Multiscale contour corner detection based on local natural scale and wavelet transform

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Abstract

A new corner detection method for contour images is proposed based on dyadic wavelet transform (WT) at local natural scales. The points corresponding to wavelet transform modulus maxima (WTMM) at different scales are taken as corner candidates. For each candidate, the scale at which the maximum value of the normalized WTMM exists is defined as its “local natural scale”, and the corresponding modulus is taken as its significance measure. This approach achieves more accurate estimation of the natural scale of each candidate than the existing global natural scale based methods. Furthermore, the proposed algorithm is suitable for both long contours and short contours. The simulation and the objective evaluation results reveal better performance of the proposed algorithm compared to the existing methods.

Keywords: Corner detection; Dyadic wavelet transform; Local natural scale

1. Introduction

In the seminal work [1], Attneave reaches the famous conclusion that information is concentrated along contours and further concentrated on the peaks of curvature points. Attneave’s work motivates the research on corner detection. As corners are sufficient to characterize a contour, the data are reduced enormously. Corners are also invariant features to translation, rotation, and scaling [2]. This property makes corners more suitable for matching problems, particularly for partial occluded object matching, while the methods based on global features can not work in this case.

A number of methods have been proposed to detect corners of different sizes and extents for contour images. In general, they can be categorized into two groups, the support region based methods [3] and the multiscale based methods [4,5]. For the support region based methods, the support region (its natural scale) is determined at each individual point. As a result, the natural scale is optimal for the corresponding point. However, the determination of the support region is based on the raw data which include quantization errors and possible noises. Thus, the methods are not robust. For the multiscale based methods, there exist scale-space based analysis and wavelet transform based analysis. Although both of them are robust due to their inherent smoothing property, the existing multiscale based methods either utilize information on one or several selected global natural scales [5] or utilize only location information in the transformed domain [4], which limits the performance.

As another multiscale analysis tool, wavelet transform (WT) analyzes the local properties of the signals well. Wavelet transform modulus maxima (WTMM) characterize the irregular structures of the signal. Consequently, it
is suitable to detect contour corner points using WT. Since, the proposed algorithm belongs to the WT-based method, we will review the existing WT-based methods briefly in the following.

In [6], Lee et al. apply the quadratic spline dyadic WT that can be implemented by a fast algorithm. The point is considered as a corner candidate if its maximum persists from the first scale to the third scale and the magnitude at the third scale is above a threshold. Then each candidate is isolated and the ratio of the WT at another two coarser scales is computed. The corner candidates are confirmed as a corner point if its ratio is close to the ratio of an ideal corner’s. In this algorithm, the post processing step is somewhat complex. Moreover, all the corner points are detected at the same scales so that some corner points are missed in the results.

In [7], Antoine et al. apply the WT directly to the coordinates of the 2D shape. The coordinates are represented as a complex function. The WTMM indicate the existence of corner candidates. The coarsest scale that an extremum lasts to is defined as its lifetime. They take the lifetime as the significance measure. Consequently, we can say that this method is only based on the position information. In [8], Hua and Liao apply the WT to the x and y coordinates separately. The values of the WTMM at the scale equal to or larger than 22 are recorded as the significance measure. We can see all the corners are detected in a single scale in this method. The wavelets with two vanishing moments are used in both [7] and [8]. This agrees with the curvature definition as the WT is a second order differential operator to a smoothed signal if the wavelet with two vanishing moments is used.

In [9], Quddus and Fahmy improve the algorithm of [6]. The WT of the orientation function is performed at four scales and the coefficients are normalized to the maximum at each scale. The WTMM at the coarsest scale are taken as corner points if the values are above the first threshold, \( r_1 \). Then those WTMM are also detected as corners if the values at the coarsest scale are above the second threshold, \( r_2(<r_1) \), and are increasing as the scales are decreasing. Finally, they check the WTMM at the first scale. The WTMM at the first scale also are recognized as corners if the values are greater than the third threshold, \( r_3 \). Although this method reduces some computation compare to the method in [6], the detection process is quite complex and somewhat heuristic. Since, this method limits the detection within four scales, it detects the corners at multiscales.

In [5], Quddus and Gabbouj propose a novel and efficient corner detection method using WT and singular value decomposition (SVD). The WT of the orientation function is performed first. Then SVD is used to detect the global natural scales in the wavelet domain. After that, the largest singular value is used in the reconstruction process and the significance measure is estimated from the average of all the global natural scales. The utilization of WT and SVD makes this method efficient and robust. They apply the method on the real and quite complicated fish contours and obtain satisfactory results in the paper. However, they have not considered the situation where the stop criterion for the selection of natural scales does not work. Moreover, the SVD increases the computational cost.

From the above description and analysis, we can see that most of the multiscale methods are operated on one or several selected global scales.

To overcome the above problems, we propose a corner detection method using WT and local natural scale for contour images. The significance measure of each candidate is considered at all the possible scales. As natural scale should be the scale that contains most or all the important information, for each candidate, the scale at which the maximum value of the normalized WTMM exists is defined as its “local natural scale”, and the corresponding measure is taken as the significance measure to differentiate the corners from the noise. Local natural scale is explored in the literature, but the goals are different. It is the first time that local natural scale is applied in contour corner detection in a multiscale framework. The inherent smoothing and localization properties of WT make this method effective and accurate. In addition, the technique is fast due to the fast implementations of the dyadic WT computations.

The paper is organized as follows. In Section 2, the proposed algorithm is presented. Section 3 shows simulation results and performance evaluation. The conclusion is given in Section 4.

2. The proposed algorithm – local natural scale based contour corner detection using WT

Corners are defined as high curvature points on a contour. As no strict mathematical definition of the curvature exists in the discrete domain, the performance of corner detection relies on the accuracy of both the curvature estimation and the scale estimation. In other words, a good curvature estimation should be measured in the spatial extent corresponding to its scale. The appropriate smoothing is necessary to remove the quantization error and noise while estimating the curvature and scale.

To estimate the curvature, we select the dyadic WT using the quadratic spline mother wavelet [10] to decompose the orientation function because it satisfies the following necessary conditions and good properties. First, the dyadic WT is shift invariant, which is a necessary condition for feature extraction. Second, the quadratic spline mother wavelet has one vanishing moment, which is a first order differential operator on a smoothed signal. Accordingly, the curvature is approximated when the transformation is applied on the orientation function. Third, the dyadic WT is complete. Thus, it provides the decomposition at a sparse set of appropriate scales, which simplifies the following analysis and computation. Lastly, it has a fast implementation algorithm, which makes the proposed algorithm computationally efficient.

In the proposed algorithm, the preprocessing steps described in [6] are adopted to get the orientation function of the contour image. Then dyadic WT is applied to the
orientation function to estimate the curvature at all possible scales because no scale should be preferred without a priori information [11]. Subsequently, the WTMM are extracted and the points with WTMM are taken as corner candidates.

Since, the “corner” is a relative conception, it depends on the shape and scale considered in the detection. At any specific scale, obtuse candidates will not be considered if there exist acute candidates. Therefore, we normalize the values of the WTMM at each scale. As a result, the curvature estimation is considered in an uniform manner in the whole framework. For different candidates at the same scale, the candidates with acute angles produce large WTMM, while the candidates with obtuse angles have small WTMM. For each candidate along different scales, the value of the normalized WTMM at a certain scale represents the “cornerity” of the candidate and the maximum value of the WTMM shows that the candidate achieves a strong “cornerity” at the corresponding scale. Consequently, the scale at which the maximum value exists should be defined as its “local natural scale” and the corresponding maximum modulus is taken as the significance measure.

The detection process of the proposed method is implemented as follows.

2.1. Step 1. Detect corner candidates

The quadratic spline wavelet \( \psi(t) \) is the first derivative of the cubic spline function \( \xi(t) \), i.e., \( \psi(t) = \xi'(t) \). Consequently, it has one vanishing moment. By denoting

\[
\mathcal{T}_s(t) = \frac{1}{\sqrt{s}} f\left(\frac{-t}{s}\right)
\]

the WT of the orientation function \( \theta \) at scale \( s \) and position \( u \) can be written as follows:

\[
W\theta(u, s) = \theta \ast \mathcal{T}_s(u) = s \frac{d}{du} \theta(\star \mathcal{S}_u)(u),
\]

where \( W\theta \) represents the wavelet transform of the orientation function \( \theta \) and \( \ast \) denotes the convolution. Eq. (2) shows that the WT (with one vanishing moment) of the orientation function is proportional to the derivative of smoothed versions of the orientation function, and therefore, is proportional to the curvature of the boundary. It measures the change of the orientation. Subsequently, the WTMM are extracted and the points with WTMM are taken as corner candidates. Modulus maximum is any point whose absolute value is more than one of its neighborhoods and not less than the other neighborhood [10], i.e.,

\[
\begin{align*}
&|W\theta(u_0, s)| > |W\theta(u_1, s)|: u_1 \text{ is one neighborhood of } u_0, \\
&|W\theta(u_0, s)| \geq |W\theta(u_2, s)|: u_2 \text{ is the other neighborhood of } u_0,
\end{align*}
\]

where \( |W\theta| \) represents the modulus of \( W\theta \). Then, the values of the WTMM are normalized with respect to the maximum value at each scale.

The range of the decomposition scales, \( 2^j \), for the WT is determined by the inherent property of the dyadic WT and the length of the signal \( N \) [10].

\[
1 < 2^j \leq N, \quad j = 1, 2, \ldots, J,
\]

where \( J \) is the maximum level of the WT. According to (4), the decomposition scales of the WT is restricted by the signal length \( N \), which makes the algorithm adaptable for both long contours and short contours.

2.2. Step 2. Detect corners at their respective natural scales, i.e., “local natural scales”

Although dyadic WT is shift invariant, there are still some shift of the local maxima due to the changes of the local properties caused by the scale changes. We adopt a bottom-up tracking for each corner candidate. The tracking is according to the distance criterion. The distance criterion is applicable for both the one-to-one tracking or two-to-one tracking. For an instance, at scale \( i \), there are two local maxima, \( p_1 \) and \( p_2 \), within a range whose positions are \( d_1 \) and \( d_2 \), respectively. At scale \( i + 1 \), there is only one local maximum, \( p_3 \), within the range at the position \( d_3 \) (we assume here, no corresponding local maximum at the outside of the range for these two corner candidates here).

If \( |d_1 - d_3| < |d_2 - d_3| \), \( p_3 \), and \( p_1 \) belong to the same candidate, vice versa.

For each corner candidate, the maximum value among all the normalized WTMM is detected. It is determined as the local natural scale at which the maximum value of the candidate exists, and the corresponding modulus value is taken as the significance measure, i.e.,

\[
M_\gamma(\cdot, 2^j) = \max\{\text{NW}(\cdot, s); s = 2^j, \quad j = 1, 2, \ldots, J\},
\]

where \( \text{NW}(\cdot, s) \) represents the normalized WTMM at scale \( s \) and \( M_\gamma(\cdot, 2^j) \) denotes the significance measure at the natural scale. The scale, \( 2^j \), is the local natural scale for the candidate. The corners are detected at the locations, where the significance measure of the candidates is greater than a predefined threshold.

The corners are detected at different scales due to their different natural scales. The proposed method provides the locations of corners as well as the local natural scale information simultaneously, which is useful for the hierarchical approximation of the original contour.

To illustrate the process, we use the test image of Fig. 7d as an example. Fig. 1a shows the normalized WTMM of the candidates at each scale. For a clear illustration, Fig. 1b shows the normalized WTMM of the corner points at each scale. The length of each segment in Fig. 1 is proportional to the measure of the “cornerity” of the pixel at the corresponding scale. We can see that at a certain scale, the acute candidates (or corners) give large values, while the normalized WTMM of different candidates (or corners) change differently along the scales. Fig. 2 shows the natural scales of the
corners as the scales with the largest normalized WTMM. Referred to Fig. 7d, it is found that detected natural scales of corners agree with their support regions, i.e., the points with larger natural scales have larger support regions, and vice versa.

3. Simulation results and performance evaluation

3.1. Subjective evaluation

The results of the proposed method and the existing methods are shown in Figs. 3–6. Fig. 7 shows another set of simulation results for the proposed method. The fixed threshold of the proposed method, which is set empirically, is 0.2 for all the simulations shown in this paper. The lengths of the curves in Figs. 3–6 are 45, 60, 102, and 120, respectively. The lengths of the curves in Figs. 7a–d are 563, 854, 872, and 1104, respectively.

From the simulation results, we see that the proposed method provides satisfactory performance for both long contours and short contours. This feature makes it more suitable for practical applications.

To give an intuitive comparison, Figs. 3–6 also show the results of Teh–Chin’s method [3], Rattarangsi–Chin’s method [4], and Quddus-Gabbouj’s method [5] on the commonly used test images. Teh–Chin’s method [3] is based on the support region determination. This method is classical but not so effective compared to the other three methods. It detects more points, including some insignificant points. Rattarangsi–Chin’s method [4] is based on scale-space analysis. The results are good in this paper. However, this method only use the location
information in the scale-space analysis. In the original paper [4], it shows some false detection on other test images. Recently, Quddus and Gabbouj propose a robust method in [5], which is based on WT. This approach requires to compute singular value decomposition (SVD) of the dyadic WT for the orientation profile of

Fig. 2. The natural scale of each corner point for Fig. 7(d) as shown by ‘×’. The vertical axis represents the logarithm of the scale with base 2, while the horizontal axis is the index of the contour pixel.

Fig. 3. The results of the “figure-8” curve. The corners are indicated by ■ and connected into polygon. (a) the proposed method, (b) Teh–Chin [3], (c) Rattarangsi–Chin [4], (d) Quddus–Gabbouj [5].
Fig. 4. The results of the “chromosome” curve. The corners are indicated by ‘■’ and connected into polygon. (a) the proposed method, (b) Teh–Chin [3], (c) Rattarangsi–Chin [4], (d) Quddus–Gabbouj [5].

Fig. 5. The results of the “semicir” curve. The corners are indicated by ‘■’ and connected into polygon. (a) the proposed method, (b) Teh–Chin [3], (c) Rattarangsi–Chin [4], (d) Quddus–Gabbouj [5].
the contour to estimate the global natural scales. It has been applied to test images used in Fig. 7 and obtains similar satisfactory results in [5] as the proposed method does. However, in a few cases the stop criterion for the selection of natural scales does not work, e.g., for “figure-8” curve. Moreover, there is some computational overhead to compute the SVD. The length of “figure-8” curve is 45 pixels and it is relatively short, which might be the possible reason that Quddus–Gabbouj’s method fails.

3.2. Objective comparison using Rosin’s method

There are a few objective evaluation methods for corner detection. Rosin’s evaluation method [12] has two advantages compared with other existing methods. First, it avoids the ground-truth definition. As no strictly mathematical definition of corners exists, it is difficult to define the ground-truth for test images. Rosin’s method takes the optimal results of the polygonal approximation as the reference so that no ground-truth is needed in the evaluation. Second, it formulates the measurements of both the Efficiency (compression ratio) and the Fidelity (error measurement) in one formula. Generally speaking, there is a trade-off between the efficiency and the fidelity for each algorithm. If we increase the number of corners detected, the efficiency is decreased, while the fidelity is increased, vice versa. Consequently, it is difficult to tell which result is good from the two measurements. Rosin’s evaluation considers these two measurements simultaneously. This unique measurement makes it possible to compare different methods that detect different numbers of corners.

The fidelity is computed as follows:

\[
\text{Fidelity} = \frac{E_{\text{opt}}}{E_{\text{appr}}} \times 100, \quad (6)
\]

where \(E_{\text{appr}}\) is the error of the polygonal approximation built with the detected corners when compared to the original contour, and \(E_{\text{opt}}\) is the error incurred by the optimal algorithm with the same number of lines as used in the evaluated algorithm.

The efficiency is computed as follows:

\[
\text{Efficiency} = \frac{N_{\text{opt}}}{N_{\text{appr}}} \times 100, \quad (7)
\]

where \(N_{\text{appr}}\) is the number of corners detected by the evaluated method, and \(N_{\text{opt}}\) represents the number of lines that the optimal algorithm would require to produce the same error as the evaluated method does.

The optimal solutions are found using dynamic programming [13] run for all necessary values.

The merit of the corner detection is calculated as follows:

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Fig. 6. The results of the “leaf curve. The corners are indicated by ‘’ and connected into polygon. (a) the proposed method, (b) Teh–Chin [3], (c) Rattarangsi–Chin [4], (d) Quddus–Gabbouj [5].
In this paper, we apply the most common ones, \( E_2 \) and \( E_{\infty} \).

\[
E_2 = \sum_{i=1}^{n} e_i^2, \tag{9}
\]

\[
E_{\infty} = \max_{1 \leq i \leq n} e_i. \tag{10}
\]

Here, \( i \) represents the index of the contour pixel and the error, \( e_i \), is the perpendicular distance from a point of the original curve to the approximating polygon constructed by connecting the corner points.

We have shown the detection results in Figs. 3–6 for a particular set of test images that has been used by most of the existing papers [3,4]. The quantitative measurements of the proposed algorithm and the existing methods [3–5] using \( E_2 \) and \( E_{\infty} \) are listed in Table 1. The measurements of the Efficiency, Fidelity, and Merit are denoted as Eff2, Fid2, and Merit2 for using \( E_2 \); Eff\(_{\infty}\), Fid\(_{\infty}\), and Merit\(_{\infty}\) for using \( E_{\infty} \).

From the results, we see that performance of all the selected methods are good. Among them, the proposed method provides better performance in general. Teh–Chin’s method achieves relatively lower efficiency and fidelity, and as a result, lower merit compared to the other three methods. Rattarangsi–Chin’s method [4] obtains better performance measurement than the proposed method for “semicir” curve using \( E_2 \). Even in this case, the performance of the proposed method is quite close to that of the Rattarangsi–Chin’s method. Quddus–Gabbouj’s method [5] gets some better results. But, as we analyzed earlier, it fails for some cases.

4. Conclusions

In this paper, we propose a new contour corner detector based on the WT and local natural scales. First, the curvature is estimated using the dyadic WT. The decomposition scales of the dyadic WT is simply imposed by the contour length. This reasonable decomposition makes the algorithm suitable for both long contours and short contours. Then, the local natural scale for each corner candidate is determined from the estimated curvature. The measure of significance is finally found in the spatial extent corresponding to its local natural scale. Due to the inherent property of the WT, the appropriate smoothing is applied to remove the quantization error and noise while estimating the curvature and local natural scales. This method is also computationally efficient as the dyadic WT is implemented by a fast algorithm. The simulation results show that the proposed method is effective for both long contours and short contours. The objective evaluation also reveals improved performance by the proposed method compared to the existing classical methods. Besides location information of the corners, the natural scale information can be obtained simultaneously, which is useful for the hierarchical approximation of the original contour.

![Fig. 7. Results of the proposed method. The corners are indicated by ‘*’.](image)
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References


Table 1
Quantitative results of the test curves using $E_2$ and $E_{\infty}$, respectively

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<th>Eff$_2$</th>
<th>Fid$_2$</th>
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