</td>
<td>85.3</td>
</tr>
<tr>
<td>420</td>
<td>76.1</td>
</tr>
<tr>
<td>430</td>
<td>82.2</td>
</tr>
<tr>
<td>440</td>
<td>86.9</td>
</tr>
<tr>
<td>450</td>
<td>81.4</td>
</tr>
<tr>
<td>460</td>
<td>76.0</td>
</tr>
<tr>
<td>470</td>
<td>82.2</td>
</tr>
<tr>
<td>480</td>
<td>71.8</td>
</tr>
<tr>
<td>490</td>
<td>78.8</td>
</tr>
<tr>
<td>500</td>
<td>75.3</td>
</tr>
<tr>
<td>510</td>
<td>72.4</td>
</tr>
<tr>
<td>520</td>
<td>90.1</td>
</tr>
<tr>
<td>530</td>
<td>89.0</td>
</tr>
</tbody>
</table>

Mean ± standard deviation: 80.5 ± 5.1

**Table 4**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density of sediment, $\rho_s$</td>
<td>1922 kg/m³</td>
</tr>
<tr>
<td>Speed of sound in the sediment, $c_s$</td>
<td>1697 m/s</td>
</tr>
<tr>
<td>Incident acoustic pressure amplitude (76 kHz), $A_1$</td>
<td>48.9 kPa</td>
</tr>
<tr>
<td>Incident acoustic pressure amplitude (114 kHz), $A_2$</td>
<td>67.0 kPa</td>
</tr>
<tr>
<td>Transmitter radius, $a$</td>
<td>40 mm</td>
</tr>
<tr>
<td>Combined attenuation coefficient of the sediment, $\sigma_{2s}$</td>
<td>4.57 Np/m</td>
</tr>
<tr>
<td>Transmission coefficient from water to the sediment, $T$</td>
<td>1.37</td>
</tr>
<tr>
<td>Transmission coefficient from the sediment to water, $T'$</td>
<td>0.63</td>
</tr>
<tr>
<td>Thickness of the sediment, $l$</td>
<td>50 mm</td>
</tr>
<tr>
<td>Receiving distance of hydrophone, $R_0$</td>
<td>280–530 mm</td>
</tr>
</tbody>
</table>
sediment [7,8]. Therefore, some corrections in their theoretical model might be required to predict the nonlinearity parameter of water-saturated sediment.

Zaitsev et al. [31] have estimated the nonlinearity parameter of water-saturated river sand from experimental results, using a wave equation for longitudinal strain in the nonlinear isotropic solid medium. The estimated nonlinearity parameter was $\beta = 74$. This value was similar to our estimated value for the water-saturated sandy sediment. However, since the processes to determine some input parameters in the model are somewhat ambiguous, the application of the model in other type sediments except the water-saturated sandy sediment may be difficult.

Belyaeva et al. [1] and Ostrovsky et al. [4] have investigated the mechanism of strong nonlinearity in porous granular media. They considered the porous granular media as media with soft and hard phases. The soft phases occupy small volumes in the media and show strong deformations for acoustic waves with finite pressure amplitude, whereas the hard phases show much less deformations. These strong deformations in porous granular media give rise to strong nonlinear acoustic responses. Generally, the contacts among individual grains in porous granular media are much softer than the grains. Therefore, the concentration of stress due to the external pressure at the contact boundaries of individual grains may cause strong nonlinearity as a result of the deformations in the media. The deformations at the contact boundaries of individual grains can ultimately be considered as the deformations of pores in porous granular media, because the contact boundary surfaces among individual grains can become a part of the surfaces of pores in the media.

Since the primary waves used in this study can become finite-amplitude waves, they can cause a deformation at the contact boundaries among individual sediment grains. Then, the nonlinear acoustic waves can be generated in sediment. Zaitsev et al. [31] mentioned that their estimated nonlinearity parameter for water-saturated sand is related with the contacts among individual sand grains. Therefore, the estimated nonlinearity parameter in this study seems similar to that estimated by Zaitsev et al. [31]. In this view point, the difference frequency wave can be usefully used to estimate the nonlinearity parameters of fluid-like granular media.

5. Conclusions

The difference frequency acoustic wave was generated as a result of the nonlinear interaction of two primary acoustic waves with finite-amplitudes at the frequencies of 76 and 114 kHz in water-saturated sandy sediment. This nonlinear interaction could be occurred at the contacts among individual sand grains. The acoustic pressure levels of the difference frequency waves were measured as a function of receiving distance of the hydrophone. The nonlinearity parameter of water-saturated sandy sediment at the difference frequency of 38 kHz was estimated with a parametric acoustic array theory. The mean value of the estimated nonlinearity parameter was $\beta = 80.5 \pm 5.1$. It was reconfirmed at the primary frequencies of 137 and 204 kHz. The reconfirmed nonlinearity parameter was $\beta = 88.6 \pm 8.7$. It agrees very well with that estimated at the difference frequency of 38 kHz.

The nonlinearity parameter estimated in this study can be used as background information for the nonlinear acoustic investigation of bottom and sub-bottom profiling in the ocean sandy sediments. Because of its lower attenuation the difference frequency acoustic wave can be usefully applied to estimate the nonlinearity parameter of ocean sediment in the ocean as well as under laboratory conditions. The difference frequency acoustic wave method can be also employed as a good method to estimate the nonlinearity parameters of fluid-like granular media. Our future work will be extended to the different type water-saturated sediments.

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