Abstract—Super-resolution image reconstruction produces a high-resolution image from a set of shifted, blurred, and decimated versions thereof. As the second image restoration, super-resolution image restoration has become an active research issue in the field of image restoration. In general, super-resolution image restoration is an ill-posed problem. Prior knowledge about the image can be combined to make the problem well-posed, which contributes to some regularization methods. In these regularization methods, however, regularization parameter was selected by experience in some cases. Other techniques to compute the parameter had too heavy computation cost. In this paper a new super-resolution algorithm is proposed to the problem of obtaining a high-resolution image from several low-resolution images that have been sub-sampled. The algorithm is based on the MAP framework, solving the optimization by proposed iteration steps. At each iteration step, the regularization parameter is updated using the partially reconstructed image solved at the last step. The proposed algorithm is tested on Lena images. The results of the experiments indicate that the proposed algorithm has considerable effectiveness in that it can not only make an automatic choice and renew the regularization parameter, but also can get the high resolution reconstruction image expectedly.

Keywords—High-resolution MAP image Reconstruction Image interpolation Motion Estimation.

I. INTRODUCTION

Constrained by the available physical conditions, Deteriorative images are caused by motion blurring, point spread blurring, under-sampling and sensor noise in imaging process. Imaging systems produce images always with a limited resolution. However, images with higher resolution are more desired for application.

Super-resolution techniques offer a possibility to produce an image with a higher resolution from a set of images with lower resolution. The underlying mechanism is implied in the fact that different sub-pixel displacement of each low resolution images contains different information of the high resolution image.

Since CCD and CMOS image sensors have been widely used, people already have got high quality images. However, due to hardware cost and fabrication complexity limitations, the current resolution level of CCD and CMOS image sensors can not make a captured image have no visible artifacts when it is magnified. It is expensive and difficult to increase the current resolution level by improving hardware performance, which is not considered effective. How to increase the current resolution level by applying low cost software tools has become a topic of very great interest.

On the one hand, in the traditional single image restoration problem only a single input image is available for processing, which result in the limited capability of resolution improving. Then a new technique, super-resolution reconstruction (SRR) is required.

On the other hand, a low cost and easy used approach is indispensable to obtain high quality images in many practical cases, including safety monitoring, health diagnosis and military surveillance.

Super-resolution techniques offer a possibility to produce an image with a higher resolution from a set of images with lower resolution. The underlying mechanism is implied in the fact that different sub-pixel displacement of each low resolution images contains different information of the high resolution image.

The pioneer work of super-resolution reconstruction may go back to 1984 by Tsai and Huang. Since then, many researchers have devoted themselves to the work in this area.

Currently, the research is focused to a few points, such as high precision sub-resolution registration algorithm; blind super-resolution methods; robust and efficient reconstruction; real-time processing techniques.

The goal of Super-resolution restoration is to reconstruct the original scene from a degraded observation. By "super-resolution"; we refer to removal of blur caused by the image system(out of focus blur, motion blur, non-ideal sampling, etc.) as well as recovery of spatial frequency information beyond the diffraction limit of the optical system. This recovery process is critical to many image processing applications. And extracting a high resolution image from some low resolution image is required in many facets of image processing. For example, in remote sensing field, where several images of the same area are given, and an improved resolution image is required; or in video processing, where single frame in video signal is generally of poor quality. Enhancement of a single image can be done by using several successive images merged together by a supper-resolution algorithm.
Although classical linear image restoration such as linear and cubic spline expansion has been thoroughly studied, they also smooth the image data in discontinuous regions, producing a larger image which appears rather blurry. So the super-resolution restoration algorithm has been studied from 60's.

Super-resolution restoration from a still image is a well recognized example of an ill posed inverse problem. Such problems may be approached using regularization methods that constrain the feasible solution space by employing a-priori knowledge. This may be achieved in two complimentary ways; (1) obtain additional novel observation data and (2) constrain the feasible solution space with a-priori assumptions on the form of the solution.

We identify three critical factors affecting super-resolution restoration.

Firstly, reliable subpixel motion information is essential. Secondly, observation models must accurately describe the imaging system and its degradations.

Thirdly, restoration methods must provide the maximum potential for inclusion of a-priori information. So study in guarantee image Super-resolution ratio restore result and is it have important meaning and value to recover as to image.

In tradition single image restoration problem only a single input image is available for processing. Super-resolution image restoration addresses the problem of producing super-resolution still image from several images, which contains additional similar, but not identical information. The additional information makes it possible that construct a higher resolution image form original data.

Super-resolution techniques can be divided into two main divisions: frequency domains and spatial domain. Frequency domain methods are earlier super-resolution methods, they can only deal with image sequences with global translational. Spatial domain methods are very flexible. At present, they are main research direction of super-resolution. Spatial methods include Iterated Back projection (IBP), Projection onto Convex Sets (POCS), Maximum a Posteriori (MAP) estimation and Maximum Likelihood (ML) estimation. Two powerful classes of spatial domain methods are POCS and MAP.

II. MAP ALGORITHM

Huang and Tsai were the first to introduce the image super-resolution technique by fusing multiple images into a higher resolution frame with improved visual quality. Their work was motivated by the need to enhance the image quality of the observed frames captured by the US satellite, Landsat. With the emergence of mobile devices, new text interpretation challenges have arisen particularly in natural scene images. Text in such scenes suffers from different degradations, including uneven lighting, optical and motion blur, low resolution, geometric distortion, sensor noise and complex back-grounds. Using multiple frames of a video sequence and static SR techniques, most of these degradations can be minimized or even suppressed, e.g. one can enhance the resolution of the image by recovering the high frequencies corrupted by the optical system.

Above all we formulate an observation model that relates the original HR image to the observed LR image. Consider the desired HR image \( \eta \) of size

\[
\eta = [\eta_1, \eta_2, \ldots, \eta_N]'
\]

\[N = M_1N_1 \times M_2N_2,\]

which is sampled at or above the Nyquist rate from a hypothetically band limited continuous scene \( M_1 \), and \( M_2 \) are the horizontal and vertical down-sampling factors, respectively. Let a set of \( K \) LR images be

\[
\xi^{(k)} = [\xi_1^{(k)}, \xi_2^{(k)}, \ldots, \xi_s^{(k)}]'
\]

\[s = N_1 \times N_2\]

During the imaging process, the observed LR image results from warping, blurring, and subsampling operators performed on \( \eta \) and is also corrupted by additive noise, we can then represent the observation model as

\[
\xi_k = DB_k M_k \eta + n_k, \quad \text{for} \ 1 \leq k \leq p
\]

where \( M_k \) is an \( S \times N \) warp matrix that maps the HR image coordinates to the LR coordinates and represents the motion that occurs during image acquisition, \( B_k \) is an \( S \times N \) blur matrix caused by the optical system, the relative motion during the acquisition period and the point spread function (PSF) of the LR sensor, \( D \) is the decimation matrix of size \( (S \times N)^2 \sqrt{L \times P} \), where \( L \) and \( P \) are the subsampling factors in the horizontal and vertical directions, respectively, and finally \( n_k \) is the associated noise.

Usually \( D \) and \( \xi_k \) are known and are inputs in the SR algorithm.

Note that the high-resolution image and the observed frames are arranged in lexicography format [6]. Rewriting the model in a simpler form:

\[
\xi_k = A_k \eta + n_k
\]

where \( A_k = DB_k M_k \) is the kernel operator.

By setting the equations for each observed frame, the equations are stacked vertically to form a system of linear equations as follows:

\[
[\xi_1 + \eta_1, \xi_2 + \eta_2, \ldots, \xi_p + \eta_p]'
\]

\[= [A_1, A_2, \ldots, A_p] \eta
\]

In this way the equation (2) can be written as

\[
\xi = A\eta + n
\]

We call model (2) the separable imaging problem for SRR. The sub-model \( \xi = A\eta \) is a CIR problem, and Eq. (4),
which reflects the term ‘multi-frame super-resolution’, is the kernel model of SRR. Applying multiframe information is the major distinction between SRR and CIR.

Super resolution is a computationally intensive problem that involves several thousand unknowns. For example, super-resolving a sequence of just 30×30 pixel LR frames into a 300×300 SR image by a factor of 4 in each direction involves 90,000 unknown pixels. As mentioned above, SR is an inverse problem and is ill-conditioned due to the obvious lack of LR frames and the additional noise. Therefore matrix a is under-determined and regularization techniques may have to be used to overcome this problem in the image super-resolution process.

An important factor that determines the quality of reconstruction depends on the construction of the kernel A, as indicated by Eq. (1), the kernel A is the product of the down-sampling, blur and motion operator. Down-sampling operator is generated based on the size of the low-resolution image and resolution enhancement factor R. The resolution enhancement factor R is the ratio of the number of pixels in the super-resolution image to the number of pixels in the low resolution image.

The MAP approach provides a flexible and convenient way to model a priori knowledge to constrain the solution. Usually, Bayesian methods are used when the probability density function of the original image can be established. Given the K LR frames of an SR image, the MAP estimator of the SR image maximizes the a posteriori distribution based on the bimodal characteristic of text.

The maximum is independent of ξ& and only the numerator need be considered.

MAP reconstruction in SR text has seen in-depth investigation by Capel and Zisserman and Donaldson and Myers. Capel and Zisserman used an image gradient penalty defined by the Huber function as a prior model. This encourages local smoothness while preserving any step edge sharpness. Donaldson and Myers used the same Huber gradient penalty function with an additional prior probability distribution based on the bimodal characteristic of text.

If the motion estimation error between images is assumed to be independent and noise is assumed to be an independent identically distributed zero mean Gaussian distribution, the conditional density can be expressed in the compact form

\[
Pr(ξ | η) = \prod \frac{1}{\sigma_ξ \sqrt{2\pi}} \exp\left(-\frac{(ξ_ξ - ξ_ξ)^2}{2\sigma_ξ^2}\right)
\]

where \(\sigma^2\) is the error variance.

\[\vdash \quad Pr(ξ) = \exp \left(-\frac{ξ^2}{2\sigma^2}\right)\]

III. REGULARIZATION TECHNIQUES

Regularization techniques can either be used during the reconstruction process or the deblurring and denoizing step as shown.

Due to the discrete ill-posedness, SRR needs well designed regularization algorithms. A variety of direct and iterative numerical regularization methods have been proposed. Most of these methods seek to either compute or approximate a certain regularized solution, namely the solution \(\hat{η}_{MAP}\) of the Tikhonov regularization.

\[\hat{η}_{MAP} = \arg \max_η \left[ \log \frac{1}{\sigma_ξ \sqrt{2\pi}} - \sum \frac{(ξ_ξ - ξ_ξ)^2}{2\sigma_ξ^2} - \frac{λ}{2} \|Qη\|_2^2 \right] \]

Let

\[f(η, ξ) = \min \left[ \|ξ - Hη\|_2^2 + κ \|Qη\|_2^2 \right](*)\]

\[\vdash \hat{η}_{MAP} = \arg f(η, ξ)\]

where \(κ\) is the regularization parameter for balancing the first term against the regularization term. The choice of \(η\) is then obtained by minimizing (*).

The necessary condition of (*) meeting the minimum is:

\[κ = \frac{(Q^T Qη)^T H^T (H(η - ξ))}{(Q^T Qη)^T (Q^T Qη)}\]

We select the conjugate gradient optimization technique for it avoids the complex computation on the Hessian matrix in the Newton gradient approach and converges faster than the steepest descent gradient approach. The conjugate gradient technique converges to a global minimum of the objective function by following the trajectory defined by the conjugate direction. The conjugate direction is determined according to the following formulas:

\[η_{k+1} = η_k + \left[H^T ξ - (H^T H + κ_{k+1} Q^T Q)η_k\right]\]

The convergence is not achieved until the relative state change for a single iteration has fallen below a predetermined threshold \(ε\), such that:

\[\left|\frac{η_{k+1} - η_k}{η_k}\right| \geq d\]

IV. CONCLUSION

Experimental results demonstrate this new technique is robust and gives very excellent reconstruction result in simulation data, actual satellite data and actual video data. Furthermore, the resulting images exhibit much sharper and clearer details than images reconstructed by the Huber-MAP estimator.
REFERENCES


