Performance Analysis of RAKE Receiver in W-CDMA Downlink

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SUMMARY

In this paper, we theoretically analyze the received signal performance during reception by a RAKE receiver at the terminal in wideband DS-CDMA (W-CDMA: wideband code division multiple access) downlink and evaluate the characteristics by a simulation. This theoretical analysis considers the long code and Walsh code used in the W-CDMA downlink, signal transmission to many users, and transmission beamforming at the base station. We derive an exact solution of the signal-to-interference-plus-noise ratio (SINR) after RAKE reception at the terminal. In this process, the interference signals exhibit correlation between the RAKE fingers of the terminal in the W-CDMA downlink. The simulation verifies the derived SINR equation. In addition, we evaluate the signal characteristics of the RAKE reception at the terminal in a fading environment and discuss the differences from the theoretical values. © 2002 Wiley Periodicals, Inc. Electron Comm Jpn Pt 1, 86(1): 24–37, 2003; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/ecja.10037

Key words: mobile communication; W-CDMA; downlink beamforming; RAKE reception; signal-to-interference-plus-noise ratio.

1. Introduction

The International Mobile Telecommunications-2000 (IMT-2000) system has adopted wideband direct sequence-code division multiple access (DS-CDMA) as one wireless access scheme, and service is slated to begin in Japan [1, 2]. In the early 1990s, the possibility of increasing the capacity by CDMA was suggested [3–5]. During that decade, a great deal of research addressed making CDMA practical. In particular, RAKE reception that separates, receives, and combines multiple delay paths is vital in CDMA and is an essential technology as a fading measure [6–14].

Previously, the analysis of CDMA characteristics often treated the extension codes of multiple users as random codes [6–9]. The performance analysis of RAKE reception was evaluated for an environment that did not have multiple users [10, 11] or treated the interference as uncorrelated Gaussian distribution between the RAKE fingers [12, 13]. Conventional analysis of the characteristics is a useful guide to practical CDMA schemes. However, if we consider improvements and better performance in future W-CDMA, theoretical analysis that is better adapted to implementation is needed.

Specifically, in contrast to no chip synchronization between users in the uplink of W-CDMA, the transmitted signal is chip synchronized on the downlink. In addition, a composite form of the long code and Walsh code is the extension code [1, 2]. Recently, tests [15] based on the W-CDMA reference standard have been conducted. However, theoretical analysis based on the reference standard is lacking, and the theoretical values that should be the indices in the tests are required.

Recently, an important research topic has become the structure of transmission beamforming in the base station from the perspective of a capacity increase in a W-CDMA downlink. However, no papers have analyzed the signal characteristics in RAKE reception at the terminal when downlink beamforming was performed at the base station. An understanding of the received signal characteristics at the terminal is important when evaluating the downlink beamforming in W-CDMA.

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In light of this situation, in this paper, we focus on the W-CDMA downlink and evaluate the received signal characteristics in RAKE reception at the terminal. Specifically, based on a chip synchronized environment, we assume that the base station uses the long code and the Walsh code, and the downlink transmits signals to multiple users. In addition, the base station uses multiple antennas and also handles transmission beamforming. In the theoretical analysis, we derive the signal-to-interference-plus-noise ratio (SINR) of the output signal in this kind of environment. Correlation is evident in the interference between the outputs of the RAKE fingers of the terminal in the W-CDMA downlink. In the performance analysis, we confirm that the derived theoretical equation agrees with the simulated values and verify the errors between the practical RAKE reception equation and the theoretical values.

In this paper, Section 2 describes the system model of the W-CDMA downlink. Section 3 describes the received signal at the terminal. Section 4 explains the SINR characteristics after RAKE reception at the terminal. In Section 5, we confirm the derived theoretical equation as well as evaluate the characteristics of the received signal in practical RAKE reception.

2. W-CDMA Downlink Model

2.1. Base station structure

Figure 1 shows the structure of the base station transmitter. A signal is transmitted to \( K \) users by the base station. The transmission spread signal \( s_k(t) = c_k(t)d(t) \) is prepared for the \( k \)-th user, for \( k = 1, \ldots, K \), where \( c_k(t) \) is the spreading code and \( d(t) \) is the information signal. Code \( c_k(t) \) changes every chip time \( T_c \) and \( d(t) \) every symbol time \( T_d \). Let \( E[\cdot] \) be the average over time \( t \), then \( E[|c_k(t)|^2] = E[|d(t)|^2] = 1 \). At the base station, the spread signal is multiplied by \( \sqrt{P_k} \) to convert the signal power. Furthermore, after the power converted signal is copied to \( M \) antennas, and each branch is multiplied by a weight, the signals are transmitted from the antennas.

Let the weight coefficient at antenna \( m \) of user \( k \) be \( w_{km} \) and the weight of user \( k \) be \( w_k = [w_{k1}, \ldots, w_{kM}]^T \), then the transmission signal \( s(t) \) of all of the users from the antennas is expressed by

\[
s(t) = \sum_{k=1}^{K} \sqrt{P_k} c_k(t)d_k(t)w_k^* \tag{1}
\]

where * denotes the complex conjugate. The transmission weights may be assumed to be either fixed weights or adaptive weights. However, the transmission weights do not change during the time needed for signal combining in one

RAKE reception at the terminal. In Eq. (1), \( M = 1 \) is equivalent to a single, omnidirectional antenna. Thus, the base station structure described above includes both transmission beamforming and no beamforming.

2.2. Transmission signal

The transmission signals are chip synchronized and symbol synchronized between the users. Let the processing gain be defined as \( G = T_d/T_c \), then the information signal \( d_k(t) \) of user \( k \) is the value of the \( q \)-th symbol at \( t = \lfloor qG \rfloor T_c, \lfloor qG \rfloor T_c + T_c, \ldots, \lfloor qG \rfloor T_c + G T_c \). The extension signal \( c_k(t) \) of user \( k \) is constructed by multiplying the Walsh code \( c_W(t) \) having the period \( T_d \) and the common long code \( c_L(t) \) for the users. \( c_k(t) \) is given by

\[
c_k(t) = c_W(t)c_L(t) \tag{2}
\]

where a different Walsh code \( c_W(t) \) is used for each user. Thus, the property given by the following equation holds for the Walsh code:

\[
\frac{1}{G} \sum_{p=1}^{G} c_{Wk_1}(pT_c)c_{Wk_2}(pT_c) = \begin{cases} 
1 & k_1 = k_2 \\
0 & k_1 \neq k_2 
\end{cases}
\]
The long code \( c_L(t) \) is shared by the users at one base station. Different long codes are used at different base stations. The long code is set to a random value from \((1, j, -1, -j)\) at each time. Thus, it has the property given by the equation

\[
E_p [c_L(pT_c) \cdot c_L((p + \Delta p)T_c)] = \begin{cases} 
1 & \Delta p = 0 \\
0 & \Delta p \neq 0 
\end{cases}
\]

where \( E_p[\cdot] \) is the average over \( p \).

### 2.3. Downlink propagation performance

As shown in Fig. 2, the downlink propagation paths between the \( M \) antennas of the base station and user \( k \) are represented by the \( L \)-path impulse response \( h_k(t) \) given by

\[
h_k(t) = \sum_{l=1}^{L} a_{k,l} \delta(t - (l - 1)T_c) 
\]

where \( a_{k,l} \) is the propagation vector of path \( l \) at user \( k \), and \( a_{k,l} \) represents the propagation coefficient at antenna \( m \) of path \( l \) to user \( k \).

### 2.4. The terminal

The terminal receives signals with a single antenna and an \( R \)-finger RAKE receiver. Figure 3 shows the structure of the RAKE receiver. In this case, the first path \((l = 1)\) is received from the base station by the first finger of the RAKE receiver. The \( r \)-th finger in RAKE reception (referred to as RAKE finger \( r \)) receives the \( r \)-th delay path. Each RAKE finger uses the extension code \( c_r(t) \) of the desired signal to despread, and the despreaded output is combined between the fingers.

Fig. 2. Propagation model between the base station and a terminal.

Fig. 3. Structure of the RAKE receiver at the terminal.

### 3. Downlink Signal Characteristics

#### 3.1. Received signal at a terminal

Let the received signal at terminal \( k \) (before despread) be \( x_k(t) \), then the received signal at the \( p \)-th sampling point is given by

\[
x_k(pT_c) = \sum_{l=1}^{L} a_{k,l}^T \delta((p - l + 1)T_c) + x_k^{(N)}(pT_c) 
\]

where \( x_k^{(N)}(t) \) is the sum of the interference components and the noise components from the other cells. In the downlink, the signals transmitted from the base station are not divided into the desired signal and the other users' signals and all have the same propagation path. In the \( L \)-path propagation model, the desired signal and the other users' signals are simultaneously included.

Let the despread signal at RAKE finger \( r \) be \( y_{k,r}(t) \), then the \( q \)-th symbol \( y_{k,r}(qT_d) \) is given by

\[
y_{k,r}(qT_d) = \frac{1}{G} \sum_{p=(q-1)G+r}^{qG+r-1} c_{k}^\ast((p - r + 1)T_c) x_k(pT_c) 
\]

where \( c_{k}^\ast((p - r + 1)T_c) \) is the component after the signal of terminal \( i \) included in path \( l \) is despread by RAKE finger \( r \) of terminal \( k \), and is given by

\[
D_{k,i}(i, l) + y_k^{(N)}(qT_d) 
\]
where $\alpha = 0$ when $l < r$, and $\alpha = 1$ when $l \geq r$. The variables $\eta_{k,r}(i,l)$ and $\varphi_{k,r}(i,l)$ represent the code correlation. The codes include a long code. $\eta_{k,r}(i,l)$ and $\varphi_{k,r}(i,l)$ are stochastic variables that are varied by symbol $q$.

### 3.2. The signal component and the interference component

RAKE finger $r$ is synchronized to delay path $l = r$. The other users’ signals in path $r$ and the desired signal of terminal $k$ have an orthogonal relationship. Thus, let $l = r$, then the following equation is satisfied for the code correlation property:

\[
\varphi_{k,r}(i,r) = 0, \quad \eta_{k,r}(i,r) = \begin{cases} 
1 & (i = k) \\
0 & (i \neq k)
\end{cases}
\]

Then, $y_{k,r}(qT_d)$ is given by

\[
y_{k,r}(qT_d) = y_{k,r}^{(S)}(qT_d) + y_{k,r}^{(I)}(qT_d) + y_{k,r}^{(N)}(qT_d)
\]

\[
y_{k,r}^{(S)}(qT_d) = \sqrt{P_k} \left( \mathbf{w}_k^\dagger \mathbf{a}_{k,r} \right) d_k(qT_d)
\]

\[
y_{k,r}^{(I)}(qT_d) = \sum_{i=1}^{K} \sum_{l=1, l \neq r}^{L} \sqrt{P_t} \left( \mathbf{w}_k^\dagger \mathbf{a}_{k,l} \right) D_{k,r}(i,l)
\]

where $y_{k,r}^{(S)}(qT_d)$ denotes the signal component, $y_{k,r}^{(I)}(qT_d)$ denotes the interference component from the home cell, and $y_{k,r}^{(N)}(qT_d)$ denotes the outer-cell-interference-plus-noise components.

### 3.3. Signal and interference powers after RAKE combining

In RAKE combining, the signal of finger $r$ is multiplied by the weight coefficient $g_{k,r}$ and combined. Let the weight be $g_{k,r} = [g_{1,r}, g_{2,r}, \ldots, g_{R,r}]^T$, then after RAKE combining, the signal power $S_k$, the interference power $I_k$, and the sum $N_k$ of the outer-cell-interference-plus-noise power are given by

\[
S_k = E_q \left[ |g_{k,r}^{\dagger} y_{k,r}^{(S)}(qT_d)|^2 \right] = g_{k,r}^{\dagger} R_S g_{k,r}
\]

\[
I_k = E_q \left[ |g_{k,r}^{\dagger} y_{k,r}^{(I)}(qT_d)|^2 \right] = g_{k,r}^{\dagger} R_I g_{k,r}
\]

\[
N_k = E_q \left[ |g_{k,r}^{\dagger} y_{k,r}^{(N)}(qT_d)|^2 \right] = g_{k,r}^{\dagger} R_N g_{k,r}
\]

respectively, where $E_q[\cdot]$ is the average over $q$. The $(r_1, r_2)$ components of matrices $R_S, R_I,$ and $R_N$ are the cross correlations of the desired signal, interference from the home cell, and the outer-cell-interference-plus-noise components between the despread signals at RAKE fingers $r_1$ and $r_2$, such that

\[
[R_S]_{r_1, r_2} = P_k (\mathbf{w}_k^\dagger \mathbf{a}_{k,r_1})(\mathbf{w}_k^\dagger \mathbf{a}_{k,r_2})^*
\]

\[
[R_I]_{r_1, r_2} = E_q \left[ y_{k,r_1}(qT_d)^{(I)} y_{k,r_2}(qT_d)^{(I)*} \right] \]

\[
[R_N]_{r_1, r_2} = E_q \left[ y_{k,r_1}(qT_d)^{(N)} y_{k,r_2}(qT_d)^{(N)*} \right]
\]

The properties of matrix $R_I$ will be discussed in the next section.

### 4. SINR Characteristics of the Received Signal

In this section, we theoretically analyze the SINR properties of the signal received at the terminal. First, we define the general correlation output needed to analyze matrix $R_I$ and define its properties. Next, we derive the equation for calculating matrix $R_I$ and use this result to formulate the output SINR for a single finger and after RAKE combining.
4.1. Definition of the general correlation output and its properties

Definition

The general correlation output for code \( c_{ki}(pT_c) \) of code \( c_{ci}((p + \Delta p)T_c) \) is defined by

\[
\xi_i(a, b, \Delta p) = \frac{1}{G} \sum_{p = qG + a}^{qG + b} c_{k}^*(pT_c)c_{i}((p + \Delta p)T_c)
\]

where \([qG + a, aG + b]\), for \( q = 1, 2, \ldots \), is the averaging interval. The extension code includes a long code, and \( \xi_i(a, b, \Delta p) \) is a stochastic variable that varies with \( q \).

The following proposition holds for a general correlation output.

Proposition 1

\[
E_q[\xi_i(a_1, b_1, \Delta p_1)\xi_i(a_2, b_2, \Delta p_2)] = \begin{cases} 
0 & \text{if } \Delta p_1 \neq \Delta p_2 \\
\frac{(b - a + 1)}{G^2} & \text{if } \Delta p_1 = \Delta p_2 \neq 0 \\
\eta_1\eta_2/G^2 & \text{if } \Delta p_1 = \Delta p_2 = 0
\end{cases}
\]

where \( \eta_1, \eta_2, a, \) and \( b \) are given by

\[
\eta_1 = \sum_{p=1}^{b_1} c_{Wk}(pT_c)c_{wi}^*(pT_c) \\
\eta_2 = \sum_{p=2}^{b_2} c_{Wk}(pT_c)c_{wi}^*(pT_c) \\
a = \max\{a_1, a_2\}, \quad b = \min\{b_1, b_2\}
\]

Proof

\[
E_q[\xi_i(a_1, b_1, \Delta p_1)\xi_i(a_2, b_2, \Delta p_2)] = \frac{1}{G^2} \sum_{p_1=1}^{b_1} \sum_{p_2=1}^{b_2} E_q[c_k((qG + p_1)T_c)] \\
\times c_i^*((qG + p_1 + \Delta p_1)T_c) \\
\times c_k^*((qG + p_2 + \Delta p_2)T_c)]
\]

\[
\zeta_W = c_{W}(p_1T_c)c_{W}^*((p_1 + \Delta p_1)T_c) \\
\times c_{Wk}(p_2T_c)c_{Wk}^*(p_2 + \Delta p_2)T_c) \\
\zeta_L = E_q[c_L((qG + p_1)T_c)c_L((qG + p_1 + \Delta p_1)T_c) \\
\times c_L^*((qG + p_2)T_c)c_L((qG + p_2 + \Delta p_2)T_c)]
\]

(i) In case of \( \Delta p_1 \neq \Delta p_2 \)

\[
p_1 = p_2 \quad \text{and} \quad p_1 + \Delta p_1 = p_2 + \Delta p_2 \quad \text{are not satisfied at the same time. In this case, one of the long code terms } c_L(\cdot) \text{ in } \zeta_L \text{ is always independent of the other terms. Thus, } \zeta_L = 0 \text{ and } E_q[\zeta_i(a_1, b_1, \Delta p_1)\zeta_i(a_2, b_2, \Delta p_2)] = 0.
\]

(ii) In case of \( \Delta p_1 = \Delta p_2 \neq 0 \)

\[
p_1 + \Delta p_1 = p_2 + \Delta p_2 \quad \text{holds only when } p_1 = p_2, \quad \zeta_L = 1. \text{ If } p_1 \neq p_2, \text{ one of the long code terms } c_L(\cdot) \text{ in } \zeta_L \text{ is always independent of the other terms, then } \zeta_L = 0. \text{ As a result,}
\]

\[
E_q[\xi_i(a_1, b_1, \Delta p_1)\xi_i(a_2, b_2, \Delta p_2)]
\]

\[
= \frac{1}{G^2} \sum_{p_1=1}^{\min\{b_1, b_2\}} |c_{Wk}(p_1T_c)|^2 \\
\times |c_{Wk}^*((p_1 + \Delta p_1)T_c)|^2 \\
= \min\{b_1, b_2\} - \max\{a_1, a_2\} + 1
\]

(iii) In case of \( \Delta p_1 = \Delta p_2 \)

\[
p_1 = p_2 \text{ are unrelated and } \zeta_L = 1. \text{ Thus, we have } \zeta_L = 1 \text{ under arbitrary values of } p_1 \text{ and } p_2.
\]

\[
E_q[\xi_i(a_1, b_1, \Delta p_1)\xi_i(a_2, b_2, \Delta p_2)]
\]

\[
= \frac{1}{G^2} \sum_{p_1=1}^{b_1} \sum_{p_2=1}^{b_2} \zeta_W \\
= \frac{1}{G^2} \left( \sum_{p_1=1}^{b_1} c_{Wk}(p_1T_c)c_{Wk}^*(p_1T_c) \right)^* \quad (10)
\]

The proposition holds.

This proposition considers the signal of user \( i \) that is \( c_{ki}((p + \Delta p)T_c) \) delayed by \( \Delta p \) chips from the synchronization point. This proposition is easy to understand if \( \xi_i(a, b, \Delta p) \) is viewed as a component of user \( i \) after the correlation output based on the signal of user \( k \) (or after despreading). Case (i) exhibits cross correlation after the output (despreading) of the correlation of two signals with different delays, and becomes uncorrelated. Case (ii) exhibits autocorrelation after the correlation output (despreading) of a signal having a different delay than the synchronization point. Case (iii) is the autocorrelation after the correlation output (despreading) of the signal having the same delay as the synchronization point. The correlation characteristics are determined by the characteristics of the long code in cases (i) and (ii). In (iii), the correlation characteristics are determined by the properties of the
Walsh code. That is, the correlation output of a signal that has a different delay than the synchronization point of the RAKE finger is set according to the properties of the long code. The correlation output of a signal that has the same delay as the synchronization point is set according to the properties of the Walsh code. In particular, when \( a = 1, b = G, \) and \( k \neq i \) in (iii), \( \eta_1 = \eta_2 = 0. \)

4.2. Cross correlation of the RAKE finger outputs

Using Proposition 1, we can derive the following proposition related to matrix \( R_f \).

Proposition 2

\[
[R_f]_{r_1,r_2} = \sum_{i=1}^{K} \sum_{(l_1,l_2) \in U_i} \frac{P_i}{G} \left( w^*_{i} a_{k,l_1} \right) \left( w^*_{i} a_{k,l_2} \right)^* \]

\[
U_i = \{ l_1, l_2 = l_1 - (r_1 - r_2) \}
\]

|\[ l_1 \neq r_1, 1 \leq l_1, l_2 \leq L \]|

Proof

Using \( D_{k,r_1}(i,l_1)D_{k,r_2}(i,l_2)^* \) of Eq. (5), the correlation element in \( \varphi_d \) is considered to be a combination of the terms \( \varphi_{k,r_1}(i,l_1), \varphi_{k,r_2}(i,l_2)^* \), \( \eta_{k,r_1}(i,l_1), \eta_{k,r_2}(i,l_2)^* \), \( \Phi_{k,r_1}(i,l_1), \Phi_{k,r_2}(i,l_2)^* \), \( \eta_{k,r_2}(i,l_2), \eta_{k,r_1}(i,l_1)^* \), and \( \eta_{k,r_1}(i,l_1), \eta_{k,r_2}(i,l_2)^* \). From Proposition 1 and Eqs. (12) and (13), the average over \( q \) of these combinations does not become zero when \( r_1 - l_1 = r_2 - l_2 \). All of the other terms have a zero average. Thus, \( \varphi_d \) is derived.

\[
\varphi_d = \frac{1}{G}
\]

holds. If substituted into Eq. (11), we obtain

\[
[R_f]_{r_1,r_2} = \sum_{i=1}^{K} \sum_{(l_1,l_2) \in U_i} \frac{P_i}{G} \left( w^*_{i} a_{k,l_1} \right) \left( w^*_{i} a_{k,l_2} \right)^*
\]

Thus, the proposition holds.

In particular, the following equation holds for a single antenna \( (M = 1, w_i = 1) \).

\[
[R_f]_{r_1,r_2} = \sum_{i=1}^{K} \sum_{(l_1,l_2) \in U_i} \frac{P_i}{G} \left( w^*_{i} a_{k,l_1} \right) \left( w^*_{i} a_{k,l_2} \right)^*
\]

4.3. The SINR of RAKE finger \( r \)

The SINR after despreading at RAKE finger \( r \) is derived. The interference power after despreading at RAKE finger \( r \) is the \((r,r)\) component of correlation matrix \( R_f \) and is given by

\[
[R_f]_{r,r} = \sum_{i=1}^{K} \sum_{l=1,l \neq r}^{L} \frac{P_i}{G} \left( w^*_{i} a_{k,l} \right)^2
\]

The signal components of the other users do not become interference due to the orthogonality of the Walsh code for path \( l = r \). The powers of the other users’ components uniformly decrease to \( 1/G \) before despreading due to the properties of the long code for the path \( l \neq r \).

Thus, the signal power \( S_{k,r} \), the interference and noise power \( I_{k,r} + N_{k,r} \), and SINR \( \Gamma_{k,r} \) at RAKE finger \( r \) of terminal \( k \) are expressed by

\[
\varphi_{k,r_1}(i,l_1), \varphi_{k,r_2}(i,l_2)^*, \eta_{k,r_1}(i,l_1), \eta_{k,r_2}(i,l_2)^*, \Phi_{k,r_1}(i,l_1), \Phi_{k,r_2}(i,l_2)^*, \eta_{k,r_2}(i,l_2), \eta_{k,r_1}(i,l_1)^* \]

$S_{k,r} = P_k |\mathbf{w}_k^i a_{k,r}|^2$

$I_{k,r} + N_{k,r}$

$= \frac{1}{G} \sum_{i=1}^{K} \sum_{l=1, l \neq r}^{L} P_i |\mathbf{w}_k^i a_{k,l}|^2 + \frac{P_N}{G}$

$\Gamma_{k,r} = \frac{GP_k |\mathbf{w}_k^i a_{k,r}|^2}{\sum_{i=1}^{K} \sum_{l=1, l \neq r}^{L} P_i |\mathbf{w}_k^i a_{k,l}|^2 + P_N}$

\( P_N \) represents the outer-cell-interference-plus-noise power before despreading at terminal \( k \). Appendix 1 presents the analysis of the outer-cell-interference-and-noise.

### 4.4. SINR after RAKE combining

Matrix \( \mathbf{R}_l \) has nonzero components even for off-diagonal elements. This indicates correlation in the interference after despreading between the RAKE fingers. In Proposition 2, the contribution to the correlation component \( \phi_{D} \) was a combination of paths \( l_1 \) and \( l_2 \) that satisfy \( r_1 - l_1 = r_2 - l_2 \). This is equivalent to the situation in which RAKE fingers \( r_1 \) and \( r_2 \) receive paths \( l_1 \) and \( l_2 \) that have equal timing offsets from the synchronization point. For example, the interference component of the delay path \( l = 2 \) despreaded by RAKE finger \( r = 1 \) and the interference component of the delay path \( l = 3 \) despreaded by RAKE finger \( r = 2 \) have the same component except the propagation coefficient differences. This leads the interference correlation between RAKE fingers. The same relationship also holds for interference component of path \( l = 3 \) at RAKE finger \( r = 1 \) and interference component of path \( l = 4 \) at RAKE finger \( r = 2 \). The cross correlation of the total interference between the RAKE fingers is represented as the total correlation for individual paths.

This phenomenon holds even when there are interference components from the other cells. However, when the interference for the other cells is constructed from many transmitting stations, the randomness of the phases of many delay paths acts to decrease the cross correlations to each other. Thus, when there are many interfering transmission stations, \( \mathbf{R}_n \) approaches a scalar multiple of the unit matrix, and correlation between the RAKE fingers is suppressed (see Appendix 1).

Earlier papers often assumed the Gaussian interference, where the correlation between the RAKE fingers was zero \([8, 9, 12]\). These results did not consider the correlation between the RAKE fingers. To explain this precisely, interference signals having Gaussian distributions and no correlation between the RAKE fingers are different problems.

For example, let us consider the interference signal to have a correlated multidimensional Gaussian distribution \([18]\). Then, even if the interference signal approaches a Gaussian distribution, the correlation between the RAKE fingers does not become zero. As shown in this paper, the interference signals are clearly correlated between the RAKE fingers. Note that Proposition 2 also holds when the interference signal does not have a Gaussian distribution.

In an environment having cross correlation, the SINR \( \Gamma_k \) after RAKE combining by the weight \( g_{RK} \) is given by

$$\Gamma_k = \frac{g_{RK}^i \mathbf{R}_S g_{RK}}{g_{RK}^i \mathbf{R}_I g_{RK} + g_{RK}^i \mathbf{R}_N g_{RK}}$$

When maximum ratio combining RAKE reception is used, the weight \( g_{MRCK} \) is given by

$$g_{MRCK} = \left[ \text{diag}[\mathbf{R}_I + \mathbf{R}_N] \right]^{-1} \mathbf{v}$$

(see Appendix 2) where \( \text{diag}[\cdot] \) denotes the matrix where all of the matrix components other than the diagonal components were set to 0. When the RAKE fingers are correlated, the output SINR of the maximum ratio combining RAKE reception is not the simple sum of the SINRs of the RAKE fingers.

In minimum mean square error (MMSE) combining RAKE reception \([16]\), the weight is given by

$$g_{MMSEK} = [\mathbf{R}_I + \mathbf{R}_N]^{-1} \mathbf{v}$$

### 5. Performance Evaluation

The theoretical equation of the SINR derived earlier is verified through a simulation. We proceed by evaluating the output SINR performance for a practical RAKE receiver and discuss the differences from the theoretical values.

#### 5.1. Simulation conditions

Table 1 shows the simulation parameters. Figure 4 shows the frame format for the performance evaluation. One frame is 10 ms and has 15 slots. One slot has 40 symbols, and is constructed from \( q_{\text{pilot}} \) QPSK modulation pilot symbols and \( q_{\text{data}} = 40 - q_{\text{pilot}} \) QPSK modulation data symbols. Each symbol uses an individual Walsh code for each user and the common long code in the cell and is spread to 64 chips. Thus, the extension gain is \( G = 64 \). The transmitted signal of each user has chip and signal synchronization.

The base station uses \( M \) element antennas separated by one wavelength as shown in Fig. 5 and transmits the
signal to $K$ users in the cell. Terminal $i$ is positioned in the direction $\theta_i = 2 \pi t_i / 100 \{ t_i = (47t_{i-1} + 1) \mod 101, \ t_1 = 100 \}$. The transmission weight for user $i$ is given by the following equation as the response vector at angle $\theta_i$:

$$w_i = u(\theta_i)$$

$$u(\theta) = \left[ e^{j2\pi r_1P(\theta)/\lambda}, \ldots, e^{j2\pi r_M P(\theta)/\lambda} \right]^T$$

(19)

where $u(\theta)$ denotes the response vector in the $\theta$ direction, $r_m = [r_{mx}, r_{my}]^T$ denotes the position vector of antenna $m$, and $P(\theta) = [\cos \theta, \sin \theta]^T$ denotes the direction vector in the $\theta$ direction. The transmission weight $w_i$ is fixed, and fixed beamforming is addressed in this performance evaluation.

The signal powers of all of the users transmitted from the base station are set to 1, that is, $P_i |w_i|^2 = 1$ and $P_i = 1/M$. The propagation path between the base station and terminal $i$ is a flat average power 4-path Rayleigh channel. Path $l$ has the arrival direction $\theta_i + (2.5 - l) \Delta \theta$ at each base station. $a_{ij}$ is given by

$$a_{ij} = \rho_{ij} \cdot u(\theta_i + (2.5 - l) \Delta \theta)$$

where $\rho_{ij}$ is the propagation coefficient of path $l$ and varies in response to the motion of the terminal. We assume a multipath environment in which the element waves that construct the arrival angles of each element path are distributed uniformly in all directions. $\rho_{ij}$ has Rayleigh distribution [17] with the maximum Doppler frequency $f_{\text{do}}$, where each path has independent distribution.

The subject of the performance evaluation is the RAKE receiver of terminal $k = 1$. The terminal uses the RAKE combining weight $g_{\text{RRK}}$ and 4-finger RAKE reception is performed. In the simulation, the combining weight $\hat{g}_{\text{MRCR}}$ of maximum ratio combining RAKE reception and the combining weight $\hat{g}_{\text{MMSE}}$ of MMSE combining RAKE reception are calculated from

$$\hat{g}_{\text{MRCR}} = \text{diag} \left[ \hat{\Phi} - \hat{\psi} \hat{\psi}^T \right]^{-1} \hat{\psi}$$

(20)

$$\hat{g}_{\text{MMSE}} = \hat{\Phi}^{-1} \hat{\psi}$$

(21)

$$\hat{\Phi} = \frac{1}{q_0} \sum_q y(qT_d) y(qT_d)^\dagger$$

(22)

$$\hat{\psi} = \frac{1}{q_0} \sum_q y(qT_d) d_k(qT_d)^*$$

(23)

$$y = [y_{1,1}(qT_d), \ldots, y_{1,R}(qT_d)]^T$$

where $\hat{\Phi}$ is the correlation matrix of the RAKE finger output, $\hat{\psi}$ is the correlation vector, and $q_0$ is the number of pilot symbols used in the calculation. The cumulative sums of Eqs. (22) and (23) represent the cumulative sums for pilot symbol $q$.

In the performance evaluation, there is no outer cell interference. Let the noise power before despreading at the terminal be $P_N$, and the desired signal-to-noise ratio (SNR) after despreading be $G_P |w_1|^2 / P_N = 25 \text{ dB}$. The simulation evaluates the SINR of the RAKE finger output or the RAKE combining output signal $z(qT_d)$. The SINR is measured based on the equation

$$\text{SINR} = \frac{|v_a|^2}{I_0 - |v_a|^2}$$

(24)

$$I_0 = \frac{1}{q_{\text{cal}}} \sum_{q=q_{\text{start}}}^{q=q_{\text{cal}}-1} z(qT_d) z(qT_d)^*$$

(25)

$$v_a = \frac{1}{q_{\text{cal}}} \sum_{q=q_{\text{start}}}^{q=q_{\text{cal}}-1} z(qT_d) d_k(qT_d)^*$$

(26)
where \( q_{\text{start}} \) denotes the SINR measurement start symbol and \( q_{\text{cal}} \) denotes the number of symbols for the SINR measurement.

### 5.2. Verification of the theoretical SINR equation

A simulation environment capable of ideal RAKE combining was set up to verify the theoretical SINR equation. In the simulation, all of the symbols in a slot were pilot symbols. By averaging the pilot symbols in three frames (total of 45 slots), RAKE combining and the SINR measurements were performed. Here, \( q_{\text{pilot}} = 40 \), \( q_{\text{start}} = 1 \), and \( q_0 = q_{\text{cal}} = 40 \times 45 \). The terminal was assumed to be stationary during the SINR measurement, and the fading did not change. When one SINR measurement ended (referred to as one point), the 1/50-period fading state changed, and the SINR was measured again in a stationary environment. Figure 6 is an example in which the SINR at each RAKE finger is evaluated by a simulation and by the theoretical equation (15) based on \( M = 2 \), \( K = 20 \), and \( \Delta \theta = 0.01 \pi \).

Figure 7 shows the calculations of the SINR after RAKE combining obtained by the simulation and the theoretical equation (16). Figure 7 also shows the approximation of the following equation where the output SINR of the maximum ratio combining RAKE reception is the simple sum of the SINRs of the RAKE fingers.

\[
\Gamma_k = \sum_{r=1}^{R} \Gamma_{k,r} \tag{27}
\]

In both figures, the simulated and theoretical values almost entirely agree. Although small errors developed in a portion of the values, these errors were caused by SINR measurement errors in the simulation. From the above results, the derived theoretical equation was accurate, which could be verified in the simulation.

In previous theoretical discussion, correlation between the interference appeared in the RAKE finger output. The simulation results match the theoretical values. Based on this, the correlation in the interference could be verified. If the approximation given by Eq. (27) is used, differences arise between the simulated and approximate values. The interference correlation between the RAKE fingers in the exact solution [Eq. (16)] varies over time in accordance with the propagation environment. Therefore, the variance in the SINR is greater in the exact solution [Eq. (16)] than in the approximation [Eq. (27)] because the exact solution includes interference correlation.

In Fig. 6, the MMSE combining RAKE receiver achieves better signal quality than the maximum ratio combining RAKE receiver. Using this interference correlation between the RAKE fingers can improve the signal quality in MMSE combining RAKE reception.

### 5.3. Performance evaluation of practical RAKE reception

RAKE reception having a practical configuration is evaluated. Here, the number of pilot symbols is \( q_{\text{pilot}} = 4 \) and \( q_{\text{data}} = 36 \). RAKE combining is performed in slot units. The weight determination for the \( N \)-th slot uses the pilot symbols totaling \( N_{\text{end}} - N_{\text{start}} + 1 \) slots in slots \( N + N_{\text{start}} \) to \( N + N_{\text{end}} \) shown in Table 2. When \( N + N_{\text{start}} \leq 0 \) or \( N + N_{\text{end}} \leq 0 \), the weight is determined considering the slot number. The simulation results match the theoretical values.
> 0, the pilot symbols in the slots in the previous frame or the following frame are used. The symbols in the $N$-th slot are stored temporarily in a buffer until the weights are determined, and the signal is combined after the weight determination. Consequently, a delay equal to the weight calculation time is generated during RAKE combining.

Based on $M = 1$, $K = 30$, and a maximum Doppler frequency of $f_d = 1, 100,$ and $200$ Hz, Figs. 8 and 9 show the calculations of the average SINR of maximum ratio combining RAKE reception and the average bit error rate (BER) as functions of the number of slots in weight determination. Similarly, Figs. 10 and 11 are the calculations of the relationships of the average SINR and the average BER of the MMSE combining RAKE receiver. The average SINR and average BER are the averages of the SINR ($q_{\text{start}} = 40n + 1$, $q_{\text{cal}} = 40$) measured at each slot and BER measured at each slot over 3000 frames. The theoretical value of the average BER was derived from the theoretical SINR equation and BER equation [18] for QPSK when the interference had a Gaussian distribution.

For a low Doppler frequency in all types of RAKE reception, the signal quality improves as the number of weight calculation slots becomes large. This is because the interference component is suppressed as the number of calculated pilot symbols $q_0$ increases and accurate weight determination becomes possible. On the other hand, for a high Doppler frequency, the signal quality degrades when...
the number of weight calculation slots exceeds a particular value. The reason is as follows: when a large number of calculated pilot symbols $q_0$ is used, temporal phase changes in the desired signal due to fading become a problem and the weight accuracy degrades. Thus, the appropriate number of calculated pilot symbols $q_0$ changes with the Doppler frequency.

In the maximum ratio combining RAKE reception in Fig. 8, if the appropriate number of pilot symbols $q_0$ is set, the degradation of the average SINR is kept to about 1 dB from the theoretical value even when $f_d = 200$ Hz. In maximum ratio combining RAKE reception, the errors with respect to the theoretical values are small even for a high Doppler frequency, and the communication quality is stable. In the MMSE combining RAKE receiver in Fig. 10, the signal quality degrades substantially as the Doppler frequency increases. This is considered to occur because accuracy is required in all of the elements in the correlation matrix $\hat{\Phi}$ in MMSE combining, and weight errors easily arise compared to maximum ratio combining RAKE reception that requires only the diagonal components. Thus, the MMSE combining RAKE receiver easily changes the signal quality by the number of weight calculation symbols and the Doppler frequency. For the maximum Doppler frequency $f_d = 200$ Hz, the average SINR degrades about 3 dB from the theoretical value.

Table 3 shows the average BER and average SINR of the maximum ratio combining RAKE reception calculated from the exact SINR solution (16) and the approximate equation (27) when $M = 1$ and $K = 30$. The exact solution [Eq. (16)] of the average SINR is larger than the approximation [Eq. (27)]. Almost no difference is seen between the two solutions for the average BER. The reason is the changes in the instantaneous SINR are larger for the exact solution [Eq. (16)] than the approximation (27), and the difference between the two becomes smaller in the average BER evaluation where errors in the small instantaneous SINRs are a problem. In the average BER evaluation, the evaluation error is relatively small even when using the conventional approximation [Eq. (27)].

5.4. The relationship between the number of antennas and the SINR

This section will evaluate the performance when the base station has multiple antennas. Figure 12 shows the average SINR after RAKE combining for terminal $k = 1$ when the base station uses $M = 1, 2$, and 3 antennas based on $f_d = 1$ Hz and transmits signals to $K$ users. As seen, when the base station uses multiple antennas, the output SINR

<table>
<thead>
<tr>
<th>M</th>
<th>K</th>
<th>$\Delta\theta$</th>
<th>$q_0$</th>
<th>Exact expression (Eq. (16))</th>
<th>Approximation (Eq. (27))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>0.01$\pi$</td>
<td>32</td>
<td>6.36 dB</td>
<td>5.82 dB</td>
</tr>
<tr>
<td>Average BER</td>
<td>0.0319</td>
<td>0.0309</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Average SINR and average BER of MRC-RAKE reception using the exact SINR equation (16) and the approximation (27) ($M = 1, K = 30, \Delta\theta = 0.01\pi$)
after RAKE combining maintains a value close to the theoretical SINR equation for a low Doppler frequency. The average SINR decreases in proportion to the number of users. Figure 12 shows unevenness in the output SINR curve for $M = 2, 3$. This result is produced by the uneven distribution of the directions of the users.

The derived theoretical SINR equation can be applied in the situation in which the long code and Walsh code are used, and the base station transmits to multiple users over multiple antennas. Thus, this can also be used as the criterion when evaluating the received signal of the terminal in the W-CDMA downlink. The criterion can be used in a system-level simulation of the entire system including downlink beamforming, and can be effective in understanding the system capacity.

6. Conclusion

In this paper, we evaluated the output SINR of RAKE reception at a terminal in the W-CDMA downlink by analysis and simulation. In the theoretical analysis, we considered the long code and Walsh code used in the downlink, signal transmission to multiple users, and transmission beamforming at the base station and derived the theoretical SINR after RAKE combining at the terminal. In the W-CDMA downlink, correlation of the interference signals was exhibited between the RAKE fingers of the terminal.

The simulation verified the agreement between the values of the theoretical SINR equation and the simulation. We evaluated the received signal characteristics in a fading environment with a practical RAKE reception configuration and discussed the differences from the theoretical values. The derived theoretical SINR equation can be used as the criterion for evaluating the received signal at the W-CDMA terminal. The derived theoretical SINR equation can be used as the criterion for evaluating the W-CDMA system capacity.

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APPENDIX

1. Correlation Characteristics in the RAKE Finger Output of Interference with Other Cells

Let base station \( n \) of the other cells, for \( n = 1, 2, \ldots \), transmit signals with weight \( w_i^{(\nu)} \), for \( i = 1, \ldots, K^{(\nu)} \), and power \( P_i^{(\nu)} \) to the \( K^{(\nu)} \) users in the cell. The propagation path between base station \( \nu \) and the terminal under consideration is represented by the propagation vector \( a_{k,l}^{(\nu)} \), for \( l = 1, \ldots, L^{(\nu)} \). The description of the propagation path is identical to Eq. (3).

The signals of the base station \( \nu \) of the other cells are extended by a long code and a Walsh code, but the long code differs for each cell. Thus, the long code for despreading of the terminal under consideration and the long codes of the other cells do not match. In this case, the paths from other cells can be handled in the same manner as delay path \( l(l \neq r) \) from the cell itself. Thus, the correlation matrix \( R_N^{(\nu)} \) after despreading of the interference from base station \( \nu \) is given by

\[
[R_N]_{r_1,r_2}^{(\nu)} = \sum_{i=1}^{K^{(\nu)}} \sum_{(l_1,l_2) \in U_i^{(\nu)}} \frac{P_i^{(\nu)}}{G} \times \left( w_i^{(\nu)\dagger} a_{k_1,l_1}^{(\nu)} \right) \left( w_i^{(\nu)\dagger} a_{k_2,l_2}^{(\nu)} \right)^* \quad U_i^{(\nu)} = \{l_1, l_2 = l_1 - (r_1 - r_2) | 1 \leq l_1, l_2 \leq L^{(\nu)} \}
\]

The following equation gives the correlation matrix \( R_N \) after despreading for the sum of the interference from the base stations of many other cells and the noise component of the terminal:

\[
R_N = \sum_{\nu} R_N^{(\nu)} + \frac{P^{(0)}_N}{G} I \tag{A.1}
\]

where \( P^{(0)}_N \) denotes the noise power before despreading at the terminal. The diagonal components of matrix \( R_N^{(\nu)} \) are always positive. The nondiagonal components become complex numbers in response to the propagation path.

However, when there are many interfering base stations, since the nondiagonal components of matrix \( R_N^{(\nu)} \) act to make the cumulative elements cancel each other, \( R_N \) gradually approaches a scalar multiple of the unit matrix.

The sum of the interference and noise power from the other cells in the output of RAKE finger \( r \) of terminal \( k \) is the \((r, r)\) component of matrix \( R_N \) and is given by

\[
[R_N]_{r,r} = \frac{P_N}{G} \tag{A.2}
\]

where \( P_N \) is the interference and noise power from the other cells before despreading and is not a function of \( r \). Thus, \( [R_N]_{r,r} \) does not depend on \( r \) and is constant.

2. Maximum Ratio Combining and MMSE Combining

In an environment with thermal noise, maximum ratio combining is defined as combining that sets the weighting coefficients to the values of coefficients proportional to the signal level of each branch divided by its average noise power [7]. In this paper, we construct maximum ratio combining that handles the interference component in the same way as the noise component. In RAKE finger \( r \), the level of signal \( k \) is proportional to \( w_k^\dagger a_{k,r} \), and the interference noise power becomes \([R_I]+[R_N]_{r,r}\). Thus, the weighting coefficient of RAKE finger \( r \) becomes \( w_k^\dagger a_{k,r}/[R_I+R_N]_{r,r} \), and the weight \( g_{MRCR} \) of Eq. (17) is derived.

If the correlations of the interference noise components between the RAKE fingers are all 0, then

\[
R_I + R_N = \text{diag}[R_I + R_N] \tag{A.3}
\]

We use the relationship between Eqs. (16), (17), and \( R_S = P_NR^\dagger \) to derive

\[
\Gamma_k = \frac{P_k R^{[\text{diag}[R_I + R_N]^{-1}v]}}{v^\dagger [\text{diag}[R_I + R_N]^{-1}v]} \tag{A.4}
\]

\[
P_k = \frac{P_k^{[\text{diag}[R_I + R_N]^{-1}v]}}{v^\dagger [\text{diag}[R_I + R_N]^{-1}v]} \tag{A.5}
\]

\[
\sum_{r=1}^{R} P_k \left[ w_k^\dagger a_{k,r} \right]^2 \tag{A.6}
\]

\[
\sum_{r=1}^{R} \Gamma_{k,r} \tag{A.7}
\]
If the interference noise components of the RAKE fingers are uncorrelated, the output SINR of the maximum ratio combining RAKE reception is represented by the sum of the SINRs of the RAKE fingers. However, if Eq. (A.3) does not hold, Eq. (A.7) usually does not hold.

MMSE combining is known as one technique to find the Wiener solution. This weight is given by Eq. (18) by using the inverse matrix of the correlation matrix of the interference noise components \( (R_I + R_N)^{-1} \) and vector \( v \) that is proportional to the signal level [16].

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