NONCOHERENT DETECTION OF CONVOLUTIONALLY ENCODED CONTINUOUS PHASE MODULATION

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Abstract

Recently digital communication systems using Continuous Phase Modulation (CPM) and convolutionally encoded Continuous Phase Frequency Shift Keying (CPFSK) have been shown to be power efficient, when an optimum coherent detector is used. The aim of this paper is twofold. Firstly, the error probability at large signal to noise ratios of convolutionally encoded CPFSK with an optimum noncoherent detector on an additive white Gaussian noise channel is considered. Secondly, we propose a limited tree search algorithm for noncoherent detection of coded and uncoded CPM schemes. The error probability of this suboptimum noncoherent detector is evaluated by means of computer simulations on an additive white Gaussian noise channel. It is shown that the degradation in error performance compared to the performance of the coherent Viterbi detector is less than one dB with a relatively simple noncoherent detector for most of the considered schemes.

1 Introduction

Spectrally efficient digital modulation schemes with constant amplitude have recently gained increased attention. One reason for this is the limited available radio frequency spectrum. A constant amplitude is favourable if the amplifiers and repeaters are non-linear. It has been demonstrated in the literature that CPM schemes are among these schemes. In this class of schemes both power and bandwidth efficient schemes can be constructed, [1].

If the optimum performance is to be obtained with a coherent Viterbi detector, the exact phase of the transmitted carrier has to be found at the receiver. As the spectrum of the transmitted signal becomes narrower, the problem of finding this carrier phase increases. It is also of great importance to find simple detectors for the convolutionally encoded CPM schemes, since the Viterbi detector becomes very complex for the very bandwidth and power efficient schemes.

The optimum noncoherent detector for CPM schemes is given and analyzed in [2,3]. Similar work considering bounds on the error probability of CPFSK signals are reported in among others [4,5]. Here, we have included also convolutionally encoded CPM-schemes. The error performance of these schemes transmitted on an additive white Gaussian noise channel is calculated by means of the minimum squared normalized equivalent Euclidean distance.

Many simple noncoherent detectors based on differential and discriminator detectors are proposed and analyzed in [6] and references therein. The simplicity of these detectors is at the expense of a quite large degradation in error performance. A dynamic programming algorithm for joint data detection and carrier phase estimation of CPM signals is proposed in [7]. The algorithm differs from the Viterbi algorithm for coherent detection only in the metric calculation. It achieves almost equally good error performance as the Viterbi detector.

In this paper we propose a limited tree search algorithm (SA) for decoding of convolutionally encoded CPM schemes in a noncoherent manner. This decoding algorithm which is referred to as SAN(β) is based on the SA(β) algorithm used with coherent detection [8]. The transmitted signal considered in this paper is schematically shown in Fig. 1. For details on this signal, see [1,8].

2 Optimum noncoherent detection of convolutionally encoded CPM

The maximum likelihood (ML) sequence detector for noncoherent detection of CPM signals is given in [1,2,3]. Assuming the data symbol \( u_0 \) in the interval \( 0 \leq t \leq T_b \) is to be detected, the optimum observation interval of the received signal \( r(t) \) for making a decision about \( u_0 \) is the interval \( t \in [-N_1 T_b, N_2 T_b] \), where \( N_1 \geq 0 \) and \( N_2 \geq 1 \) [2]. The detector must find the sequence of data symbols \( \hat{u} \) which maximizes the likelihood function

\[
A[r(t)] = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi N_0}} \exp \left[ -\frac{1}{2N_0} \int_{-N_1 T_b}^{N_2 T_b} r(t) dt \right] dt \quad (1)
\]

and select \( \hat{u}_0 \) as an estimate of the transmitted data symbol. Here it is assumed that the estimated carrier phase of the received signal \( \tilde{\phi}_0 \) is randomly distributed in \([0, 2\pi]\). The noise is additive white Gaussian with one-sided power spectral density \( N_0 \). For details, see [2]. In [2,3] it is shown that this error probability for large \( E_b/N_0 \).
can be written in a way similar to the coherent case, namely

$$P_e \sim Q \left( \sqrt{\frac{d_{2,\text{min}}^2 E_b}{N_0}} \right) ; \text{ large } E_b/N_0 \quad (2)$$

where $d_{2,\text{min}}^2$ is the minimum squared normalized equivalent Euclidean distance in the signal set. The squared normalized equivalent Euclidean distance between the sequences $\mathbf{a}$ and $\mathbf{b}$ is given by

$$d_2^2(\mathbf{a}, \mathbf{b}) = R \left( N_r + N_z - \sqrt{c(\mathbf{a}, \mathbf{b}) + s(\mathbf{a}, \mathbf{b})} \right) \quad (3)$$

where $R$ is the overall rate, [8,9]

$$c(\mathbf{a}, \mathbf{b}) = \frac{1}{T} \int_{-N_r T}^{N_r T} \cos [\varphi(t, \mathbf{a}) - \varphi(t, \mathbf{b})] dt$$

$$s(\mathbf{a}, \mathbf{b}) = \frac{1}{T} \int_{-N_r T}^{N_r T} \sin [\varphi(t, \mathbf{a}) - \varphi(t, \mathbf{b})] dt \quad (4)$$

and $n_r(\mathbf{a})$ and $n_z(\mathbf{a})$ are both zero mean Gaussian random variables. We will refer to this distance many times in this paper and will mostly refer to it as equivalent Euclidean distance or equivalent distance. Similarly the squared normalized Euclidean distance is referred to as Euclidean distance. A tight upper bound, $d_{2,\text{min}}^2$, of $d_{2,\text{min}}^2$ can easily be found. For further details, see [2,3,9].

2.1 Numerical Results

The minimum equivalent Euclidean distance has been calculated for many rate 1/2 codes combined with both binary and 4-level CPFSK. All these results are found in [9]. In [2,3] it is shown that uncoded CPM schemes at large $E_b/N_0$ perform equally well with optimum noncoherent detection as with optimum coherent detection at least with a large observation interval. If this is the case also for coded schemes, the optimum codes with coherent detection will of course be optimum also with noncoherent detection. The emphasis in this paper is therefore on codes that are optimum with coherent detection [8].

With a binary mapping rule, it is well known that the (4,3) encoder is optimum for modulation indices less than or equal to 0.75 with coherent detection. The upper bounds on the equivalent Euclidean distance for various values of $N_1$ for this code are shown in Fig. 2. The upper bound on the Euclidean distance, $d_{2,B}^2$, is given as a reference. The minimum equivalent Euclidean distance for this case is shown in Fig. 3 for various observation lengths. It is seen that a quite long observation interval is needed for the noncoherent detector to perform almost equally well as the coherent detector.

3 The SAN(B)-algorithm for noncoherent detection of convolutionally encoded CPM

Many reduced state search algorithms for coherent detection have been proposed, for which the error performance is close to the performance of the optimum detector implemented by means of the Viterbi algorithm, see [10] and references therein. However, in the noncoherent case, using the Viterbi algorithm is not optimum since the likelihood function is non-linear. Thus, the tentative decisions made by the Viterbi algorithm at each state in the code trellis are not necessarily the optimum deci-

45.3.2.
sions. This problem is avoided if the decoding is done in the code tree where no tentative decisions have to be made. The problem is that the number of sequences grows exponentially with the length of the observation interval. Now we propose a limited search algorithm which works in a code tree.

In the derivation of the metric (object function) to be used by the detector, we will use the notation $x$ to denote the number of branches leaving each node in the code tree (or state in the trellis), for details, see [9]. Throughout this section, the transmitted sequence and the candidate sequence at the detector are denoted $y$ and $\tilde{y}$, respectively, with the corresponding coded sequences $\tilde{x}$ and $\tilde{\tilde{x}}$, respectively. In the following we assume that the reader is familiar with the state description, the code trellis and the code tree of a convolutionally encoded CPM scheme. The interested reader is referred to [1], for details on these matters.

The received signal

$$r(t) = s(t, x, y) + n(t) \quad (5)$$

where $n(t)$ is a white Gaussian process with one-sided power spectral density $N_0$ and $\varphi_0$ is an unknown randomly distributed carrier phase, is observed by the detector. Now assume that a decision on the transmitted information symbol $u_m$ is to be made, using an observation interval $-\infty < t \leq (n + \tau_2)T$. The likelihood function for the optimum sequence detector over this observation interval is given in Eq. (1) simply by exchanging $N_1T_1$ with $-\infty$ and $N_2T_2$ with $(n + \tau_2)T$. Now, we define the complex correlation $\xi_n(\tilde{x})$ of the code tree of the received signal $\tilde{r}(t)$ and the complex envelope of an estimated candidate sequence up to time $(n + \tau_2)T$ as

$$\xi_n(\tilde{x}) = \int_{-\infty}^{(n + \tau_2)T} \tilde{r}(t) \exp[-j\varphi(t, \tilde{x})] dt \quad (6)$$

and the increment of the complex correlation as

$$\xi^m_n(\tilde{x}) = \int_{(n + \tau_2 - 1)T}^{(n + \tau_2)T} \tilde{r}(t) \exp[-j\varphi(t, \tilde{x})] dt \quad (7)$$

Now, it can easily be shown that maximizing Eq. (1) is equivalent to maximizing the absolute value of the complex correlation $\xi_n(\tilde{x})$ up to time $(n + \tau_2)T$, given by

$$|m_n(\tilde{x})| = |\xi_n(\tilde{x})| = |\xi_{n-1}(\tilde{x}) + \xi^m_n(\tilde{x})| \quad (8)$$

where $| \cdot |$ denotes the absolute value of the complex argument. From now on we will refer to $m_n(\tilde{x})$ as the object function.

It is easily seen that $\xi^m_n(\tilde{x})$ is obtained by feeding the complex envelope of the received signal into a complex filter matched to $\exp[-j\varphi(t, \tilde{x})]$ [1]. Thus, all the object functions are obtained from a filter bank with $M^k$ complex filters in a way similar to the object functions of a coherent Viterbi detector [1].

Now the SAN(B) algorithm will be proposed for an efficient noncoherent detection of CPM schemes. The name SAN(B) is an abbreviation of Search Algorithm For Noncoherent detection using B candidate sequences. This algorithm is a generalized form of the algorithm SA(B) used for coherent detection of CPM schemes in [10].

Let us assume that a decision on $u_{m-1}$ has been done and that we have a list of $B$ candidate sequences $\tilde{y}$, that survived from the previous step of the algorithm. Furthermore, assume that the corresponding object functions and complex correlations are $m_{n-1}(\tilde{y})$ and $\xi_{n-1}(\tilde{y})$, respectively. Now, the algorithm works in the following way to estimate $u_m$.

1. All $B$ candidate sequences are extended one decoding interval into the code tree. This leads to a total of $KB$ sequences. The corresponding object functions $m_n(\tilde{y})$ are calculated by calculating $\xi^m_{n}(\tilde{y})$, and adding to it the corresponding stored functions $\xi_{n-1}(\tilde{y})$ thereby obtaining $\xi_n(\tilde{y})$. This leads to $KB$ different object functions.

2. Put every new sequence in one of $2^M$ or $MN_s$ different sub-lists for the coded or uncoded schemes, respectively, where all sequences in one sub-list have equal information symbols $u_i$ over the interval $(n + N_1 - N_2)T \leq t \leq (n + N_1)T$, i.e. the last $N_1$ information symbol intervals.

3. Choose the sequence in every sub-list which have the largest object function, $m_n(\tilde{y})$, and place these sequences, in decreasing order of their object functions, in a main-list. This leads to a total of $2^{MN}$ or $MN_s$ sequences for the coded or uncoded schemes, respectively.

4. Choose the $B$ sequences at the top of the this main-list as the candidate sequences for the next step of the algorithm and save them. Note that $B \leq 2^{MN}$ or $B \leq MN_s$ for the coded or uncoded schemes, respectively.

5. Choose the symbol $\tilde{u}_m$ from the sequence at the top of the main-list, i.e. the sequence having the largest object function, as an estimate of the transmitted symbol $u_m$. This means that the decoding depth is $N_2$ information symbols.

6. Return to 1 for the decoding of the next symbol $u_{m+1}$.

When the first symbol $u_0$ is to be detected and all previous symbols are assumed to be 1's, the algorithm is initialized by setting all symbols in one stored sequence to 1. The complex correlation $\xi_n(\tilde{y})$ and the object function $m_n(\tilde{y})$ are both initialized to 0. Thus, in the first few intervals, there will only be at most $2^k$ candidate sequences stored by the algorithm, where $k$ is an interval counter. However, in just a few symbol times, $k$ will reach $B$ and the algorithm will store $B$ sequences for each decision.

Except for step 2, this algorithm is similar to the SA(B) algorithm when used in a code tree. Step 2 is very important when the algorithm is used with noncoherent detection. A noncoherent detector cannot distinguish between signals having parallel phase trajectories only differing by a constant phase value. These signals with parallel phase trajectories have equal values on the object function. This is a problem for a limited search algorithm, since this algorithm only keeps the $B$ sequences with the $B$ largest object functions. Thus, if there are several sequences which corresponds to erroneous signals with parallel phase trajectories, they contain the same symbols and therefore also correspond to equal decisions. These sequences which contains the same information may fill the main-list. Thus, the number of different saved sequences in the algorithm is less than $B$, and the performance is degraded. This is clearly seen in simulations when step 2 is omitted. By introducing step 2 we avoid having these kind of sequences in the main-list and the problem becomes smaller. For details see [9].

3.1 Simulated symbol error probability

The error performance of the SAN(B)-algorithm is evaluated by means of computer simulations for both uncoded CPM schemes and convolutionally encoded CPM schemes. For all considered schemes we also show an estimate of the asymptotic error
performance of the maximum likelihood sequence detector for noncoherent detection as given by $Q(\sqrt{d_{e,B}^2 E_b/N_0})$.

First two uncoded schemes are considered. It is the binary 3RC scheme and the quaternary 2RC scheme. In Fig. 4, the simulations of the 3RC scheme, with modulation indices $h = 1/2$ and $h = 2/3$, are shown. The symbol error probability of quaternary 2RC with $h$ equal to 1/3 and 2/5 is shown in Fig. 5.

The results above and in [9] show, that the error performance of the SAN($B$) algorithm for noncoherent detection of most uncoded CPM schemes is very close to the performance of the coherent Viterbi detector. The decoding delay is in all cases comparable for both detectors.

We have also applied the SAN($B$) algorithm for noncoherent detection to some convolutionally encoded CPFSK schemes. The results for the binary CPFSK scheme with a (4,3)-code are shown in Fig. 6. The Viterbi detector has 24 states when the modulation index $h$ is 1/3, and 8 states when $h=1/2$. In Fig. 7 the simulated bit error probability of quaternary CPFSK combined with the (7,5)-code and using a natural quaternary mapper is shown for $h=1/4$ and $h=1/3$. The optimum Viterbi detectors have 16 and 24 states respectively for these schemes.

The performance when a Viterbi detector using the same object function as the SAN($B$) algorithm is simulated for some of these coded schemes. The results are not shown, but the error probability almost coincides with the performance of the coherent Viterbi detector.

4 Discussions and conclusions

In this paper the performance at large signal to noise ratios for a maximum likelihood sequence detector performing noncoherent detection of rate 1/2 convolutionally encoded CPFSK schemes are studied. The performance is given by means of the minimum squared normalized equivalent Euclidean distance. Furthermore, we propose a new limited tree Search Algorithm for Noncoherent detection using $B$ candidate sequences, SAN($B$), to be used for both uncoded and convolutionally encoded CPM schemes. The performance of this detection algorithm is evaluated by means of computer simulations of the error probability.

In summary, it is shown that the asymptotic error performance with optimum noncoherent detection of coded CPFSK is almost as good as with optimum coherent detection for most schemes. The observation interval have, however, to be quite long.

The performance of the SAN($B$) algorithm has been simulated for several uncoded CPM schemes and convolutionally encoded CPFSK schemes. The obtained results show that both power and bandwidth can be saved simultaneously, compared to MSK, with a noncoherent detector for convolutionally encoded CPM. For most of the simulated schemes, a degradation of 0-1 dB at $P_e = 10^{-3}$ compared to optimum coherent detection can be obtained if the parameters in the algorithm are chosen correctly.

It is difficult to compare the complexity of the Viterbi algorithm and the SAN($B$) algorithm since they work differently. Our feeling is, however, that for the interesting schemes, i.e. the very power and bandwidth efficient schemes, the SAN($B$) algorithm is less complex than the Viterbi algorithm. The main
advantage of the SAN(B) algorithm is however that there is no need for carrier recovery.

There are still some more work to be done on this algorithm. The problem of getting rid of the sequences corresponding to parallel phase trajectories have to be further addressed. The performance of the algorithm on channels with time varying carrier phase and different types of fading should also be considered.

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References


