A Doppler Parameters Estimation Technique for Squint SAR Imaging

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Abstract—An essential part of SAR processing is the estimation of the Doppler parameters of the received data—the Doppler centroid and the Doppler frequency rate. This paper proposes a Doppler parameters estimation technique for squint SAR imaging, where the estimation of the Doppler frequency rate is dependent on that of the Doppler centroid. It estimates the Doppler centroid based on the relationship of the Doppler centroid and the range cell migration (RCM), and estimates the Doppler frequency rate using an improved Radon ambiguity transform (RAT) method based on the data aligned according to the RCM trajectory. This technique needs no prior knowledge of the radar platform velocity and the beam squint angle, and shows good performance of estimating the Doppler parameters. Simulation results demonstrate the validity of the method.

I. INTRODUCTION

In SAR imaging, high azimuth resolution is achieved by coherently processing the Doppler histories of the return signal, where several important parameters such as the Doppler centroid and Doppler frequency rate are required. Unreliable Doppler parameters estimations affect registration and focusing, and raise the noise and ambiguity levels in the processed image, sometimes to the point of seriously affecting image quality [1].

Some methods based on time-frequency analysis have attracted considerable attention and proven themselves to be effective in the detecting and focusing moving targets [2]-[5]. But these methods can not be used in the estimation of the Doppler frequency rate for squint SAR directly. Because they are all based on the assumption that the RCM is not exceed one range resolution cell during the synthetic aperture time, which can not be held in squint SAR configuration usually. Furthermore, when noise and clutter exist, using time-frequency-based methods to estimation the Doppler frequency rate on the data without range cell migration correction may totally fail.

This paper extends the time-frequency-based methods to squint SAR mode. Two characteristics must be considered for squint SAR imaging: one is the Doppler ambiguity, and the other is the RCM. The basic idea of the proposed technique is using the estimated ambiguity-free Doppler centroid to correct the RCM, and then estimating the Doppler frequency rate on the data realigned according to the RCM trajectory. The Doppler ambiguity resolver proposed in [6] shows good performance of estimating the absolute Doppler centroid and needs no prior information of radar platform velocity and the squint angle. In addition, when the Doppler centroid is estimated, the RCM is corrected. After the RCM correction (RCMC), the data realigned on the range cell can be seen as a linear frequency-modulated (LFM) signal. Several methods for analysing a LFM signal in the time-frequency domain have been proposed so far. Considering that the RAT [8] provides equivalent performance to the Radon Wigner transform (RWT) while reducing the search space from two dimensions to only one, we use an improved RAT to estimate the Doppler frequency rate. The improved RAT method replaces finding the peak of the RAT energy by fitting a Gaussian function to find the energy concentration of the RAT. This method can refine the estimation accuracy in the presence of noise. The proposed Doppler parameters estimation technique can be performed on the received data, and needs no knowledge of the radar platform velocity and the squint angle as priori information. It is suitable to low to medium squint SAR mode where the range curvature is neglectable.

The paper is organized as follows. Section 2 introduces the geometry and the signal characteristics of airborne squint stripmap SAR. Section 3 proposes a Doppler parameters estimation scheme for squint SAR imaging, which uses the method based on RCMC to estimate the Doppler centroid and the improved RAT to estimation the Doppler frequency rate.
Section 4 gives the simulation results to verify the accuracy and efficiency of the method. Section 5 concludes the paper.

II. GEOMETRY AND SIGNAL CHARACTERISTICS OF SQUINT STRIPMAP SAR

Fig. 1 shows the geometric relationship of an airborne squint stripmap mode SAR. The instant slant range distance \( R(t_m) \) between the radar and the point target P on the ground can be expressed as

\[
R(t_m) = \sqrt{R_0^2 + (v t_m)^2 - 2 R_0 v t_m \sin \theta}
\]

(1)

where \( t_m \) is the azimuth time along the radar flight path, \( R_0 \) is the initial slant range at \( t_m=0 \), \( v \) is the constant velocity of the radar platform, and \( \theta \) is the squint angle.

A. Range Cell Migration (RCM)

Equation (1) is expanded into Taylor’s series

\[
R(t_m) = R_0 - v \sin \theta \cdot t_m + \frac{v^2 \cos^2 \theta}{2 R_0} t_m^2 + \frac{v^2 \cos^2 \theta \sin \theta}{2 R_0} t_m^3 + \frac{v^4 \cos^2 \theta \theta(-1+5 \sin^2 \theta)}{8 R_0^3} t_m^4 + \cdots
\]

(2)

The variation of slant range \( R(t_m) \) with time \( t_m \) is called range cell migration (RCM). In general, terms up to quadratic order in (2) will be sufficient to account for RCM effects in the range compressed profile. The linear term and quadratic term in (2) account for the range walk and range curvature, respectively. Then the absolute value of the maximum range walk is

\[
|\Delta R_{\text{max}}| = |v \sin \theta \cdot T_s|
\]

(3)

where \( T_s \) is the synthetic aperture time. The absolute value of the maximum range curvature is

\[
|\Delta R_{\text{max}}| = \left| \frac{v^2 \cos^2 \theta \left( \frac{T_s}{2} \right)^2}{2 R_0} \right|.
\]

(4)

Let \( \rho \) be the range resolution bin. Here we suppose the maximum range curvature satisfies \( \Delta R_{\text{max}} < \rho / 2 \), and then the range curvature can be neglected [9]. For many short band and low to medium squint SAR systems, this assumption is reasonable. Thus the RCM can be approximated by

\[
R(t_m) \approx R_0 - v \sin \theta \cdot t_m.
\]

(5)

Range walk may cross many range resolution cells in squint SAR cases and is needed to be corrected.

B. Doppler Ambiguity

Terms up to quadratic order in (2) are sufficient to account for azimuth processing in many low or medium squint SAR cases too. Then the form of the return signal can be approximated to

\[
s(t, t_m) = \psi \left( i \left( \frac{2 R(t_m)}{c} \right) \exp \left[ -j 4 \pi \left( \frac{R_0 + \frac{1}{2} f_d t_m - \frac{1}{4} f_d^2 t_m^2}{\lambda} \right) \right] \right)
\]

(6)

where

\[
f_{dc} = 2 v \sin \theta \overline{\lambda}
\]

(7)

\[
f_{df} = -\frac{2 v^2 \cos^2 \theta}{\lambda R_0}
\]

(8)

are Doppler centroid and Doppler frequency rate respectively, \( \psi() \) is the range-compressed profile, \( \hat{t} \) is the range time, \( \lambda \) is the wavelength corresponding to the radar carrier frequency, and \( c \) is the speed of light.

It should be noted that in high-squint mode, the cubic-phase term and even the higher-order phase terms cannot be neglected, though that is beyond our consideration in this paper.

Because the azimuth data are sampled by the pulse repetition frequency (PRF), which introduces a periodic replication of the Doppler spectrum, the Doppler ambiguity occurs. That is to say, the Doppler centroid (7) can be expressed as

\[
f_{dc} = f_{dc,\text{base}} + M_{\text{amb}} \cdot \text{PRF}
\]

(9)

where \( f_{dc,\text{base}} \) is the baseband Doppler centroid, and \( M_{\text{amb}} \) is the ambiguity number. Separate estimators are needed for each part.

III. DOPPLER PARAMETERS ESTIMATION TECHNIQUE

A. Doppler Centroid and Range Cell Migration

From (5), the RCM is approximated by

\[
\Delta R_{\text{w}} = -v \sin \theta \cdot t_m.
\]

(10)

If the velocity of the radar platform and the squint angle are unknown or not accurate enough, the RCM must be estimated using measurements made from the received SAR data. From (7) and (10), the RCM can be expressed as

\[
\Delta R_{\text{w}} = -\frac{\lambda f_{dc}}{2} t_m.
\]

(11)

Thus if the Doppler centroid is estimated, the RCM can be corrected through multiplying the range compressed SAR data in the range frequency domain by a linear phase

\[
H_1(f, t_m) = \exp \left[ j 4 \pi \frac{\Delta R_{\text{w}}(t_m)}{c} (f - f_c) \right]
\]

(12)

where \( f_c \) is the range frequency and \( f_c \) is the carrier frequency.

The range cell migration correction (RCMC) processing provides a simple method to estimate ambiguity-free Doppler centroid, which is to be discussed in Section III-B, and after the RCMC, the range-compressed data realign according to the RCM trajectory, on which the RAT can be performed.

B. Doppler Centroid Estimation Based on RCMC

From (9), the absolute Doppler centroid can be expressed in two parts. In baseband Doppler centroid estimation, some algorithms, such as the correlation Doppler estimator [10], can give reliable estimates. In the ambiguity number estimation, reference [6] proposes a simple search scheme. This scheme substitutes a preliminary estimate \( M_{\text{amb}} \) of the ambiguity number and the estimated baseband Doppler centroid.
\( \tilde{f}_{R0} \) into (9) to get the initial estimate of the absolute Doppler centroid \( \tilde{f}_{d0} \), and then substitutes \( \tilde{f}_{R0} \) into (11) and (12) to correct the RCM. If the RCM is accomplished, the ambiguity number is got. If not, vary the estimate of the ambiguity number and repeat the above steps until the RCM is corrected well.

After estimating the Doppler centroid and correcting the RCM, the return signal \( s(\mathbf{t}, \mathbf{m}) \) becomes

\[
s(\mathbf{t}, \mathbf{m}) = \psi \left[ \mathbf{t} - \frac{2R_0}{c} \right] \exp \left[ -j \frac{4\pi}{\lambda} \left( R_0 - \frac{\lambda}{4} f_{d0} \mathbf{t} \cdot \mathbf{m} \right) \right].
\]

Convert the radar data to power units and integrate the energy over the azimuth axis to obtain a curve of energy versus range. Check the range cell with the maximum energy and use it to estimate the Doppler frequency rate. The data on the range cell can be seen as a LFM signal

\[
s(t_m) = A \exp \left( -j \frac{4\pi}{\lambda} (R_0 + j\pi f_{d0} t_m) \right)
\]

with time during \( T_\tau \), where \( A \) is a constant depends on the radar cross section (RCS) of the target.

C. Doppler Frequency Rate Estimation Based on Improved RAT

The ambiguity function (AF) of a complex signal \( r(t) \) is a 2D function in Doppler frequency shift \( \dot{\xi} \) and time-delay \( \tau \) and is defined by

\[
A_\mathcal{R}(\tau, \dot{\xi}) = \int_{-\infty}^{\infty} r(t+\tau/2) r^*(t-\tau/2) e^{-j2\pi \dot{\xi} t} dt.
\]

Then the modulus of the AF of (14) is

\[
|A_\mathcal{R}(\tau, \dot{\xi})| = \begin{cases} \sqrt{T_r \pi} |A| \sin c(|\dot{\xi} - f_{d0} \tau| T_r / 2) & \text{if } |\dot{\xi} - f_{d0} \tau| T_r / 2 \leq T_r, \\ 0 & \text{otherwise} \end{cases}
\]

where \( \sin c(x) = \sin(\pi x) / \pi x \). This means that in ideal conditions the energy is concentrated along a line characterized by the chirp rate \( f_{d0} \) in the time frequency plane \((\tau, \dot{\xi})\).

In order to detect the line in the noise background, the Radon transform can be applied. The Radon transform calculates the integral of every line in an image \( g(x,y) \) and is defined as

\[
\hat{g}(\rho, \phi) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) \delta(x - \rho \cos \phi - y \sin \phi) dx dy
\]

where \( \delta \) is the Dirac delta function, and the integration angle \( 0^\circ \leq \phi < 180^\circ \).

From (16), since the AF of the LFM signal (14) passes through the origin in the \((\tau, \dot{\xi})\) plane, the Radon transform with parameter \( \rho \) set to 0 is applied to (16). The transformed result is highly concentrated at the actual skew angle of the line, which is specified by the delta function \( \delta(\dot{\xi} - f_{d0} \tau) \) in the ambiguity plane.

Traditional methods are finding the peak of the Radon transform energy to find the energy concentration. However, when noise exists, the accuracy is challenged and needs further refining. To get better estimation sensitivity, here a Gaussian function fitting approach [6][7] is introduced. Calculate the variance of the differential of the transform slices along \( \rho \) for each angle in the Radon transform, and use a Gaussian function to fit the variance curve to find the angle corresponding to the greatest energy concentration. The Gaussian function with four unknown parameters is

\[
G(x) = A_\mathcal{G} \exp \left( -\frac{(x-\mu)^2}{2\sigma^2} \right) + C
\]

where \( x \) is the independent variable and there are four unknown parameters, the Amplitude, \( A_\mathcal{G} \), the mean or peak location parameter, \( \mu \), the standard deviation, \( \sigma \), and a pedestal, \( C \). The peak location parameter \( \mu \) is the estimate of the skew angle of the line specified by the delta function \( \delta(\dot{\xi} - f_{d0} \tau) \). Then the Doppler frequency rate estimation is got.

IV. SIMULATIONS

Simulations are performed to test the effectiveness of the proposed method in different SNRs. The positions and the RCS’s of the scatterers are randomly generated with uniform distribution. The simulation parameters are: the number of scatterers \( N_s = 50 \), \( \theta = 10^\circ \), \( \text{PRF} = 1000\text{Hz} \), \( v = 150\text{m/s} \), \( R_0 = 15\text{km} \), range bandwidth \( f_{BW} = 150\text{MHz} \), carrier frequency \( f_c = 15\text{GHz} \), pulse width \( T_p = 1\mu s \). We add complex Gaussian noise to range compressed data and estimate the Doppler parameters. Here, SNR is defined as the ratio between the power of the most prominent target and noise power.

To measure the effect of the estimation error of the Doppler centroid, we defined the relative error of the Doppler centroid estimation by

\[
\text{RE}_{f_{d0}} = \sqrt{\frac{E[|\tilde{f}_{d0} - f_{d0}|^2]}{\text{a bandwidth}}}
\]

where \( E[\cdot] \) denotes the expectation value, \( \tilde{f}_{d0} \) and \( f_{d0} \) are estimated value and the actual value of the Doppler centroid respectively, and \( \text{a bandwidth} \) is the azimuth bandwidth, here which is equal to PRF. To measure the effect of the estimation error of the Doppler frequency rate, we calculate the quadratic phase error (QPE) [1] caused by the estimation error of the Doppler frequency rate \( \Delta_{\dot{\xi}_0} \)

\[
QPE = \pi \cdot \Delta_{\dot{\xi}_0} \left( \frac{T_r}{2} \right)^2.
\]

In addition, to investigate the performance of the estimation, the bias and standard deviation of the estimation error of the Doppler parameters \( \Delta_{\dot{\xi}_0} = \tilde{f}_p - f_p \) are defined by

\[
\Delta_{\dot{\xi}_0} = E(\tilde{f}_p - f_p)
\]
\[ \sigma_{f_c} = \sqrt{E\left(\left(\hat{f}_c - E\left(\hat{f}_c\right)\right)^2\right)} \quad (22) \]

where \( \hat{f}_c \) and \( f_c \) are estimated value and the actual value of the Doppler parameters respectively.

The results of the Monte-Carlo simulation are based on 50 runs for each SNR and are shown in Fig.2-7. It can be seen that the performance of the estimation is robust if SNR is higher than 5dB. As seen in Fig.4 and Fig.7, the accuracy of the Doppler centroid estimation is within 5% of the azimuth bandwidth, and the QPE caused by the estimation error of the Doppler frequency rate is kept to within \( \pi/2 \), which are sufficient to the SAR processing [11][1].

At last, Fig.8 gives a SAR image of 25 point targets equally spaced based on the estimated Doppler parameters. Here SNR=10dB, and the radar parameters are set as above.
Fig. 7 QPE caused by the estimation error of the Doppler frequency rate. Enlarged plot is shown in the inset.

Fig. 8 Result of imaging processing based on simulated parameters

V. CONCLUSIONS

This paper proposes a Doppler parameters estimation technique which using the method based on RCMC to estimate the Doppler centroid and using the improved RAT to estimate the Doppler frequency rate. This technique is suitable to the cases where the range curvature and the cubic-phase and higher-order phase can be neglected. It can be easily extended to the detection and focusing of moving targets.

REFERENCES


