Modelling shallow water wave generation and transformation in an intertidal estuary

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Abstract

An experiment is described in which wave growth was measured in Manukau Harbour, a New Zealand estuary with relatively large fetches and extensive intertidal flats. Wave spectra were obtained from pressure sensors and current meters placed at six sites across the estuary. The SWAN third-generation spectral model was then used to simulate wave transformation during a part of the study period during which consistent south-westerly winds blew along the instrument transect. The simulations incorporated refraction by currents using output from a circulation model of the estuary. Measured wave variance spectra were compared with the model results, and the contributions of the various processes represented by source terms within the model were compared. It was found that, along with whitecapping, bed friction and exponential growth from wind input, four-wave nonlinear interactions played a dominant role. Some limitations were noted in the discrete interaction approximation which the SWAN model uses to compute the four-wave nonlinear interaction term. © 1999 Elsevier Science B.V. All rights reserved.

Keywords: Wave model; Shallow water; Wave spectrum; Nonlinear interaction

1. Introduction

Further progress in modelling the transformation of wind-generated waves in shallow water is dependent on the collection of a suitable body of field data, and detailed testing of models against that data. The TMA study (Bouws et al., 1985) of the Texel, MARSEN and ARSLOE datasets has provided an equivalent for shallow coastal waters

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of the JONSWAP (Hasselmann et al., 1973) experiments which provided much of the foundation for deep-water models. More recently Young and Verhagen (1996a) have made a major contribution to our understanding of fetch-limited wave growth with their studies in the relatively controlled shallow water environment of Lake George, Australia. By using measurements at multiple sites they were able to study the evolution of wave spectra with fetch into depth-limited conditions (Young and Verhagen, 1996b). Both the TMA and Lake George studies supported the idea due to Kitaigorodskii (1983) of a spectrum with a high-frequency tail scaling as (wavenumber) \(^{-3}\). Young et al. (1996) were further able to identify enhanced directional spreading in shallow water, associated with transfer of energy by nonlinear interactions.

To go beyond the description of wave conditions by parametric spectral forms we need to isolate the dynamical processes that shape these spectra. To this end, numerical models provide a framework within which we can relate our understanding of physical processes to measurements made under often complex environmental conditions. At present, wave forecasting at the oceanic and continental shelf scale is commonly carried out with the third-generation WAM wave model (WAMDIG, 1988), which is operationally available at a large number of institutes around the world. While WAM has been adapted for shallow-water effects, its use of an explicit numerical scheme for propagation means that for the high spatial resolution needed in coastal applications, the stability requirements demand an impractically short time step. There is also the need to simulate more complex phenomena such as depth-limited wave breaking and interactions with the changing currents and tides found in coastal and estuarine environments. Models are now being developed specifically for the coastal zone, including the third generation SWAN model (Holthuijsen et al., 1993; Ris et al., 1994). Some testing of the SWAN model has been carried out against the Lake George data set, as well as in coastal applications such as the Friesche Zeegat, where the North Sea barrier islands provide more complex intertidal bathymetry, and currents are present (Booij et al., 1996; Holthuijsen et al., 1997).

These simulations have so far concentrated for the most part on predictions of integral parameters such as significant height and mean period. A more detailed examination of the evolution of wave spectra in such a simulation can offer further insight into the individual processes taking part.

This paper presents some results of a modelling study using data from a wave evolution experiment conducted in an extensive intertidal estuary with non-uniform currents and strong tidal currents. In the following sections, both the SWAN model and the field study are summarised, then comparisons of measured and predicted energy spectra are made. The contributions of the various processes of spectral growth, transformation and dissipation are examined.

Particular attention is given to the transfer of energy through nonlinear interactions. Of the various physical processes associated with the growth, transformation and dissipation of the wave spectrum, the four-wave interaction has the distinction of providing considerable debate and difficulty in spite of having a well-defined functional form (Herterich and Hasselmann, 1980) free of empirical parameters. Unfortunately the computational complexity of its formulation has obliged the developers of working spectral models such as WAM and SWAN to employ major simplifications through the
discrete interaction approximation of Hasselmann et al. (1985). This approximation was calibrated to provide satisfactory results for a class of directional spectra of the JONSWAP form in deep water, but its validity for a wide range of conditions has had limited testing. One of the aims of the present work is to examine the accuracy with which it represents energy transfer through the fetch-limited growth of wave energy in a complex shallow-water environment.

2. Field programme

Manukau Harbour is a large (368 km$^2$) semi-enclosed estuary opening to the west coast of the North Island of New Zealand (Fig. 1). A mean tidal range of 2.0 m (neap) and 3.4 m (spring) results in the drying of extensive intertidal banks, covering 40% of the estuary at low spring tide, and results in strong tidal currents in several major channels draining the estuary (Bell et al., 1998). The maximum (north–south) fetch is 26 km, but the predominant winds are from the south-west and north-east, blowing across fetches of up to 18 km.

For six weeks in December 1996–January 1997, wave records were obtained from pressure sensors and current meters deployed on bottom-mounted frames at six sites along a transect across the southern part of the estuary (Neilson, 1998). The transect was aligned at 53°T, to match the prevailing south-westerly winds. At three of the sites (TR3, TR4, TR6 in Fig. 1), locally-developed ‘Dobie’ wave recorders were used, mounted at elevations of 0.83 m, 1.31 m, 0.21 m from the bed respectively. These contain a pressure transducer accurate to ±0.3 psi logged by a microprocessor, each of which was programmed to record at 4 Hz for 512 s bursts every half-hour, computing and storing wave spectra internally. At the remaining sites (TR1, TR2, TR5), InterOcean S4 current meters were mounted at 1.25 m, 2 m and 2 m elevations respectively. These recorded both pressure and velocity at 2 Hz for 9 min bursts every 30 min (TR2) or 60 min (TR1, TR5).

A bathymetric survey of the measurement transect and surrounds was conducted after the wave study. Depths of 3–5 m at high-water were typical over most of the instrumented transect, but this was crossed in places by channels of 10–15 m depth (Fig. 2a).

Winds over water were recorded at two locations at 10 m elevation: at a platform erected offshore from Wiroa Island near the north-east end of the transect and, for a short period, in mid-estuary North of the transect (Fig. 1). Averaged velocities were recorded every 10 min. Data were also available from the permanent station at the nearby Auckland Airport and from a temporary anemometer on Matakapu Pt (Fig. 1). The latter site was affected by sheltering from nearby trees and hills forming Awhitu Peninsula, consistently reducing wind speeds there to approximately 50% of speeds measured over water, although directions were consistent with those observed elsewhere (Fig. 3).

The short wave periods (0–4 s) observed in the estuary present some problems in wave measurement with pressure gauges. Time series data from the wave recorders were used to compute variance spectra by Fast Fourier Transform. Assuming linear wave
Fig. 1. Location map of Manukau Harbour, showing bathymetry contours (bottom panel) relative to water level at the wave hindcast time (16:00 NZST on 15 December 1996). The top left panel shows the domains for wave (——) and hydrodynamic (---) modelling. Wave measurements were made at sites TR1 to TR6. Wind data were recorded at the following locations: Wiroa platform (W), Central Harbour (C), Matakau Pt (M) and Airport (A).
Fig. 2. (a) Bathymetry profile along the instrument transect. Depths are shown relative to the water level at the wave model hindcast time (16:00 NZST on 15 December 1996), which was 3.34 m above chart datum (dotted line). Instrument elevations are shown at sites TR1 to TR6. (b) Significant height along the instrument transect hindcast assuming homogeneous winds (solid line), and spatially-varying winds (dotted line).

theory, the variance spectrum of a pressure signal at a distance $z$ below the surface in water of depth $d$ is related to the surface variance spectrum $F(f)$ by

$$F_p(f, z) = \left[ \frac{\rho g \cosh k(d-z)}{\cosh kd} \right]^2 F(f)$$

(1)

where $f$ is frequency, $k$ is wavenumber, $\rho$ is the density of water and $g$ ($= 9.81 \text{ m s}^{-2}$) is gravitational acceleration. The attenuated signal becomes indistinguishable from instrument noise and random turbulence at high frequencies, and it is usual to specify a cut-off frequency, above which the spectrum is removed in computing integral wave parameters from pressure spectra transformed to the surface. The evaluation of integral surface wave parameters from spectral moments will depend on the treatment of this cut-off.
Fig. 3. Winds observed around the hindcast period at the platform near Wiroa Island at the northeastern end of the measurement transect and at Auckland Airport (+), Central Harbour (×) and Matakawau Pt (○). Observations are at 10 m elevation.

For the purposes of comparing measured and modelled spectra, however, the opposite approach can be taken. Rather than amplify a possibly noisy pressure spectrum to the surface, the transfer function (1) was used to attenuate the well-defined model spectra down to instrument level. Data from pressure sensors typically have an approximately ‘white’ noise spectrum, the level of which can be reasonably easily estimated by eye (in Fig. 6, for example). This is more difficult in a spectrum transformed to the surface.

All six wave recording instruments were functioning during the second week of the deployment. In that week, a 36-h period was noted of consistent 5–10 m s⁻¹ south-westerly winds, close to the transect alignment (Fig. 3). During that time, wave heights in the range 20–60 cm were observed at the downwind sites, with periods varying in the range 1.5–2.5 s, positively correlated with water depth (Neilson, 1998).

3. The SWAN model

Spectral wave models describe the evolution in position (x, y) and time (t) of the spectral density $F(\omega, \theta, x, y, t)$ of surface wave energy at frequency $\omega$ and direction $\theta$. The SWAN model is based on the radiative transport equation

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x} \left( c_x N \right) + \frac{\partial}{\partial y} \left( c_y N \right) + \frac{\partial}{\partial \omega} \left( c_\omega N \right) + \frac{\partial}{\partial \theta} \left( c_\theta N \right) = \frac{S}{\sigma}$$

(2)

for the wave action $N$, related to the spectral density $F$ by

$$N(\omega, \theta, x, y, t) = \frac{F(\omega, \theta, x, y, t)}{\sigma}$$

(3)
where \( \sigma \) is the ‘intrinsic’ frequency, in a reference frame co-moving with the current. The effects of refraction by currents and bottom variation are incorporated through the co-moving derivative terms defined by \( c_s = D x / D t \), and similarly for the other variables. \( S \) represents the sum of separate source terms. The model accommodates the processes of wind generation, white-capping, bottom friction, quadruplet wave–wave interactions, triad wave–wave interactions and depth-induced breaking (Ris et al., 1994).

Wave growth by wind is described by a combination of linear and exponential terms:

\[
S_{in}(\sigma, \theta) = \Lambda + BF(\sigma, \theta)
\]  

The exponential growth term is based on the linear expression of Snyder et al. (1981), modified by Komen et al. (1984) to scale by friction velocity \( U_* \) instead of a wind speed at defined elevation. The linear growth term \( \Lambda \) as described by Cavaleri and Malanotte-Rizzoli (1981) is also included to initiate wave action from a zero-energy state.

The whitecapping term is derived from the model of Hasselmann (1974) which considers whitecaps as randomly distributed pressure pulses. The dissipation coefficient was assigned the value \( C_{\text{sc}} = 2.36 \times 10^{-5} \) (Komen et al., 1984). The process of the wave energy dissipation at the seabed may be based on the empirical JONSWAP form (Hasselmann et al., 1973), with a friction coefficient \( C_{\text{bottom}} = 0.038 \text{ m}^2\text{s}^{-3} \).

The shape and evolution of a wind wave spectrum are largely controlled by nonlinear interactions which transfer energy between frequency ranges. In deep water, four-wave interactions provide the lowest order contribution to the transfer rate, for which Hasselmann (1962) derived an expression based on a perturbation analysis. Computation of this term (discussed in more detail in Section 5) has to date proven numerically prohibitive for typical modelling applications, so a discrete interaction approximation (Hasselmann et al., 1985) for the four-wave interaction is applied within the SWAN model.

In very shallow water, triad wave–wave interactions can also become near-resonant, and can transfer energy from lower frequencies to higher frequencies. A parameterisation of this effect is included in SWAN using the lumped triad approximation (Eldeberky and Battjes, 1995; Eldeberky, 1996). The triad term only becomes significant for depths which are small relative to wave height and wave length. For typical wave heights and frequencies observed in the experiment, this is only likely to occur occasionally on steeper shoals. Similarly depth-induced breaking, which is treated by the Eldeberky and Battjes (1996) spectral formulation for random waves based on the bore model of Battjes and Janssen (1978), will have limited influence.

The numerical schemes in SWAN (Ris et al., 1994) are developed from those used in the second-generation HISWA model (Holthuijsen et al., 1989). In the present application, a rectilinear spatial grid is used, on which propagation is handled by implicit upwind schemes in geographic space, supplemented by a second-order central approximation in spectral space. This implicit scheme is quite diffusive (Ris et al., 1994, 1996) so would produce progressively greater error over large propagation distance. Even for WAM’s first order upwind explicit scheme, intended for such applications, this can result in excessive dissipation of swell (Bender, 1996). But for the coastal applications for which SWAN is intended this is of less consequence than the advantage of an...
unconditionally stable computation, regardless of the space and time intervals. The discretised spectrum is partitioned into four quadrants, and a sequence of four forward marching sweeps is used to propagate the waves of each quadrant with the upwind scheme. This cycle is carried out iteratively for each time step, to allow boundary conditions to be matched between quadrants. This is also the case for a stationary computation, as in the present study, in which time dependence is removed and a single equilibrium solution obtained.

4. SWAN simulation

A simulation was established on a $213 \times 129$ cell grid covering the southern and central parts of Manukau Harbour with 100 m resolution. The grid is oriented at 24$^\circ$ to true North (Fig. 1). Bathymetry data was gridded over the entire harbour at 100 m resolution by linear kriging, using data digitised from hydrographic charts supplemented by the survey conducted along with the field programme. Spectra were computed at 12 equally spaced propagation directions and 25 logarithmically spaced frequencies, between $f_1 = 0.12$ Hz and $f_{25} = 1.18$ Hz, covering typical mean frequencies for this enclosed estuary. It is assumed that the spectrum contains no energy below $f_1$, but a diagnostic $f^{-5}$ tail is applied above the high frequency cut-off.

As the model was to be run in stationary mode, a hindcast time of 16:00 NZST on 15 December was selected at which near-equilibrium wave conditions could be expected. This was a period of consistent wind, staying within a 1.5 m s$^{-1}$ range of speeds and a 15$^\circ$ band of directions for the preceding 4 h at each of the anemometer sites (Fig. 3). With little spatial variation evident in winds observed over water, the wind observed at the Wiroa platform was applied throughout the model domain. At the hindcast time this was 7.13 m s$^{-1}$ from 215$^\circ$T, 18$^\circ$ from the alignment of the instrument transect. Under these conditions, there was no need to model the northern part of the estuary as wave generation there will have no effect at the transect.

With a relatively large tidal range, large surface area and moderate depths, Manukau Harbour is subject to strong tidal currents, with mean velocities of greater than 50 cm s$^{-1}$ being recorded at the current meter sites. Currents can reach 80–100 cm s$^{-1}$ in the channels (Bell et al., 1998). These are sufficient to produce plainly visible refraction effects, especially at the margins of the major channels. To incorporate currents in the wave model, the hydrodynamic model 3DD (Black, 1983, Black et al., 1993) was employed. This provides for 2 and 3-dimensional modelling of homogeneous and stratified water bodies by solving the vertically-averaged momentum and continuity equations using an explicit leap-frog finite difference scheme, while Lagrangian or Eulerian transport solutions are applied in stratified environments.

The vertical salinity structure of Manukau Harbour can be classed as well-mixed. Vant and Williams (1992) measured profiles in depths ranging from 5–35 m and found that the top–bottom salinity variation averaged 0.2 ppt. Hence a 2-dimensional hydrodynamic simulation was used. This was established on a 200 m grid spanning the whole estuary and forced at the harbour entrance by a sea-level boundary derived by lagged regression from tidal records at Onehunga (Fig. 1), and by winds measured at the Wiroa platform. Hindcast depth-averaged currents (Fig. 4) were calibrated against velocities...
Fig. 4. Measured currents (+) at sites TR2 (top panels) and TR5 (bottom panels) in Manukau Harbour, compared with those hindcast by the hydrodynamic model 3DD (solid lines). The wave hindcast time (15/12/96 18:00 NZST = day 349.666) is marked by the solid vertical lines. The current meter at TR2 emerged from the water at low tide, resulting in the anomalies in the velocity data at 12-hourly intervals from day 348.8.
measured at TR2 and TR5 (the instrument frame at TR1 was damaged shortly after the wave hindcast period).

The wave hindcast time was close to peak ebb (Figs. 4 and 5). Current flows at this time were predominately transverse to the wind in the southern part of the estuary, but with a notable following component in and around the Waiuku Channel at the western (upwind) end of the fetch. Conversely, currents near the eastern Papakura channel had a component opposing the wind and wave direction.

The SWAN model was run in stationary mode to identify an equilibrium state under these conditions. The model predicted wave heights generally increasing downwind over deeper sections of the estuary, but the influence of bed friction becomes evident in the distribution of hindcast wave heights over the banks (Fig. 2b, solid line). This can be seen for example where the trend for wave height to increase with fetch is reversed in crossing the shallow bank extending through TR3 and the Central anemometer site (Fig. 1), and again on encountering the bank on which TR4 was placed. Maximum predicted heights were below 50 cm, while mean periods were mostly 1–2.5 s, and predicted wave directions were aligned with the wind over most of the estuary.

Measured and hindcast spectra at each of the 6 transect sites are shown in Fig. 6. By site TR3 (7 km fetch) a clearly developed spectrum has evolved, with down-fetch wave heights in the range 30–40 cm, and peak frequencies around 0.4 Hz. A low energy peak at 5-s period is also seen at TR4 and TR5, which is likely to be associated with remnant waves propagating up the main channel from the entrance (Neilson, 1998), rather than

![Diagram](image)

Fig. 5. Currents in Manukau Harbour at 16:00 NZST on 15 Dec. 1996, hindcast by the hydrodynamic model 3DD. Arrows are plotted for every second cell of the 200 m 3DD grid, within the region used for the wave model. The instrument transect and 5 m and 20 m depth contours are also marked.
Fig. 6. 1-dimensional wave spectra at 16:00 NZST on 15 Dec. 1996 at sites TR1–3 (left panels) and TR4–6 (right panels). The spectra hindcast by SWAN with homogeneous wind (run 9S) are shown at the surface (dotted lines) and transformed to instrument depth (solid lines) for comparison with measured spectra (+). Dashed lines show 1-dimensional spectra at instrument depth hindcast with spatially variable wind (run 11S).

the local generation processes included in the model. At lower frequencies, spectral densities at all sites drop below the instrument noise floor. Although the coast outside Manukau Harbour is subject to energetic swell conditions, the geometry of the estuary allows negligible swell energy to penetrate to its southern parts.

At the upwind site (TR1) the predicted spectrum is still entirely below the noise level of the wave recorder. Further downwind at TR2 the model is overestimating wave
growth. Hills of up to 140 m elevation forming Awhitu Peninsula (Fig. 1) provide some wind sheltering on the western side of Manukau Harbour, so the assumption of uniform wind speed equal to that measured down-fetch is likely to be an overestimate there. To simulate this shielding effect, a second hindcast was carried out with spatially variable winds, interpolated from measurements at the Matakawau Pt, Central and Wiroa anemometers (Table 1). A piecewise linear vector interpolation was applied in the $x$ direction, assuming no variation in the $y$ direction. This simple prescription will underestimate wind speed in the western part of the estuary by giving excess weight to the locally-shielded land site. A more realistic distribution may well be provided by applying a sheltering formulation such as that of Taylor and Lee (1984). However for present purposes it was considered more useful to provide scenarios which serve as approximate upper and lower bounds on the actual wind field. Indeed the two simulations were found to bracket the observed spectra on the growing forward face (Fig. 6) where, as is so often the case, an accurate description of the wind field is the limiting factor in the performance of the wave model. Further downwind, the model estimates below the spectral peak converge at the eastern end as the reliability of the wind field prescription improves.

This trend for the hindcast spectrum to move closer to the data with increasing fetch is not seen in the frequency range above the peak. By TR3 the equilibrium range has become established, and at this stage it is well described by the hindcast. Further downwind, the energy in this band becomes overpredicted. This part of the hindcast is not directly sensitive to the uncertainty in wind input, so the role of other processes needs to be considered in order to explain this discrepancy.

To investigate the relative importance of the processes, the various SWAN model source terms were computed at the transect sites, based on directional spectra predicted in the homogeneous wind simulation. Fig. 7 shows these in directionally-integrated form.

Wave energy was sufficiently developed for the linear wave growth component $A$ of the wind input source term (4) to be negligible in comparison with the exponential growth term. With significant wave heights below 0.5 m and periods below 2.4 s (Fig. 2b, Table 2), triad interactions were of negligible magnitude, and it was also found that depth induced wave breaking was negligible at all sites. This is to be expected for a semi-enclosed waterway dominated by locally-generated windsea, and had previously been noted by Booij et al. (1996) in SWAN simulations of Lake George data (Young and Verhagen, 1996a) with and without the inclusion of this term.

<table>
<thead>
<tr>
<th>Sites (west to east)</th>
<th>Speed (m s$^{-1}$)</th>
<th>Direction (°T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matakawau Pt</td>
<td>3.47</td>
<td>220</td>
</tr>
<tr>
<td>Central</td>
<td>6.15</td>
<td>224</td>
</tr>
<tr>
<td>Wiroa</td>
<td>7.13</td>
<td>215</td>
</tr>
<tr>
<td>Airport</td>
<td>7.20</td>
<td>230</td>
</tr>
</tbody>
</table>
Fig. 7. Integrated action density \( N(\alpha, \theta) d\theta \) and source terms contributing to \( S/\sigma \) (Eq. (2)), as hindcast by the SWAN model with homogeneous winds for 16:00 NZST on 15 Dec. 1996 at sites TR1–3 (left panels) and TR4–6 (right panels). Terms shown are: exponential wind input (---), whitecapping (- - -), four-wave nonlinear interactions (- - -) and bottom friction (- -). Other terms computed (triad nonlinear interactions, depth-induced breaking and linear wind input) were found to be negligible at the scale plotted.
Table 2
Water depths and hindcast wave parameters at the instrument sites at 16:00 NZST, 15/12/96, assuming homogeneous wind. $R$ is the shallow water scaling factor (Eq. (8)) used in the discrete interaction approximation for four wave nonlinear interactions

<table>
<thead>
<tr>
<th>Site</th>
<th>Depth (m)</th>
<th>$H_{1/3}$ (m)</th>
<th>$T_{\text{mean}}$</th>
<th>$f_{\text{peak}}$ (Hz)</th>
<th>$k_d d$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TR1</td>
<td>3.73</td>
<td>0.23</td>
<td>1.38</td>
<td>0.61</td>
<td>5.54</td>
<td>0.996</td>
</tr>
<tr>
<td>TR2</td>
<td>4.14</td>
<td>0.32</td>
<td>1.65</td>
<td>0.50</td>
<td>4.19</td>
<td>0.983</td>
</tr>
<tr>
<td>TR3</td>
<td>2.41</td>
<td>0.31</td>
<td>1.81</td>
<td>0.41</td>
<td>1.76</td>
<td>0.838</td>
</tr>
<tr>
<td>TR4</td>
<td>2.88</td>
<td>0.34</td>
<td>1.97</td>
<td>0.38</td>
<td>1.75</td>
<td>0.838</td>
</tr>
<tr>
<td>TR5</td>
<td>5.27</td>
<td>0.39</td>
<td>2.15</td>
<td>0.34</td>
<td>2.52</td>
<td>0.897</td>
</tr>
<tr>
<td>TR6</td>
<td>2.34</td>
<td>0.37</td>
<td>2.11</td>
<td>0.38</td>
<td>1.49</td>
<td>0.862</td>
</tr>
</tbody>
</table>

Over the mid-estuary shoals between TR3 and TR4, bed friction becomes significant in limiting growth in the forward face of the spectrum, then in absorbing wave energy near TR6 at the downwind edge of the estuary. Over most of the fetch, however, whitecapping is the dominant dissipation mechanism. This term has similar shape but opposite sign to the wind growth term, in this case balancing approximately 50% of the latter. To first order then, the shape of the predicted wave spectrum may be expected to have a similar sensitivity to any errors in the scale of the whitecapping term as it has to uncertainty in wind input. As noted above, this is relatively small in the equilibrium range.

The remaining contribution of significant magnitude comes from four-wave nonlinear interactions. At all sites, this term is computed to provide a positive input at and below the spectral peak, of magnitude similar to or greater than that provided by wind input. It is also predicted to have a negative lobe of equivalent magnitude removing energy from the spectral tail. Because of the important role quadratic interactions of this size can play in reshaping the spectrum, further attention is given to them in Section 5.

Source terms were also computed for the simulation with spatially-variable winds (Fig. 8). The slower initial growth resulted in waves arriving at the mid-estuary banks with less energy in the lower frequency range than for the homogeneous wind simulation. As a result, bed friction does not play such a role in reducing wave energy over the shoals, and the variation of wave height with fetch (Fig. 2b) is closer to the monotonic increase expected for a uniform depth simulation. Where the spectrum is well-developed, the relative magnitudes of the major processes are similar to those obtained in the homogeneous case.

5. Four-wave interaction source terms

Hasselmann (1962) derived the transfer rate to and from a spectral component arising from interactions with sets of three other spectral components. The resulting source term takes the form of an integral over the phase space of interacting quadruplets:

$$S_{ijkl}(k_4) = \int G(k_1, k_2, k_3, k_4) \delta(k_1 + k_2 - k_3 - k_4) \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4)$$

$$\times \left[ N_1 N_2 (N_3 + N_4) - N_3 N_4 (N_1 + N_2) \right] dk_1 dk_2 dk_3$$

(5)
Fig. 8. Integrated action density $|N(\alpha, \theta)|d\theta$ and source terms contributing to $S/\sigma$ (Eq. 2), as hindcast by the SWAN model with spatially variable winds for 16:00 NZST on 15 Dec. 1996 at sites TR1–3 (left panels) and TR4–6 (right panels). Terms shown are: exponential wind input (-----), whitecapping (- - - -), four-wave nonlinear interactions (---) and bottom friction (○ ○). Other terms computed (triad nonlinear interactions, depth-induced breaking and linear wind input) were found to be negligible at the scale plotted.
where \( N_i = N(k_i, x, t) \) is the action density for the \( i \)th wavenumber vector \( k_i \) at position \( x = (x, y) \) and time \( t \), and \( G(k_1, k_2, k_3, k_4) \) is the interaction coefficient derived by Hasselmann (1962). The Dirac delta functions \( \delta \) in Eq. (5) select the resonance conditions, which are associated with conservation of energy and momentum in the interaction (Hasselmann, 1963).

To reduce the computational demands involved in implementing (5) in SWAN, as in WAM, a discrete interaction approximation (Hasselmann et al., 1985) is applied, in which only two quadruplets are summed. These both have the same set of frequencies:

\[
\begin{align*}
\sigma_1 &= 1.25 \sigma \\
\sigma_2 &= 0.75 \sigma \\
\sigma_3 &= \sigma_4 = \sigma
\end{align*}
\]

In both quadruplets, the third and fourth wave vectors coincide, while the others lie at angles \( \theta_1 = -11.5^\circ \) and \( \theta_2 = 33.6^\circ \) to this common direction, for the first quadruplet, or at \( \theta_1 = 11.5^\circ \) and \( \theta_2 = -33.6^\circ \) for the second. The source term is evaluated for infinite depth \( (S_{a3,4}) \), then adjusted for finite depth by an empirical scaling (Herterich and Hasselmann, 1980; Hasselmann and Hasselmann, 1981)

\[
S_{a34} = R(k_p d) S_{a14.4}
\]

\[
R(k_p d) = \begin{cases} 
1 + \frac{5.5}{k_p d} \left( 1 - \frac{5k_p d}{6} \right) \exp\left( -1.25k_p d \right), & k_p d \geq 0.5 \\
\exp(-4.43), & k_p d < 0.5
\end{cases}
\]

where \( k_p \) represents the peak wavenumber, taken as \( k_p = 0.75 \bar{k} \) (Komen et al., 1984).

The full four-wave interaction term (5) can, in principle, be integrated to arbitrary accuracy without recourse to such an approximation. A procedure to do so has been presented by Hasselmann and Hasselmann (1985) in the form of the EXACT-NL model. This uses principles of detailed balance, rotational symmetry and exchange symmetry to replace the six-dimensional integral (5) with a five-dimensional summation. An efficient piecewise-linear discretisation is chosen to provide maximal resolution near the centre \( (k_1 = k_2 = k_3 = k_4) \) of the interaction region. This allows interactions to be computed with a numerical cost acceptable for single point applications, though still prohibitive for application in an extended spatial domain.

The four-wave interaction source terms were calculated with EXACT-NL based on the directional spectra at the transect sites obtained in the SWAN hindcast with homogeneous winds (Fig. 9). These show some marked differences from terms calculated within SWAN from the same spectra using the discrete interaction approximation (DIA, Eq. (6)). The main positive and negative lobes are generally in similar positions, but with variations in magnitude. Also a secondary positive lobe at high frequency is more pronounced, providing for a greater transfer to high frequencies than is accounted for by the DIA: this energy will subsequently be dissipated rather than contributing to growth near the peak. At the two windward sites TR1 and TR2, EXACT-NL provides a much stronger interaction than the DIA. The effect of this on spectral shape is difficult
Fig. 9. Nonlinear four-wave interaction source terms $S_{int}$ (integrated over direction) computed by the SWAN model using the Discrete Interaction Approximation (---) and by the EXACTNL model (○ ○ ○), based on SWAN hindcast directional spectra at sites TR1–3 (left panels) and TR4–6 (right panels), assuming homogeneous winds. Transfer rates computed by EXACTNL for the same spectra but assuming a depth of 30 m are also shown ( - - ).

to assess, however, given the low energy of the measured spectra at depth, and the uncertainty in wind input on the western side of the estuary.
The nearest agreement between the EXACT-NL and DIA source terms is found at TR3, which also shows the closest match in the high frequency tail of modelled and measured spectra. Further downwind at TR4, TR5 and, to a lesser extent, TR6, the DIA begins to overpredict the transfer of energy into the frequency band from the spectral peak to 0.5 Hz, corresponding to the overpredicted spectral tails.

It should be pointed out that the discrepancy between the DIA and EXACTNL transfer rates does not appear to arise directly from the treatment of depth-dependent effects, as the largest differences occur at sites TR1 and TR2 where \( k_d d > 3 \) of wave conditions (Table 2). To observe the effect of finite depth, transfer rates were recomputed by EXACTNL for the same spectra, but with the true depth replaced by a nominal value of 30 m (Fig. 9). At the sites TR3–TR6 where \( k_d d \) is in the ‘intermediate’ depth range \( 0.7 < k_d d < 3 \), the magnitude of the transfer rate in the main positive and negative lobes is found to be slightly larger when computed with the true depth than for the deep water values. This will act to slightly offset the loss of low frequency energy in shallow water due to bottom friction by enhanced transfer from higher frequencies. However, even at these ‘intermediate’ depth sites the effect is minimal.

As noted above, there is a distinct lower frequency peak in the measured spectra at TR4 and TR5, which could not be reproduced in SWAN by local generation. It is possible that energy in this band may interact with higher frequency components through the four-wave interaction. This possibility was tested by re-running EXACT-NL with the TR5 hindcast spectrum augmented by energy around a peak at 0.2 Hz, 105°, approximating the observed spectrum. The additional contribution to the nonlinear interaction acts to transfer energy from the 0.3–0.5 Hz range to both the 0.2–0.3 Hz and 0.5–0.7 Hz bands, in approximately equal measure. It was found however that with such a low-energy second peak, the magnitude of this additional exchange was negligible, at only 0.2% of the exchange associated with the main peak.

6. Discussion

In spite of having an explicit functional form for the four-wave nonlinear interaction (Herterich and Hasselmann, 1980) that could in principle be determined with arbitrary accuracy, sheer computational limits have obliged the use of the discrete interaction approximation of Hasselmann et al. (1985) in operational wave models. It is somewhat disappointing then that this approximation, which was calibrated to provide satisfactory results for a class of directional spectra of the JONSWAP form, can introduce significant inaccuracy when applied to a more general class of spectra.

The challenge remains to find either faster methods of computing the exact nonlinear transfer rate, or more accurate approximations to it. Starting from the EXACT-NL algorithm of Hasselmann and Hasselmann (1985), further efficiencies have been identified by Thacker (1982) and by Snyder et al. (1993) who have employed a piecewise-constant discretisation and transferred much of the computation to a spectrum-independent pre-processing stage. This has been incorporated in a distributed wave transformation model (Snyder et al., 1998) which provides a factor of 10 gain in accuracy relative to the discrete interaction approximation at the expense of a factor 600 loss in speed.
The methods based on EXACT-NL use a five-dimensional discretisation of the Boltzmann integral (5). As the integrand of this expression is only non-zero on a three-dimensional surface it would seem plausible that progress may still be made by further tailoring the discretisation to this resonant surface. Alternatively, further attention could be given to extending the discrete interaction approximation by a systematic optimisation of the choice of quadruplets to be included.

For shallower water conditions \( k_d < 0.4 \) than the present study has addressed, the weak nonlinear approximation assumed by the four-wave interaction (5) becomes questionable (Hasselmann and Hasselmann, 1985). As well as needing to consider triad interactions, higher-order terms of the perturbation expansion may be needed (Hasselmann, 1962). Lin and Perrie (1997) have extended the analysis to five-wave interactions, and found that these dominate the four-wave term for conditions where \( k_d < 3.6 \) and the nonlinearity parameter \( \varepsilon = \frac{(k_d H_w)}{(3 + \tanh^2 k_d d)} / \frac{(4 \tanh^2 k_d d)}{4} > 0.3 \). In the present study, \( \varepsilon \) was generally well below this range, so we did not have an opportunity to test the role of higher-order terms.

The development of improved computation methods for the nonlinear term can be expected to foreshadow developments in the specification of other processes. Earlier generations of wave model were able to operate quite successfully with either a simply parameterised nonlinear term, or none at all (SWAMP, 1985), by virtue of the empirical nature of other terms. The whitecapping term in particular has been determined largely on the basis of closing the energy balance, and tuning its parameterisation to observations of fetch-limited wave growth. Hence there is a need to provide working models with an efficient, accurate nonlinear transfer algorithm so that calibration of less well-known terms will provide information more directly about the underlying dynamics.

Notwithstanding the potential for improved representation of individual processes within the SWAN model, it has been found capable of providing a good representation of wave transformation in the complex environment of a tidal estuary with strong currents and variable bathymetry. The model was able to simulate wave growth within the bounds of our knowledge of the input windfield, and offers a suitable platform for further investigation of coastal and estuarine wave processes.

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