Online Identification of Low-Frequency Oscillation in Power System based on Fuzzy Filter and Prony Algorithm

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Abstract—Prony algorithm can be used in online identification of power system low-frequency oscillation, but it is sensitive to the noise of the analysis data which will affect the analysis precision extremely. In this paper, a fuzzy filter and Prony algorithm based hybrid method is presented to identify the modes of low-frequency oscillation. Because the fuzzy filter can cancel the mixed noise rapidly and easily by fuzzy logic, the dominant mode of the low-frequency oscillation can be obtained relatively easily by Prony algorithm this time. Simulation results on active power oscillation of branch 302245 in Center China Power Grid (CCPG) and comparison results of different methods demonstrate the proposed method can provide more precise analysis results and prove its validity.

Index Terms—low-frequency oscillation, wide-area measurement system (WAMS), fuzzy filter, Prony algorithm.

I. INTRODUCTION

In modern large-scale power systems, inter-area low-frequency oscillation is becoming a serious bottleneck for increasing power transfer [1],[2]. So identification the characteristics of the concerned inter-area power swings, including oscillatory frequencies, damping coefficients, is important step before applying damping control. The traditional low-frequency analysis methods mainly include numerical simulation [1],[2] and off-line calculation [3],[4] which are hard to meet the requirements of online calculation. Reference [5] has pointed out that how to obtain the characteristics of low-frequency oscillation by the measured data is the problem need to research in online real-time analysis. The appearance of synchronized phasor measurement based Wide-area measurement system (WAMS) proposes the new idea [6],[7] for the dynamic security analysis of large scale interconnect power system. Reference [8] uses kalman filter and the discrete data provide by WAMS to calculate the electro-mechanical oscillation modes, and reference [9] uses the FFT and the wavelet methods to analyze the electro-mechanical oscillation modes by the data from WAMS. All these algorithms can hardly extract the attenuation characters. Prony algorithm [10] can use a linear combination of exponential functions to analyze the sampling data of equal interval samples, and it can give out the frequency, damping, amplitude and phase of the signal conveniently. So, we can analyze the low-frequency oscillation modes directly by the Prony algorithm and the spot measurements from WAMS, which will meet the requirement of online calculation. Reference [11] has presented a WAMS measurements and improved Prony algorithm based power system low-frequency oscillation analysis method which uses singular value deposition (SVD) method to reduce the order of signal model and then uses Prony method to analyze the low-frequency oscillation electro-mechanical modes of the reduced-order model. But Prony algorithm is sensitive to the noise of the analysis data which will affect the analysis precision extremely, which will make the analysis orders are different with the actual orders and will affect the analysis precision. Although the Prony algorithm has ability to cancel the noisy by least mean square (LMS) algorithm, reference [12] has pointed out that Prony algorithm cannot obtain correct analysis results when signal noise ratio (SNR) is below 40dB. In references [13] and [14], the kalman filter and the low-pass filter were used for pre-filters to omit the noise. But the kalman filter needs to build a system model which is hard to set up because the practical signal is not steady sometimes. To low-pass filter or band-pass filter, it needs to transform between the time domain and the frequency domain and also needs to make certain the limit frequency and the bandwidth. From the analysis mentioned, we can see the traditional noise cancellation methods, including kalman filter, wiener filter, adaptive filter and wavelet method, need complicated calculation and need making certain correlative coefficients and system model. These filters have different filtering effect to different signals, which may make them hard to deal with the hybrid noise signal. References [15]-[18] have proposed a fuzzy logic based filtering method which uses fuzzy logic rules to judge whether there is noise in signal by comparing the signal value of one moment with their values of the anterior and posterior moments. The process of judgment and
noise cancellation is very simple to realize and easy to understand and it doesn’t need any complicated calculation formulas. References [15]-[18] have demonstrated the excellent filtering effect in noise cancellation of one and two dimensions discrete signal.

In this paper, a fuzzy filter and Prony algorithm based hybrid method is presented to identify the electro-mechanical modes of low-frequency oscillation. Because the fuzzy filter doesn’t need complex calculation and can cancel the mixed noise rapidly and easily by several fuzzy logic rules, the more accurate dominant modes of the low-frequency oscillation can be obtained relatively easily by Prony algorithm. Simulation results on active power oscillation of branch 302245 in Center China Power Grid (CCPG) and comparison results of the proposed method and the traditional method proved its validity and feasibility.

II. FUZZY LOGIC BASED FILTERING METHOD

Let $s(n)$ be a digitized input signal in the range $[0, L-1]$. Let $s_0*T(n)$ be the sample to be processed at time $n$, and let $W = \{s_j\}$ be the set of $M$ neighboring samples which belong to a window centered on $s_0$: $W = \{s_1, s_2, ..., s_M\} = \{s(n-M/2), s(n-1), s(n+1), s(n+M/2)\}$. The input variables of the filter are defined as the amplitude differences given by the following relationship:

$$x_j = s_j - s_0 \quad (1 \leq j \leq M)$$  \hspace{1cm} (1)

Since $s(n) \in [0, L-1]$, we have $x_j \in [-L+1, L-1]$. The output variable $y$ is the correction term which must be added to the sample $s_0$ in order to obtain the new resulting sample $s'_0$:

$$s'_0 = s_0 + y$$  \hspace{1cm} (2)

References [15]-[17] have pointed out that the triangular-shaped fuzzy function should be adopted when we use fuzzy logic to cancel the noise. The triangular-shaped fuzzy membership function can be depicted in Fig.1. This membership function has two parameters: here, $c$ and $w$ represent the center and the half-width of the fuzzy set, respectively.

![Fig. 1. The definition of triangular-shaped membership function](image)

Because the input and output variables are both defined as the amplitude differences of signals of different moments. So, three fuzzy member functions can be adopted here to depict the relationship of $x_j$ and $s_0$, or $s_0$ and $s'_0$. The three fuzzy sets are labeled positive (PO), zero (ZE), and negative (NE), as represented in Fig.2.

![Fig. 2. The membership function of fuzzy sets for noise cancellation](image)

As Fig.2 demonstrated, the typical rulebase of the fuzzy filter includes two symmetrical sub-rulebases and one ELSE-rule. Basically, the operation of the fuzzy rulebase can be described by the following statements:

- **IF** the value of a sample is higher than those of its neighbors, **THEN** decrease its amplitude;
- **IF** the value of a sample is lower than those of its neighbors, **THEN** increase its amplitude;
- **ELSE** don’t change it.

References [15]-[17] have discussed two fuzzy filters’ design for impulse noise and mixed noise, respectively. To the impulse noise, we should only consider the two neighboring samples defined by $W_{i}=\{s_i, s_{i+1}\}$. The input variables $x_1$ and $x_2$ are consequently defined by formula (1). When considering the mixed noise, a wider neighborhood is adopted. The set of six neighboring samples defined by $W_{i}=\{s_1, s_2, s_3, s_4, s_5, s_6\}$ is $\{s(n-3), s(n-2), s(n-1), s(n+1), s(n+2), s(n+3)\}$ and corresponding input variable $x_i (i=1,...,6)$ are used to build up a fuzzy filter to the mixed noise. Reference [18] has pointed out by simulation that when the number of neighboring samples is 4, the relatively simple fuzzy logic can also do well in the process of mixed noise cancellation. So, we define $W_{i}=\{s_1, s_2, s_3, s_4\}$ as four neighboring samples, and the fuzzy logic for mixed noise cancellation can be expressed as follows:

- **IF** $(x_1, PO) \AND (x_2, PO) \AND (x_3, PO) \AND (x_4, PO)$ **THEN** $(y, PO)$
- **IF** $(x_1, PO) \AND (x_2, PO) \AND (x_3, PO) \AND (x_4, NE)$ **THEN** $(y, PO)$
- **IF** $(x_1, PO) \AND (x_2, NE) \AND (x_3, PO) \AND (x_4, PO)$ **THEN** $(y, PO)$
- **IF** $(x_1, NE) \AND (x_2, PO) \AND (x_3, PO) \AND (x_4, PO)$ **THEN** $(y, PO)$
- **IF** $(x_1, NE) \AND (x_2, NE) \AND (x_3, NE) \AND (x_4, PO)$ **THEN** $(y, PO)$
- **IF** $(x_1, NE) \AND (x_2, NE) \AND (x_3, NE) \AND (x_4, NE)$ **THEN** $(y, PO)$
- **ELSE** $(y, ZE)$

Here, $(y, PO)$ and $(y, NE)$ denote the input variable $j$ is positive or negative, respectively. $(y, PO)$, $(y, NE)$ and $(y, ZE)$ also
denote the output variable is positive, negative or zero, respectively.

let \( \lambda_1 \) and \( \lambda_2 \) be the degrees of activation of the first five and the last five IF... THEN rules, respectively. These degrees are evaluated by means of the following relationships [19]:
\[
\lambda_1 = \text{MIN}\{m_j: j = 1, \ldots, 4; i = 1, \ldots, 5\;
\lambda_2 = \text{MIN}\{m_j: j = 1, \ldots, 4; i = 6, \ldots, 10\}
\]

Here, \( m_{ij} \) denotes the degree of the membership when the input variable is \( x_i \) in subrule \( i \). Let \( \lambda_0 \) denote the degree of activation of the ELSE-rule: this degree is here evaluated by using the following relationship [19]:
\[
\lambda_0 = 1 - (\lambda_1 + \lambda_2)
\]

So, the output \( y \) can be evaluated by using the following relationship [19]:
\[
y = c_{PO}w_{PO}\lambda_0 + c_{ZE}w_{ZE}\lambda_2 + c_{AZ}w_{AZ}\lambda_2
\]

From Fig.2, we have
\[
c_{PO} = -c_{ZE} = L - 1 \\
c_{AZ} = 0 \\
w_{PO} = w_{ZE} = w_{AZ} = L - 1
\]

Thus, formula (7) becomes
\[
y = (L - 1)(\lambda_1 - \lambda_2)
\]

III. Fuzzy Filter and Prony Algorithm Based Electro-Mechanical Oscillation Analysis

![Flowchart](image)

The principle of fuzzy filter and Prony algorithm based low-frequency oscillation analysis method proposed in this paper is demonstrated by Fig.3. The content of the fuzzy filter has been introduced in part II, and in this part we will introduce the improved Prony algorithm.

Supposing the sample signals \( s(0), s(1), \ldots, s(N-1) \) coming from the WAMS system, in Prony algorithm, the approximate value of the measured signal can be described as follow:
\[
\hat{s}(n) = \sum_{i=0}^{\infty} A_i e^{j\omega_i n + j\phi_i} \tag{9}
\]

Here, \( A_i \) is the amplitude, \( \phi_i \) is the phase in radians, \( \omega_i \) is the frequency in Hz, \( \alpha_i \) is the damping factor and \( \Delta t \) represents the sample interval in seconds.

In order to ascertain the actual order \( p \), we should set up a sample matrix \( R \).

\[
R_y = \begin{bmatrix}
r(1,0) & r(1,1) & \ldots & r(1,p) \\
r(2,0) & r(2,1) & \ldots & r(2,p) \\
\vdots & \vdots & \ddots & \vdots \\
r(p,0) & r(p,1) & \ldots & r(p,p) \\
\end{bmatrix}
\]

Here
\[
r(i,j) = \sum_{n=0}^{N-1} s(n-j)s^*(n-i) \tag{11}
\]

where \( N \) is the number of total samples, \( p \) is the calculation order which usually equals \( \lceil N/2 \rceil \) (where the operator \( \lceil \cdot \rceil \) means round numbers).

By using singular value decomposition (SVD) [10,11], we can make certain the effective rank order \( p \), and the coefficients \( a_1, a_2, \ldots, a_p \) can be also made certain by
\[
\begin{bmatrix}
r(0,0) & r(0,1) & \ldots & r(0,p) \\
r(1,0) & r(1,1) & \ldots & r(1,p) \\
\vdots & \vdots & \ddots & \vdots \\
r(p,0) & r(p,1) & \ldots & r(p,p) \\
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1 \\
\vdots \\
a_p \\
\end{bmatrix} = \begin{bmatrix}
r(0) \\
r(1) \\
\vdots \\
r(p) \\
\end{bmatrix} \tag{12}
\]

In term of the coefficients \( a_1, a_2, \ldots, a_p \), obtained from formula (12), the Prony polars can be obtained by the following formula (13):
\[
1 + a_1z^{-1} + \ldots + a_pz^{-p} = 0 
\]

Using recursive formula (14) and (15), we can get coefficients \( b_1, b_2, \ldots, b_p \),
\[
\hat{s}(n) = \sum_{i=1}^{\infty} a_i\hat{s}(n-i), \quad n = 0, 1, \ldots, N-1 \tag{14}
\]

\[
\begin{bmatrix}
1 & 1 & \ldots & 1 \\
z_1 & z_2 & \ldots & z_p \\
\vdots & \vdots & \ddots & \vdots \\
z_1^{N-1} & z_2^{N-1} & \ldots & z_p^{N-1} \\
\end{bmatrix}
\begin{bmatrix}
b_1 \\
b_2 \\
\vdots \\
b_p \\
\end{bmatrix} = \begin{bmatrix}
\hat{s}(0) \\
\hat{s}(1) \\
\vdots \\
\hat{s}(N-1) \\
\end{bmatrix} \tag{15}
\]

So, the amplitude \( A_i \), phase \( \phi_i \), frequency \( f_i \) and the damping factor \( \alpha_i \) can be calculated by
\[
\begin{bmatrix}
A_i \\
\phi_i \\
\alpha_i \\
f_i
\end{bmatrix} = \begin{bmatrix}
|b_1| \\
\arctan |\text{Im}(b_1)/\text{Re}(b_1)| \\
|\text{ln}|z_i|/\Delta t \\
\arctan |\text{Im}(z_i)/\text{Re}(z_i)|/2\pi \Delta t
\end{bmatrix} \tag{16}
\]

IV. Simulation Results

The proposed methods were tested by the active power oscillation of branch 302245 in CCPG. By using PSASP 6.2 (Power System Analysis Software Package), the active power oscillation curve can be obtained by simulation. By adding 10% white noise to the curve, we can get the supposing active power measurements (shown in Fig.4) of branch 302245 by WAMS. Supposing the sample interval being 0.01s and the samples of the first two second coming from WAMS, Fig.5 has shown the different filtering effect by fuzzy filter. In Fig.5, figure (a) shows the original measurements, figure (b) shows the filtering results after first filtering and figure (c) shows the filtering results after the twice filtering. The filtering results
have shown that the fuzzy filter can get fine filtering effect by twice filtering.

Fig. 4. Active power curve of branch 302245 in CCPG

(a)original samples

(b)error square analysis

Fig. 7. The Prony analysis approximate results of signals performed twice by fuzzy filtering

TABLE I

<table>
<thead>
<tr>
<th>order</th>
<th>amplitude</th>
<th>damping</th>
<th>frequency</th>
<th>energy</th>
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</thead>
<tbody>
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<td>6</td>
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TABLE II

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<th>frequency</th>
<th>energy</th>
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<tr>
<td>2.6e+000</td>
<td>-1.2e+000</td>
<td>0.0e+000</td>
<td>2.7e+001</td>
<td></td>
</tr>
</tbody>
</table>

Table I and Table II listed the results of SVD and Prony analysis of the two conditions, respectively. The results have indicated that the SVD can obtain the lower order of electromechanical oscillation modes when the relative precise samples can be obtained by twice fuzzy filtering. The relative
low order can help us grasp the dominant modes of the low-frequency oscillation.

From Table 1, in the Prony analysis results of the original samples, because the difference of the amplitude of each oscillation mode is not obvious, it is hard to find out the dominant oscillation frequencies and oscillation modes. There are three frequencies from this analysis, in which the frequency 8.1 Hz has exceeded the traditional defined range of low-frequency which is usually from 0.2 to 2.5 Hz. In Table II, not only the amplitude of each mode is relatively obvious, but the order of oscillation modes is relatively small, we can easily find out the dominant mode and dominant oscillation frequency which is 0.59 Hz. By simulation of PSASP, we can find oscillation frequency participated by branch GND1# on incident bus of branch 302245 is 0.602 Hz. In addition, the dominant oscillation mode is the fourth oscillation mode which is 0.59 Hz.

Simulation results have indicated that the Prony algorithm can offer the more accurate approximation curve with lower order after the noise cancellation by fuzzy filter.

V. CONCLUSION

Because Prony algorithm is very sensitive to noise, it is difficult to cancel the noise partly or completely by the LMS algorithm of itself. So, it is important to process noise of the samples in advance. Fuzzy filter is very simple in principle, easy to implement, and it needn’t any complicated calculations. Due to these merits of fuzzy filter, this paper uses fuzzy filter to pre-process the samples at first, and then uses Prony algorithm to analyze the oscillation modes. Simulation results have indicated that the Prony algorithm can offer the more accurate approximate curve with lower order after the noise cancellation by fuzzy filter.

VI. REFERENCES


VII. BIOGRAPHIES

Duhu Li was born in Hubei Province in the People’s Republic of China, on January 3, 1978. He graduated from the Hubei polytechnic University, Wuhan, China in 1999 and received MSc degree from the Wuhan University, Wuhan, China in 2003, and now he is PhD student in Electrical Engineering department of Huazhong University of Science and Technology (HUST). His main field of interest includes power system stability analysis and control.

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