Abstract — We consider, in this paper, a space-time coded (STC) orthogonal frequency-division multiplexing (OFDM) system with multiple transmitter and receiver antennas over correlated frequency- and time-selective fading channels. We propose a maximum-likelihood (ML) receiver for STBC-OFDM systems based on the expectation-maximization (EM) algorithm. We then propose a low-density parity-check (LDPC)-code-based STC-OFDM system to interface with the EM module. Using this encoder scheme together with the EM-based receiver, we built a novel turbo receiver which is particularly well suited for fast-time varying channels with a modest computational complexity.

Keywords: EM, LDPC, STC, STBC, OFDM, SAGE, Correlated fading channels.

I. INTRODUCTION

Space-time coding (STC) techniques, including space-time trellis coding (STTC) and space-time block coding (STBC) integrate the techniques of antenna array spatial diversity and channel coding and can provide significant capacity gains in wireless channels. A lot of papers have investigated their use particularly for wireless flat-fading channels [1]. However, many wireless channels are frequency-selective in nature, for which the STC design problem becomes a complicated issue. On the other hand, the orthogonal frequency-division multiplexing (OFDM) technique transforms a frequency-selective fading channel into parallel correlated flat-fading channels. Hence, in the presence of frequency selectivity, it is natural to consider STC in the OFDM context [2].

We propose here an iterative turbo-based receiver including the expectation-maximization (EM) algorithm exchanging extrinsic information with an outer channel decoder. The EM algorithm, which approximates the likelihood optimal demodulation, enables to reduce the error floor exhibited by suboptimum receivers when they have to cope with time-varying channels.

For the outer channel code, following the approach of Lu & al [3], we propose a low density parity-check (LDPC) code. These kinds of codes, first addressed by Gallager [4], have the following advantages for the STC-OFDM system considered here: 1) the LDPC decoder usually has a lower computational complexity than the turbo-code decoder. 2) The minimum distance of binary LDPC codes increases linearly with the block length with probability close to 1. 3) LDPC codes do not show an error floor, which is suitable for short-frame applications. 4) Due to the random generation of parity-check matrix, the coded bits have been effectively interleaved; therefore, no extra-interleaver is needed.

The combination of the EM-based demodulator and the LDPC yields to an iterative turbo-receiver which is indeed a promising solution to highly efficient high data rates transmission over time-and frequency-selective fading channels.

II. SYSTEM MODEL

We consider an STC-OFDM system with $K$ subcarriers, $N$ transmitter antennas and $M$ receiver antennas, signaling through frequency- and time-selective fading channels. Each STC codeword spans $P$ adjacent OFDM words and each OFDM word consists of $(NK)$ STC symbols, transmitted simultaneously during one time slot. It is first assumed that the fading process remains constant during each OFDM word but varies from one OFDM word to another, and the fading processes associated with different transmitter-receiver antenna pairs are uncorrelated. The $(NK)$ information bits (design the set of all possible STC symbols) are encoded by a rate $R = 1/N$ LDPC encoder into $(NPK)$ coded bits and then the binary LDPC coded bits are modulated into $(NPK)$ STC symbols by an MPSK modulator. These $(NPK)$ STC symbols, which correspond to an STC code word, are split into $N$ streams; the $(PK)$ STC symbols of each stream are transmitted from one particular transmitter antenna at $K$ subcarriers and over $P$ adjacent OFDM slots (see Fig. 1). Unlike turbo codes there's no need of extra-interleaver due to the use of a high density encode matrix which is akin to a self built-in random interleaver.

![Figure 1. Transmitter structure of an LDPC-based STC-OFDM system with multiple antennas](image)

At the receiver, the signals are received from $M$ receiver antennas. After matched filtering and symbol-rate sampling, the discrete Fourier transform (DFT) is then applied to the received discrete-time signals to obtain
\( y_i[p] = \sum_{j=1}^{N} X_j[p, i] H_{i, j}[p] + z_i[p] \) \hspace{1cm} (1)

\[
y_i[p] = X[p] H_i[p] + z_i[p], \quad i = 1, \ldots, M, \quad p = 1, \ldots, P
\]

with

\[
X[p] = \text{diag}[x_1[p], \ldots, x_N[p]]_{K \times K}
\]

\[
X_j[p] = \text{diag}[x_j[p, 0], \ldots, x_j[p, K-1]]_{K \times K}
\]

\[
H_{i, j}[p] = [H_{i, j}^H[p, 0], \ldots, H_{i, j}^H[p, K-1]]_{N \times 1}
\]

where \( H_{i, j}[p] \) is the \((NK)\) vector containing the complex frequency responses between the \(i\)th receiver antenna and all \(N\) transmitter antennas at the \(p\)th OFDM slot. \( x_j[p, k] \) is the symbol transmitted from the \(j\)th transmitter antenna at the \(k\)th subcarrier and at the \(p\)th OFDM slot, and \( z_i[p] \) is the ambient noise which is circularly symmetric complex Gaussian with covariance matrix \( \Sigma_z \).

Consider the channel response between the \(j\)th transmitter antenna and the \(i\)th transmitter antenna, the time-domain channel impulse response can be modeled as a tapped delay line given by

\[
h_{i, j}(k; t) = \sum_{k=0}^{L-1} a_{i, j}(k; t) \delta(t - \frac{k}{\Delta f})
\]

where \( L := \tau_{\text{max}} + 1 \) denotes the maximum number of resolvable paths, with \( \tau_{\text{max}} \) being the maximum multipath spread and \( \Delta f \) being the tone spacing of the OFDM system and \( a_{i, j}(k; t) \) is the complex amplitude of the \(k\)th tap, whose delay is \( k/\Delta f \). For OFDM systems with proper cyclic extension and sample timing with tolerable leakage, the channel frequency response between the \(j\)th transmitter antenna and the \(i\)th receiver antenna at the \(p\)th time slot and at the \(k\)th subcarrier can be expressed as

\[
H_{i, j}[p, k] = h_{i, j}(p \Delta f, k) \]

\[
= \sum_{k=0}^{L-1} h_{i, j}[k; p] e^{-2\pi j k / K} = w_{i, j}[k] h_{i, j}[p]
\]

where \( h_{i, j}[k; p] = a_{i, j}(k; p) \), \( \Delta f \) is the duration of one OFDM slot, \( h_{i, j}[p] = [a_{i, j}(0), \ldots, a_{i, j}(L-1; p T)]^T \) is the \(L\) vector containing the time responses of all the taps, and \( w_{i, j}[k] = [e^{-2\pi j k / K}, \ldots, e^{-2\pi j (L-1) / K}]^T \) contains the corresponding DFT coefficients. Using (3), the signal model in (1) can be further expressed as

\[
y_i[p] = X[p] W h_i[p] + z_i[p], \quad i = 1, \ldots, M, \quad p = 1, \ldots, P
\]

with \( W := \text{diag}[W_1, \ldots, W_F]_{HM \times NL} \),

\[
W_f := [w_f(0), w_f(1), \ldots, w_f(K-1)]^T_{K \times 1}
\]

and

\[
h_i[p] := [h_{i, 1}^H[p], \ldots, h_{i, N}^H[p]]^T_{KL \times 1}
\]

III. ML RECEIVER BASED ON THE EM-ALGORITHM

A. Derivation of EM algorithm

In this section, we consider the ML receiver design for STC-OFDM systems. As in a typical data communication scenario, communication is carried out in a burst manner (a burst = \(L_b\) OFDM words) with the first \(L_b\) OFDM words \((p=0, \ldots, L_b)\) containing known pilot symbols.

For notational simplicity, here we consider an STC-OFDM system with two transmitter antennas and one receiver antenna. Without CSI, the maximum-likelihood (ML) detection problem is written as

\[
\hat{X}[p] = \arg \max_{X[p]} P(X[p] | y[p]) \quad p = L_b + 1, \ldots, L_p
\]

\( (X[0]) \) contains pilot symbols. The optimal solution to (5) is of prohibitive complexity. We next propose solving (5) iteratively according to the expectation-maximization (EM) algorithm (5). Two steps are required.

E-step: Compute

\[
Q(X|X') = \mathbb{E}[\log p(y | X, h) | y, X']
\]

M-step: Solve

\[
X^{(i+1)} = \arg \max_X Q(X|X^{(i)}) + \log P(X)
\]

where \( X^{(i)} \) denotes hard decisions of the data symbols at the \(i\)th EM iteration and \( P(X) \) represents the priori probability of \( X \), which is fed back by the channel decoder from the previous turbo iteration. In the E-step, the expectation is taken with respect to the "hidden" channel response \( h \) conditioned on \( y \) and \( X \).

\[ \hat{h}(y, X^{(i)}) = N_z(\hat{h}, \hat{\Sigma}_z) \quad (8) \]

with

\[ \hat{h}(y, X^{(i)}) = N_z(\hat{h}, \hat{\Sigma}_z) \]

where

\[ \hat{\Sigma}_z = \hat{\Sigma}_z - \mathbb{E}(h h^H) = \text{diag}[\hat{\sigma}_{1,1}^2, \ldots, \hat{\sigma}_{L,1}^2, \ldots, \hat{\sigma}_{L,L}^2] \]

and

\[
\hat{\Sigma}_z = \text{diag}[\mathbb{E}(h h^H)]
\]

are average power of the \(i\)th tap related with the \(j\)th transmitter antenna;
Within each turbo iteration, the above E-step and M-step are iterated I times. At the end of the I<sup>th</sup> EM iteration, the extrinsic <i>a posteriori</i> LLRs of the LDPC code bits are computed and then fed to the soft LDPC decoder (see part IV). At each OFDM subcarrier, two transmitter antennas transmit two STC symbols, which correspond to (2log<sub>2</sub>N) LDPC code bits. Based on (12), after I EM iterations, the extrinsic <i>a posteriori</i> LLR of the <i>j</i><sup>th</sup> LDPC code bit at the k<sup>th</sup> subcarrier is computed at the output of the MAP-EM demodulator as follows:

\[
Z^j[d^j(k)] = \log \frac{P[d^j(k) = +1]}{P[d^j(k) = -1]}
\]

\[
= \log \frac{\sum_{\omega \in C^j} P[x(k) = x]}{\sum_{\omega \in \bar{C}^j} P[x(k) = x]}
\]

\[
= \log \frac{\sum_{\omega \in C^j} \exp[-q(x(k))] + \log P(x)}{\sum_{\omega \in \bar{C}^j} \exp[-q(x(k))] - X^j[d^j(k)]}
\]

where \(C^j\) is the set of \(x\) for which the <i>j</i><sup>th</sup> LDPC coded bit is +1 and \(\bar{C}^j\) is similarly defined. The extrinsic <i>a priori</i> LLRs \(X^j[d^j(k)]\) are provided by the soft LDPC decoder at the previous turbo iteration (where \(P\) denotes the previous turbo iteration; at the first turbo iteration, \(X^j[d^j(k)] = 0\)). Finally, the extrinsic <i>a posteriori</i> LLRs \(X^j[d^j(k)]\) are sent to the soft LDPC decoder, which in turn iteratively computes the extrinsic LLRs \(X^j[d^j(k)]\) and then feeds them back to the MAP-EM demodulator and thus completes one turbo iteration. Fig. 1 depicts the turbo receiver.

### Initialization of MAP-EM Demodulator

The performance of the MAP-EM demodulator (and hence the overall receiver) is closely related to the quality of the initial value of \(X^{[0]}[p]\). Except for the first turbo iteration, \(X^{[0]}[p]\) is simply taken as \(X_{\text{hard}}[p]\) from the previous turbo iteration: at the output of LDPC, symbols hard decisions are taken based on the extrinsic information. This implies redecoding the symbols probabilities based on the bit LLRs. The initial estimate of \(X^{[0]}[p]\) is based on a particular version of the EM algorithm named SAGE algorithm [6]. We choose this algorithm since it is largely simpler than the conventional MMSE detection while exhibiting similar performances. SAGE algorithm proceeds as follows:

(1) The observed data \(y[p]\) is viewed as incomplete data and \(X_{\omega}[p]W_{h}[p] + z[p], \ n = 1, \ldots, N\) as complete data.

\[
\hat{X}[p] = \text{arg max}_{\hat{X}} [Q(\hat{X})] + \log P(X)
\]

\[
= \text{arg max}_{\hat{X}} \left[ -\sum_{k=0}^{K-1} q(x(k)) - \log P(x(k)) \right]
\]

\[
X^{[i+1]}(k) = \text{arg min}_{x(k)} \left[ q(x(k)) - \log P(x(k)) \right]
\]

\[
k = 0, 1, \ldots, K - 1
\]
Initialization: For \( 1 \leq n \leq N \), \( \hat{z}_n^{(0)} = x_n w_n^{(0)} \).

At the \( k \)-th iteration (\( k = 0, 1, 2 \ldots \)):

\[
\hat{y}_n^{(k)} = \frac{1}{\tau} \sum_{m=1}^{N} \hat{z}_m^{(k)}
\]

\[
\hat{h}_n^{(k+1)} = w_n^{-1} \hat{y}_n^{(k)}
\]

\[
\hat{z}_n^{(k+1)} = x_n w_n^{(k+1)}
\]

For \( 1 \leq m \leq N \) and \( m \neq n \), \( \hat{z}_m^{(k+1)} = \hat{z}_m^{(k)} \).

A proper selection of the initial value of \( \hat{h}_n \) is fundamental for the convergence speed of the algorithm. A good estimate can consist in \( \hat{h}_n^{(0)} = w_n^{-1} x_n y_n \). This implies that all the signals transmitted from other than the \( n \)-th antenna to be zero. In practice, this estimation algorithm is used with a training sequence of length \( L_0 \) symbols and the final initial value is averaged over the \( L \) values of \( \hat{h}_n \).

IV. Turbo Receiver

We propose a modified LDPC-based STC-OFDM transmitter structure depicted in Fig. 1 [3]. First proposed by Gallager [4] and recently re-examined in [7], low-density parity-check (LDPC) codes have been shown to be a very promising coding technique for approaching the channel capacity in AWGN channels. An LDPC code is a linear block code characterized by a very sparse parity-check matrix. The parity check matrix \( P \) of an \((N, K, t)\) LDPC code of rate \( R = K/N \) is an \((N-K) \times N\) matrix, which has \( t \) ones in each column and \( j > t \) ones in each row. Apart from these constraints, the ones are placed at random in the parity check matrix. When the number of ones in every column is the same, the code is known as a regular LDPC code; otherwise it is called irregular LDPC code. Similar to turbo codes, LDPC codes can be efficiently decoded by a suboptimal iterative belief propagation algorithm which is explained in detail in [4].

V. Simulation Results

In this section, we provide computer simulation results to illustrate the performance of the proposed LDPC-based STC-OFDM system in frequency- and time-selective channels. The correlated fading processes are generated by using the methods in [8]. In the simulations, the available bandwidth is 1 MHz and is divided into 64 subcarriers. These correspond to a subcarrier symbol rate of 15.6 kHz and OFDM word duration of 64 \( \mu \)s. In each OFDM word, a guard-time interval of 6 \( \mu \)s is added to combat the effect of inter-symbol interference, hence \( T = 70 \mu \)s. For all simulations, two information bits are transmitted from each of the six subcarriers at each OFDM slot, therefore the information rate is \( 2 \times 64/70 = 1.82 \) bits/sec/Hz. All the LDPC codes used in the simulations are designed with column weight \( t = 3 \) in the parity-check matrices.

A. Performance with unknown CSI

In the following simulations, the receiver performance with unknown CSI is shown. The system transmits in a burst manner, each data burst includes training symbols pilots at the beginning and the packet size corresponds to LDPC generator matrix size. The length of the training sequence is chosen according to channel frequency selectivity and Doppler rates. Simulations are carried out in two-tap (two equal-power taps at 1 ms and 4 ms) frequency- and time-selective fading channels on each link (2 antennas). The maximum Doppler frequency of fading channels is 150 Hz (with normalized Doppler frequency \( 0.044 \)). The turbo receiver performance of a regular LDPC code is shown in Fig. 2 and Fig. 3 shows the results obtained in the case of an irregular LDPC code. TurboDD denotes the turbo receiver with pilot/decision-directed channel estimates as described in [9], and TurboEM denotes the turbo receiver with the MAP-EM demodulator already described in Section III-A. When CSI is not available, the proposed TurboEM receiver significantly reduces the error floor exhibited by TurboDD receiver. In simulations, the turbo receiver takes three turbo iterations; and at each turbo...
iteration, the MAP-EM demodulator takes three EM iterations. At the cost of few pilot insertion and a modest complexity, the proposed turbo receiver with the MAP-EM demodulator is shown to be a promising technique, especially in fast fading conditions.

![Figure 2](image)

**Figure 2.** BER of a regular LDPC based STC-OFDM system with multiple antennas \((N = 2, M = 1)\) in two tap \((L = 2)\) frequency-selective channels, without CSI

![Figure 3](image)

**Figure 3.** BER of an irregular LDPC-based STC-OFDM system with multiple antennas \((N = 2, M = 1)\) in two tap \((L = 2)\) frequency-selective channels, without CSI

**VI. Conclusion**

In this paper, we have considered the design of iterative receivers for STBC-OFDM systems in unknown wireless dispersive fading channels. We have proposed a low-complexity ML receiver for STBC-OFDM systems based on the EM algorithm and a LDPC channel decoder to interface with the EM algorithm. Compared with the conventional space-time trellis code (STTC), LDPC-based STC can significantly improve the system performance by efficiently exploiting both the spatial diversity and selective-fading diversity in wireless channels. Compared with the recently proposed turbo-code based STC scheme, LDPC-based STC exhibits lower receiver complexity and more flexible scalability. Furthermore, we have considered the design of an iterative receiver when no CSI is available. Simulation results demonstrate that the use of the EM algorithm alleviates the error floor frequently encountered on fast-time varying channels. It is then straightforward to conclude that this architecture is a promising technique for highly efficient data transmission over dispersive fading channels.

**References**


