Low Frequency Oscillation Analysis and Damping Based on Prony Method and Sparse Eigenvalue Technique

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Abstract—The power system economy and safety demand is likely to force more and more power systems to interconnect. But the power transfer capability in the tie-lines is largely limited by small signal instability. A technique for low frequency oscillation analysis of bulk interconnected power system by combining sparse eigenvalue analysis with Prony method is presented in this paper, which uses Prony analysis to provide approximation of the oscillation modes for initial shift points of the sparse eigenvalue analysis. Based on the result of the sparse eigenvalue analysis, the PSSs are installed on selected generators to damping the inter-area low frequency oscillation modes; time-domain simulations and sensitivity analysis of the power flow in the tie-line verify their performances. The simulation results of a practical power system demonstrate its effectiveness.

I. INTRODUCTION

NOWAYS, many power systems face the problem of troublesome power oscillations in the range of 0.1 to 2.5 Hz due to heavy load condition or system interconnection [1]-[5]. The low frequency oscillation (LFO) has been studied for a long time and there have been some methods to analyze power system small signal stability. In [6], the oscillation mode has been analyzed by sampling the real-time power signal. A frequency-domain approach, combined with time-domain simulations is used in the analysis [1].

Eigenvalues and eigenvectors, i.e., oscillation modes and corresponding mode shapes, which have been widely used to identify oscillation patterns of a particular mode and determine the power system stabilizer locations to damping the critical oscillation modes, can be obtained by eigenvalue calculation. However, for full-state eigenvalue analysis, the size of the model that can be analyzed is limited to approximately 500 states. Selective eigenanalysis methods, such as Inverse Iterations to Rayleigh Quotient Iterations (II/RQI) algorithm and Simultaneous Iteration (SI) algorithm, etc., which take advantage of the sparsity of the network, have been used for large complex systems [7]. However, the reliability and convergence of the algorithm largely rest on the initial shift points. Prony method has recently been applied to analyze large scale model time-domain simulations results [8]. The method approximates time-domain signals by finite sum of damped sinusoids with specific frequencies, amplitudes, phases, and damping [9]. Because only the output is analyzed, the size of the model is not limited; nevertheless this method is perturbation dependent [8].

This paper presents a technique for low frequency oscillation analysis of bulk interconnected power system by combining eigenvalue analysis with Prony method. This technique uses Prony analysis to provide approximation of the oscillation modes for initial shift points of the selective eigenvalue analysis. The simulation results of a practical power system demonstrate its effectiveness.

The paper is organized as follows. The description of the interconnected power system is discussed in section II. Section III describes the technique of low frequency oscillation analysis. Section IV shows the simulation results, and Section V draws the conclusion.

II. DESCRIPTION OF THE POWER SYSTEM

A. Shandong Power System

The geographical diagram of Shandong power system (SD) is showed in Fig.1. Normally, power from the west areas is transferred to the east areas. The system has an installed capacity of 25757 MW. The main transmission system is formed by 500 kV and 220 kV transmission lines.

B. The Interconnected Power System

A simplified diagram of the interconnected power system network structure is showed in Fig.2. North China power system (NC) consists of several regional utilities, such as Jing-Jin-Tang power system, south Hebei power system, Shanxi power system, and west Inner Mongolia power system (dashed trace in Fig.2). On March 1st, 2005, SD was interconnected with NC through Xin’an-Liaocheng 500 kV double-circuit transmission lines.

NC and North-east China power system (NEC) were weakly interconnected by a 500 kV double-circuit line (Suizhong-Jiagjiaing) on May 11, 2001. It is the first trans-regional AC interconnected power system in China. One year later, On April 22, 2002, Central China power system (CC) interconnected with Chuanyu power system (Sichuan and Chongqing systems, CY). These two areas are two major hydro-generation centers and rich in hydro power capacity (e.g., the biggest hydro power plant in the world—the Three Gorges Hydro Power Plant in CC and Ertan Hydro Power Plant in CY).
NC also interconnected with CC by a 500 kV double-circuit line (Xin’an-Huojia) on November 18, 2003. At last, the interconnection of NC and CC forms a bulk power system with an area of 14 provinces and municipalities, a distance of about 4600 km transmission line from the north to the south, and an installed capacity of 140 GW. The bulk transmission system is mainly formed by 500 kV and 220 kV transmission lines.

III. LOW FREQUENCY OSCILLATION ANALYSIS TECHNIQUE

A. Eigenvalue Analysis

The electrical power system can be described by a set of nonlinear differential equations that represent the dynamic behavior of the system, and a set of algebraic equations. Thus, the state space model may be written as a differential algebraic model as

$$\dot{x} = f(x, y)$$

$$0 = g(x, y)$$

where $x \in R^n$ are the system state variables, and $y \in R^m$ are the algebraic variables.

For small disturbances, the behavior of the system can be linearized around the nominal operating conditions $(x_0, y_0)$, thus the linearized model of system can be obtained by eliminating the algebraic variables $y$ and may be described by the following state space representation as

$$x = Ax$$

where $A$ is the $n \times n$ system matrix.

The solution of the system in terms of eigenvalue $\lambda_i$, left eigenvector $v_i$ and right eigenvector $u_i$ may be written as

$$x(t) = \sum_{i=1}^{q} v_i^T x_0 u_i \exp(\lambda_i t)$$

Therefore, the output of this system is given as

$$y(t) = Cx(t)$$

where $C$ is the output matrix of appropriate dimensions.

Participation factor indicates how much a certain state participates in a certain mode. The participation of state $j$ in the $i$th mode is given by

$$PF_{ji} = u_i v_j$$

Therefore, the machines with higher participation factors in the mode of interest are good candidates for applying control via the power system stabilizers (PSSs).

B. Prony Method

Prony analysis is a technique of approximating signals by finite sum of damped sinusoids with specific amplitude $A_i$, damping $\sigma_i$, frequency $f_i$, and phases $\phi_i$. It is an extension of Fourier analysis in that damping information as well as frequency information is obtained. The Prony analysis estimates $y(t)$ in (5) by a linear combination of exponent functions (in the least-squared-error sense) in the form

$$\hat{y}(t) = \sum_{i=1}^{q} A_i \exp(\sigma_i t) \cos(2 \pi f_i t + \phi_i)$$

where $q$ is the number of sinusoids, the $i$th eigenvalue of the linear prediction model is $\lambda_i = \sigma_i + f_i$, and the corresponding damping ratio is defined as $\zeta = \sigma_i / \sqrt{\sigma_i^2 + \omega_i^2}$.

C. LFO Analysis and Damping

Applying the Prony method to the eigenvalue analysis results a possible way to perform small signal stability analysis of interconnected power system as follows:

1) Determination of suitable models for generators, AVRs, and governors;
2) Identification of the inter-area oscillation modes by using the Prony analysis of active power flow in the tie-line;
3) Determination of oscillation modes and corresponding mode shapes by sparse eigenvalue technique;
4) Enhancement of the interconnected system damping by PSSs and verification of their performances by...
5) Impact on the oscillation analysis due to active power transfer change in the tie-line.

In step 1), mathematical models of generators, AVR, and governors are obtained for every generating unit in the interconnected system by means of tests [1]. Step 2) is carried out by using Prony analysis of active power flow in the tie-line. Based on the Prony solution in the previous step, the sparse eigenvalue technique [7] is used to search out the corresponding system eigenvalues and eigenvectors for low frequency oscillation analysis in a small domain which is adjacent to the Prony solution in step 3). In step 4), with the help of the participation factor and mode shape, the PSSs are installed on selected generating units to enhance the damping of the interconnected system; and time-domain simulation could be used to verify their performance. At last, impact on the oscillation modes due to the active power transfer change in the tie-line is analyzed.

Before the small signal stability calculation, the start search point and the end search point (i.e., the “initial shift points” in [9]), which determine the span of the search start points of the iterations, should be specified for the sparse eigenanalysis algorithms, such as II/RQI and SI. Due to system state variables increase greatly along with the interconnection and low frequency oscillation modes are rich in the range of 0.1 and 2.5 Hz, these two points should be close to the approximation of the oscillation modes of the Prony analysis so as to trace them accurately. The number of the search points which distribute uniformly between the start search point and the end search point, and the number of eigenvalues that carried out from each search point by the algorithm should be destined for the algorithms too. In order to decrease the possibility of lost eigenvalues, these two number should be as large as possible, but this means longer calculation time.

According to the frequency range and system states by which the modes are affected, the oscillation modes are classified as local and inter-area. This study mainly pays attention to the inter-area low frequency oscillation modes as they have more severe damage to the interconnected power system. Among the eigenvalues that carried out by the small signal stability calculation, the oscillation patterns of these inter-area modes are identified using the mode shapes of rotor speed signals. These oscillation patterns provide unique finger prints for each of these inter-area modes [4]. In an inter-area oscillation mode, a group of generators swing against another group of generators almost in the opposite direction. PSS locations and initial parameter setting are determined by the participation factors and mode shapes. Time-domain simulation and sensitivity analysis of the power flow in the tie-line are used to verify the performance of the PSSs.

IV. SIMULATION RESULTS

For a practical operation condition, the Xin’an-Huoja tie line between NC and CC is tripped out, the interconnected power system that only includes NC and NEC. The generators in NC are represented by the six order model, and those in SD and NEC are represented by the five order model. Loads in SD are modeled as 65% constant power and 35% constant impedance, NC contains 60% induction motors and 40% constant impedance load, and NEC has fifty-fifty induction motors and constant impedance load. In SD, the 500 kV Jinan-Zibo transmission line is tripped out to simulate the stressed operating condition.

A. Prony Analysis

Assume that a small perturbation, one of the Shen’er Power Plant (in south Hebei power system) generators’ output is cut down by 5%, is simulated in the Prony analysis. The active power transfer in one of the 500 kV Xin’an-Liaocheng double-circuit tie-lines between SD and NC is analyzed by Prony method.

For the interconnected system without PSS, consider an operation condition that the active power flow in the tie-line is equal to zero, the Prony solution for an amplitude threshold of 0.0005 is given in Table I. It can be found that modes 3 and 7 are two inter-area modes. Mode 3 is dominant, with negative damping, other modes with larger amplitude have higher damping.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Amplitude</th>
<th>Damping</th>
<th>Freq.(Hz)</th>
<th>Damping Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.006833</td>
<td>-12.59324</td>
<td>0.000000</td>
<td>1.000000</td>
</tr>
<tr>
<td>2</td>
<td>0.004722</td>
<td>-0.293629</td>
<td>0.404631</td>
<td>0.114731</td>
</tr>
<tr>
<td>3</td>
<td>0.003585</td>
<td>0.144342</td>
<td>0.261356</td>
<td>-0.087560</td>
</tr>
<tr>
<td>4</td>
<td>0.003562</td>
<td>-0.199005</td>
<td>0.022879</td>
<td>0.810629</td>
</tr>
<tr>
<td>5</td>
<td>0.001191</td>
<td>-0.528544</td>
<td>0.891446</td>
<td>0.093947</td>
</tr>
<tr>
<td>6</td>
<td>0.000930</td>
<td>-0.931743</td>
<td>0.967584</td>
<td>0.151491</td>
</tr>
<tr>
<td>7</td>
<td>0.000814</td>
<td>-0.169953</td>
<td>0.613198</td>
<td>0.044068</td>
</tr>
<tr>
<td>8</td>
<td>0.000562</td>
<td>-1.208451</td>
<td>0.311245</td>
<td>0.525674</td>
</tr>
</tbody>
</table>

B. Eigenvalue Analysis

To find out the participating generators that involved in each of the above two inter-area oscillation modes, sparse eigenvalue analysis is used. Take mode 3, a negative damping oscillation mode, for example, II/RQI here is used to search out the corresponding oscillation mode and mode shape. As the eigenvalue of this mode carried out by the Prony method is 0.144342±j1.642148. So the start search point and the end search point can be specified as 0.130000±j1.500000 and 0.150000±j1.700000, respectively. The number of the search points and eigenvalues that carried out from each search point by the algorithm are 2 and 4, respectively. Table II shows the eigenvalues calculated by the II/RQI algorithm (cross out the same ones).
Analyzing the data in Table II, it is easy to identify that mode 2 with an eigenvalue of 0.149542±j1.652818 and a frequency of 0.26Hz is the same mode as Prony solution mode 3, while other modes are local oscillation modes have relative larger damping ratios. The corresponding mode shape of mode 2 is showed in the left part of Fig.3. It's clearly seen from the mode shape that for this mode, generators of SD and NC swing against another group formed by the generators of NEC, just like two ends of a “strip” swinging each other from the north to south in geography. The generators in NEC and SD have larger oscillation magnitudes and participation factors.

In the right part of Fig.3, there lists some generating units which have relative larger participation factors, such as Yimin, Yuanbao, Harbin, Hegang, Shuangliao and Baishan in NEC, Zouxian, Huade and Liaocheng in SD, etc. Because there are a great number of generating units involved in this mode, the participation factor of a specific generator is small.

Similarly, the corresponding eigenvalue, frequency, and damping ratio of mode 7 in Table I are searched out by the II/RQI algorithm. They are -0.137104±j3.924921, 0.624671Hz, and 0.034910, respectively. According to the modal analysis, it is found that, for mode 7, generators of Shanxi and south Hebei power system, swing against generators of west Inner Mongolia and SD power system. Although generators in SD participate in this oscillation, the participation factors are much smaller.

Comparing the results of mode 3 and mode 7 that carried out by Prony method and sparse eigenvalue analysis, it can be seen that differences exists in the frequency and damping of the oscillation modes of the two methods. This may be due to the Prony parameters, i.e., the number of sampling data points and the order of the linear prediction model are not very properly optimized. Further insight into the parameter optimization of Prony analysis could be found in [8].

C. Damping Enhancement

According to the analysis above, the system has two major inter-area oscillation modes. There are a great number of generating units in different areas involved in each inter-area oscillation mode which is influenced by global states of the power system Therefore, it’s more difficult to damping the inter-area low frequency oscillation than the local modes which are largely determined and influenced by local states. At present, the most common control measures in use today are employment of the PSSs. Fig. 4 shows the transfer function diagram of the PSS used in the analysis. The PSS uses the active power deviation of the generating unit as input signal and mainly consists of the amplifying block, the washout block, and two lead-lag compensation blocks, etc.

![Fig. 4. The transfer function diagram of PSS](image)

For mode 3, generators which are listed in right part of Fig. 3 that have larger participation factors are better PSS candidates. For mode 7, Zhunger, Tuoketuo, and Daqi in west Inner Mongolia power system are selected to install PSSs. At last, 49 generators are selected to install PSSs according to the mode shape and participation factors.

Under the operation mode that the active power flow in the tie-line is equal to zero, Fig. 5 shows the active power flow oscillation in one of the 500 kV Xin’an-Liaocheng double-circuit tie-lines. In the figure, the results for the system with and without PSSs are compared. For the system without PSS, the active power oscillation is about 25 MW at 25s, and the oscillation frequency is about 0.25Hz, after the PSS installation, the power oscillation in the tie-line is very small. It is clear that the interconnected system damping is significantly improved by PSSs.

![Fig. 5. Tie-line active power flow with PSSs versus without PSSs](image)
D. Sensitivity Analysis of Tie-Line Power Flow

Three operating conditions are considered corresponding to three active power transfer levels in the Xin’an-Liaocheng tie-lines between SD and NC: zero power transfer, SD transfers 600 MW to NC and NC transfers 600 MW to SD. For each operating condition, interconnected system with and without PSSs are simulated. In total, there are six scenarios. The eigenvalue analysis results of the two inter-area oscillation modes under different active power transfers in the tie-line are listed in Table III.

<table>
<thead>
<tr>
<th>Operation Condition</th>
<th>Mode 3 (0.26 Hz)</th>
<th>Mode 7 (0.62Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Freq.(Hz)</td>
<td>Damping</td>
</tr>
<tr>
<td>0MW, without PSSs</td>
<td>0.263054</td>
<td>-0.090109</td>
</tr>
<tr>
<td>0MW with PSSs</td>
<td>0.246239</td>
<td>0.084524</td>
</tr>
<tr>
<td>+ 600 MW, without PSSs</td>
<td>0.261001</td>
<td>-0.098605</td>
</tr>
<tr>
<td>+ 600MW, with PSSs</td>
<td>0.244729</td>
<td>0.073052</td>
</tr>
<tr>
<td>- 600 MW, without PSSs</td>
<td>0.263495</td>
<td>-0.084396</td>
</tr>
<tr>
<td>- 600MW, with PSSs</td>
<td>0.245831</td>
<td>0.084524</td>
</tr>
</tbody>
</table>

+: means SD transfers power to NC
:- means NC transfers power to SD

Analyzing the data in Table III, it can be found that, for mode 3, the damping decreases in a small extent as the power transfer increasing from SD to NC. For mode 7, there is only a small change either in frequency or in damping under different active power transfers in the tie-line. That is to say, the power flow in the tie-line between SD and NC has greater impact on mode 3 over mode 7, so increment of power transfer from NC to SD or reduction of power transfer from SD to NC could improve the damping of mode 3. Further, PSSs show good robustness and ability to enhance the damping of all the critical inter-area low frequency oscillation modes under different operating conditions, with the lowest damping ratio being 0.073052.

V. Conclusion

A technique for low frequency oscillation analysis of bulk interconnected power system by combining sparse eigenvalue analysis with Prony method is presented in this paper, which uses Prony analysis to provide approximation of the oscillation modes for initial shift points of the sparse eigenvalue analysis. This technology not only grants the reliability and convergence of the sparse eigenvalue calculation, but also has no limitation to the size of the system scale. Based on the technique, two inter-area low frequency modes with negative or weak damping in a practical China interconnected power system are analyzed. With the help of the mode shape and participation factor, PSSs are installed on the selected generators to enhance the damping of the inter-area oscillation modes. The simulation results demonstrate the effectiveness of the proposed method.

REFERENCES